The Evolution of Preferences for Conflict

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Abstract

We consider preference evolution in a class of conflict models with finite populations. We show that whereas aggregate conflict effort is always the same in evolutionary equilibrium, larger populations have greater individual subjective costs of conflict effort.

1 Introduction

Perhaps more than other characteristics, the propensity for aggressive behavior in humans and animals must reasonably have been subject to evolutionary pressure. Peaceful exchange in a market is a very recent invention in human history—and arguably still not the most common form of economic transaction—and the archetypal interaction among strangers in the evolutionary setting is likely to have involved violence or the threat of violence. (See, e.g., Otterbein [8] on the archaeological and anthropological evidence.) In this paper we study how the subjective utility cost of expending effort on appropriative activities evolves, with special attention to the effect of group size.

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In line with an existing literature on preference evolution, we shall assume that individuals behave rationally and strategically given their preferences. The distribution of preference types in the population then evolves according to the objective evolutionary fitness outcomes that the behavior of individuals gives rise to. Unlike most of the received literature, however, we shall use as our concept of evolutionary stability the finite-population criterion developed by Schaffer [9]. The assumption of a finite population seems especially appropriate here.

Ely and Yilankaya [3] show that in infinite populations and the Maynard Smith model of an evolutionary game, preference evolution leads to an equilibrium of the fitness game. In a finite-population game this is *not* necessarily the case, as we shall see in this paper.

Applications of preference evolution include Güth and Yaari [4], Bester and Güth [1], and Wärneryd [12]. In this paper, we study a class of conflict models known as *contests* in economics. The abstract literature on contests includes Dixit [2] and Wärneryd [13]. Earlier contributions to the literature on evolution in contests include Hehenkamp et al [6], who study evolution at the level of conflict expenditures directly, and Leininger [7], who studies the evolution of interdependent preferences in contests.

2 Contests

Consider a contest for a prize of value 1, with $n > 2$ risk neutral contestants and success function

$$p_i(x_1, x_2, \ldots, x_n) := \begin{cases} x_i/X & \text{if } X > 0 \\ 1/n & \text{otherwise,} \end{cases}$$

where x_i is the effort of contestant i and $X := \sum_j x_j$.

This specific functional form for the conflict technology was introduced, in an *ad hoc* manner, early on in the economic theory of conflict (see Haavelmo [5]), and has proved particularly popular in the study of rent-seeking contests (Tullock [11]). More recently it was axiomatized by Skaperdas [10], who shows that it essentially follows from assumptions of anonymity, decomposability, and homogeneity of degree zero.

Suppose contestant i's subjective cost of exerting effort x_i is $c_i x_i$, where $c_i \in [\underline{c}, \infty)$, with $\underline{c} > 0$. His payoff function is then

$$u_i(x_1, x_2, \ldots, x_n) := p_i - c_i x_i,$$

which is strictly concave in x_i.

We first note that there can be no equilibrium in which no player exerts positive effort. For suppose everyone except player i exerted zero effort. Then player i would win with probability one if he exerted positive effort. Since this effort could be arbitrarily small, there must be some positive effort level such that this would be profitable.

Consider the first partial derivative of contestant i's expected utility with respect to x_i, given the efforts of everyone else. It is

$$\frac{\partial u_i(x_1, x_2, \ldots, x_n)}{\partial x_i} = \frac{X - x_i}{X^2} - c_i.$$

This derivative is non-positive at $x_i = 0$ if we have $c_i \geq 1/X$. Hence contestant i rationally spends 0, or is *inactive*, if we have $c_i \geq 1/X$. If contestant i is active, we must have that

$$x_i = X - X^2 c_i. \tag{1}$$

Let A be the set of n_A contestants who are active in equilibrium. Writing $C := \sum_A c_j$, we have that

$$X = \sum_{i \in A} x_i = n_A X - X^2 C,$$

i.e., that

$$X = \frac{n_A - 1}{C}.$$

When we later go on to consider the evolutionary stability properties of a particular subjective cost of conflict effort, we shall be concerned with situations in which a homogeneous population is invaded by exactly one mutant with different cost. Consider, therefore, a contest in which one contestant with cost c

faces $n-1$ contestants with common cost $c^\star$. Suppose the c-player is inactive in equilibrium. Then we have $C=(n-1)c^\star$ and $X=(n_A-1)/C=(n-2)/((n-1)c^\star)$. Since (1) implies that active players with the same cost exert the same effort in equilibrium, this is the unique equilibrium if and only if we have that

$$c \geq \frac{n-1}{n-2}c^\star.$$

Consider next the possibility of an equilibrium such that the $c^\star$-players are inactive. In such an equilibrium, it would have to be the case that the c-player exerts positive effort. But for every such positive effort level, there is a strictly lower positive effort level that still lets him win with probability one, but at lower cost. Hence the c-player could not be playing a best reply, and hence there is no such equilibrium.

Finally, consider the possibility of an equilibrium such that some subset of the $c^\star$-players are active, the rest inactive. Let $n_A^\star \geq 1$ be the number of active $c^\star$-players. Total effort in such an equilibrium would be

$$X=\frac{n_A^\star}{c+n_A^\star c}.$$

Consider the first order condition of one of the inactive players. In order for it to be a best reply for him not to exert effort, it would have to be the case that

$$\frac{1}{X}-c^\star=\frac{c}{n_A^\star}+c^\star-c^\star \leq 0,$$

an impossibility.

We summarize these observations as follows.

Lemma 1 *There are only two types of equilibrium when there is one player with cost c and $n-1$ players with cost $c^\star$. If we have $c \geq (n-1)c^\star/(n-2)$, the c-player is inactive. Otherwise all players are active.*

3 Evolution

Now suppose the actual *fitness* cost per unit of effort is the same for everybody and equal to 1. Again considering a situation with one player with subjective cost c and $n-1$ players with subjective cost $c^\star$, the fitness of a c-contestant is then in equilibrium

$$f(c) := \begin{cases} (1-X)(1-cX) & \text{if } c < (n-1)c^\star/(n-2) \\ 0 & \text{otherwise.} \end{cases}$$

Define $X_{\sim c} := (n-2)/((n-1)c^\star)$, total equilibrium effort when the c-player is inactive. The equilibrium fitness of a $c^\star$-player is then

$$f(c^\star) := \begin{cases} (1-X)(1-c^\star X) & \text{if } c < (n-1)c^\star/(n-2) \\ (1-X_{\sim c})(1-c^\star X_{\sim c}) & \text{otherwise.} \end{cases}$$

Define $\Delta f(c, c^\star) := f(c) - f(c^\star)$. An evolutionarily stable effort cost c^{ESS} in the sense of Schaffer [9] in this setting is a solution to the fixpoint inclusion

$$c^\star \in \arg\max_c \Delta f(c, c^\star).$$

Hence we are first interested in the value of c that maximizes $\Delta f(c, c^\star)$ for a given $c^\star$. We have that

$$\Delta f(c, c^\star) = \begin{cases} (1-X)X(c^\star - c) & \text{if } c < (n-1)c^\star/(n-2) \\ -(1-X_{\sim c})(1-c^\star X_{\sim c}) & \text{otherwise.} \end{cases}$$

Note that $\Delta f(c, c^\star)$ is continuous at $c = (n-1)c^\star/(n-2)$. Consider the first partial derivative of the lower part of $\Delta f(c, c^\star)$ with respect to c. It is equal to zero at

$$c^{\max}(c^\star) := \frac{c^\star(n-1)(1-n(c^\star-1))}{n-1+nc^\star}.$$

Furthermore, we have that

$$\left.\frac{\partial^2 \Delta f(c, c^\star)}{\partial c^2}\right|_{c=c^{\max}(c^\star)} = -\frac{(n-1+nc^\star)^4}{8(c^\star)^3(n-1)^2 n^3} < 0.$$

We also have that

$$c^{\max}(c^\star) \lesseqqgtr c^\star \text{ as } c^\star \gtreqqless \frac{n-1}{n}.$$

Define

$$\underline{c}^\star := \frac{n-1}{n} - \frac{2}{n(n-1)}.$$

When we have $c^\star \leq \underline{c}^\star$, the value of c that maximizes $\Delta f(c, c^\star)$ is such that the c-player is inactive in equilibrium.

Define

$$\overline{c}^\star := \frac{n^2 - \underline{c}\, n - 1 + \sqrt{\left(1 + \underline{c}\, n - n^2\right)^2 - 4\, \underline{c}\, n (n-1)^2}}{2\,(n-1)\, n}$$

When we have $c^\star \geq \overline{c}^\star$, the value of c that maximizes $\Delta f(c, c^\star)$ is $c = \underline{c}$. That is, $\overline{c}^\star$ is a threshold such that if everyone else's subjective cost exceeds it, then from fitness point of view the optimal cost to have is the lowest possible. Note that if, as we shall assume, $\underline{c}$ is close to zero, then $\overline{c}^\star$ is close to $(n+1)/n$.

Hence we have that

$$\arg\max_c \Delta f(c, c^\star) = \begin{cases} [(n-1)c^\star/(n-2), \infty) & \text{if } c^\star \leq \underline{c}^\star \\ \{c^{\max}(c^\star)\} & \text{if } \underline{c}^\star < c^\star < \overline{c}^\star \\ \{\underline{c}\} & \text{otherwise.} \end{cases} \tag{2}$$

Note that $\underline{c}^\star < (n-1)/n$. Clearly, a value of $c^\star$ less than or equal to $\underline{c}^\star$ can never be a fixpoint of (2). Likewise, $c^\star \geq \overline{c}^\star$ cannot be a fixpoint either. Hence $c^\star = (n-1)/n$ is the unique fixpoint, and the unique evolutionarily stable value of the cost parameter.

We summarize this result as follows.

Proposition 1 *The unique evolutionarily stable value of the subjective cost parameter is $c = (n-1)/n$.*

It follows that in an evolutionarily stable population, total effort spent on conflict is always 1. That is, the value of the prize is always completely dissipated in fitness terms. Nevertheless, individual conflict effort is decreasing in population size, and the subjective effort cost approaches the objective fitness cost as population size approaches infinity.

To get some intuition for this result, consider a population in which $n-1$ individuals have subjective cost equal to 1, but a single individual has cost $c = (n-1)/n$. The equilibrium fitness of an individual whose subjective cost is equal to the fitness cost is then $(1-X)^2$, whereas the fitness of the c-individual is $(1-X)(1-cX)$. Since we have $c<1$, the c-individual has greater fitness than the others.

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