Conflict financing

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Abstract

Consider a two players contest where efforts are financed through debt and the loser partially defaults on such debt. A risk neutral representative investor allocates his funds into three assets: the debt of the two contestants and a riskless asset. If contestants are price makers and risk neutral their expected utilities are equivalent to the ones of an all pay contest. Both for risk neutral and risk averse contestants the equilibrium efforts are always higher when they are price makers than when they are price takers. As a consequence in contrast with the classical duopsony model the contestants are worse off by being price makers. In a winner pay contest with price making risk averse contestants the efforts in a symmetric equilibrium are higher than when they are risk neutral, coherently with the results in Skaperdas and Gan (1995).

Recently there has been increasing interest in winner pay contests (see Skaperdas and Gan (1995), Wärneryd (2000) and Yates (2011)). In this paper we analyze contests where, as in a winner pay contest, the loser is (partially or completely) exempted from paying the costs of his effort. The novelty of this setting is that contestants finance their effort by issuing debt. This extension to winner pay contests seems natural once interpreting the exemption of the loser from paying as limited liability. The financial market is modeled as follows. A risk neutral representative investor allocates his funds into three assets: the debt of each contestant and a riskless asset.

Let us briefly summarize the results. In a contest with limited liability of the loser and with efforts financed issuing debt if the contestants are risk neutral and price makers in the financial market their expected utilities are equivalent to the ones of an all pay contest. This result is simple but powerful, since it allows to analyze the complex situation of a contest financed issuing debt as a straightforward all pay contest.

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Let us now consider the results with risk averse contestants. The analysis with risk averse contestants concentrates exclusively on the cases in which the loser is fully exempted from paying. Winner pay contests with risk averse contestants have been analyzed in Skaperdas and Gan (1995), showing that the higher the risk aversion of the contestants the higher their efforts in a symmetric equilibrium. Such result is valid in our setting for price taker contestants. In this work we show that also with price making contestants the same holds.

Both for risk neutral and risk averse contestants it is always the case that efforts in a symmetric equilibrium are higher when they are price makers than when they are price takers. The intuition is that the contestants anticipate their interest rate to decrease with their win probability. This brings them to a double competition in increasing efforts: higher effort increases the likelihood of winning and also indirectly decreases the interest rate, which makes it easier to further increase the effort. In our setting then price making behavior exacerbates competition among the contestants, making them worse off. This result is in contrast with the classical duopsony model, where price making behavior softens competition and makes the firms better off. The reason of this opposite result is that in our setting the supply is downward sloping, while in a classical setting the supply is upward sloping.

For completeness let us briefly summarize the classical duopsony model. Consider two big firms $i \in \{1, 2\}$ which are in a duopsony: they duopolize the demand of a certain product. The profit of firm i is

$$u_i(X) = x_i - p(X)x_i$$

where x_i is the demand of firm i. The supply of such product is such that its price increases with aggregate demand: $p(X) = x_1+x_2$. Profit maximization and symmetry of equilibrium demands brings to $x_i = 1/3$. Equilibrium demands under perfect competition are identified instead by marginal revenue equal to marginal cost, taking price as given. Under perfect competition the equilibrium demand of firm i is $x_i = 1$, which is higher than the one under price making behavior. The profit under price making behavior is $u_i = 1/9$ while under price taking behavior is 0. We can conclude that in a classical duopsony model price making behavior softens competition, making the equilibrium demands and the price lower and hence making the duopsonists better off. We will see that in our framework the opposite happens: price making behavior exacerbates competition and makes the contestants worse off.

The present framework may be useful to analyze R&D contests among oligopolists, whose R&D investments are financed issuing debt in the financial market. Examples of such situations are the R&D battles among high tech firms as Airbus and Boeing or Apple and Google, which are fought not only in their laboratories but also in the financial market in convincing the investors of the superiority of one firm over the other. A similar situation is the one of the speculative financial sector. Two financial institutions compete in gathering superior information for speculating in the financial market. The gains of one are the losses of the other. For such financial institutions gathering information is costly, and the resources for supporting such activity are collected by issuing debt in the financial market.

Analogous situations of conflict both in the physical world and in the financial market could be found in war financing in Europe before the 20th century. Early international bankers where willing to finance both sides of conflicts among countries, and since the controls on capital markets where still limited they were allowed to do it. Several examples of such situation are found during the 30 Years war or the Napoleonic wars. With the 20th century the states increased their controls on capital flows and made sure that their citizens would not lend funds to their enemies.

The paper develops as follows. The first section analyzes the model with risk neutral contestants, and the second section with risk averse ones. The third section discusses the assumptions and suggests lines of further research. The fourth section concludes.

1 Risk neutral contestants

Two risk neutral contestants (players $i \in \{1,2\}$) compete for winning a unique prize normalized to 1. The contestants choose their effort, and higher effort increases their probability of winning the contest. They finance their efforts by issuing debt which is bought by a representative investor (player $i = 3$). The contestants are price makers in the financial market, in the sense that they anticipate the effect of their choices on the interest rates. The representative investor is price taker. The winning contestant never defaults on his debt, but the losing contestant defaults on a share of his debt. Because of partial default of the loser, the winning probabilities play a role in the investor' portfolio choices. If the investor finances both contestants then the expected returns of their debts have to be equal: if a contestant is more likely to win than the opponent then his debt should have a lower interest rate than the one of the opponent. Each contestant will then take in to account that by increasing his effort he will be more likely to win and will also pay a lower interest rate, making his effort cheaper.

Let us see in detail the contestants problem. The utility of contestant i is

$$u_i(x_i, x_{-i}) = \begin{cases} 1 - R_i(x_i, x_{-i})x_i & if w = i \\ -\alpha R_i(x_i, x_{-i})x_i & otherwise \end{cases}$$

where $x_i > 0$ is his effort and $x_{-i} > 0$ is the effort of his opponent. The contestant collects money by issuing debt and then transforms it $1:1$ into effort. $\alpha \in [0, 1)$ is the share of debt which is always paid back to the investor, while the rest is defaulted if the contestant loses the contest. $R_i(x_i, x_{-i})$ is the gross interest rate on contestant i debt, which is a function of efforts since the contestant is price maker. The function $R_i(x_i, x_{-i})$ is determined by solving the investor problem. The expected utility of contestant i is

$$E\left[u_i(x_i, x_{-i})\right] = P_i(x_i, x_{-i})\left[1 - (1-\alpha)R_i(x_i, x_{-i})x_i\right] - \alpha R_i(x_i, x_{-i})x_i$$

where $P_i(x_i, x_{-i}) \in [0, 1]$ is his win probability. The win probability is assumed to be concave in x_i and to fulfill the property of anonymity.

Let us see in detail the investor problem. The investor is risk neutral and owns $W > 0$ units of money. Let us assume W large enough such that the demands of funds of the contestants are always fulfilled. The investor can invest his funds in three assets: the debt of each of the contestants and a risk free asset with gross return normalized to 1. The investor is price taker, hence he does not consider the effects of his investment choices on interest rates. The expected utility of the investor is

$$E\left[u_3(y)\right] = \sum_{i=1}^{2}\left[\alpha + (1-\alpha)P_i(x_i, x_{-i})\right]R_i(x_i, x_{-i})y_i + (W - y_1 - y_2)$$

where $y_i \geq 0$ is his demand of contestant i debt and $1 - y_1 - y_2 \geq 0$ is his investment in the risk free asset. $P_i(x_i, x_{-i})$ is the win probability of contestant i: it is relevant for the investor since it determines contestant i default probability on debt. $R_i(x_i, x_{-i})$ is the gross interest rate on contestant i debt.

Being risk neutral the investor allocates all his resources in the asset with the highest expected return. If he invests in all three assets then their expected returns have to be equal to each other, hence for contestant i

$$\left[\alpha + (1-\alpha)P_i(x_i, x_{-i})\right]R_i(x_i, x_{-i}) = 1 \tag{1}$$

From this condition we can derive how contestant i effort affects his interest rate, which is

$$R_i(x_i, x_{-i}) = [\alpha + (1-\alpha)P_i(x_i, x_{-i})]^{-1}$$

Market clearing requires that for each contestant i the supplied funds y_i equal the demanded funds x_i. Market clearing is always possible if W is sufficiently large and the interest rates fulfill the previous condition.

Let us go back to the contestants problem, focusing on contestant i. We now know that he will anticipate his interest rate to be affected by his effort according to the last equation. If contestant i takes into account the effects of his choice of effort on the interest rate his expected utility is then

$$E\left[u_i(x_i, x_{-i})\right] = P_i(x_i, x_{-i}) - x_i$$

which is equivalent to the expected utility of a player in an all pay contest. The following proposition summarizes the results we have just seen.

Proposition 1 *Assume risk neutral and price maker contestants. If the investor allocates strictly positive funds in all three assets then the gross interest rate on the debt of contestant i is*

$$R_i(x_i, x_{-i}) = [\alpha + (1-\alpha)P_i(x_i, x_{-i})]^{-1} \tag{2}$$

and the expected utility of contestant i is

$$E\left[u_i(x_i, x_{-i})\right] = P_i(x_i, x_{-i}) - x_i \tag{3}$$

which is equivalent to the expected utility of a player in an all pay contest.

Equation 2 shows that the higher the win probability of a contestant the lower his interest rate. Moreover the magnitude of this effect is stronger the higher the degree of limited liability when loosing the contest. Equation 2 is the supply function of loanable funds for contestant i, and is downward sloping. This is anomalous, since usually the supply function is modeled as upward sloping. This fact will drive the differences of our setting from the classic duopsony model.

The following proposition compares the efforts in a symmetric equilibrium when contestants are price makers with the ones when they are price takers. In this proposition as in most other ones the issues of existence and uniqueness of the analyzed equilibria are not contemplated. Such issues are extensively discussed in the third section. Although the uniqueness of equilibria is generally far from obvious, it is the case that the equilibria studied in all the propositions exist and are unique for the popular Tullock contest success function.

Proposition 2 *Suppose the contestants are price makers, risk neutral and their win probabilities fulfill anonymity and are concave in their own effort. For all* $\alpha \in [0,1]$ *their efforts in a symmetric equilibrium are always higher when they are price makers than when they are price takers.*

Proof: By proposition 1 equilibrium efforts when contestants are price makers are the same as in an all pay contest with cost function x_i. The symmetric equilibrium efforts $x_1 = x_2 = \hat{x}$ are then identified by

$$\frac{\partial}{\partial x_i}\left[P_i(x,x)\right] = 1 \tag{4}$$

If the contestants play a symmetric equilibrium then their equilibrium win probabilities are $P_i = 1/2$. We have already seen in the proof of proposition 1 that if the investor allocates funds in all three assets then their expected returns have to be equal, hence $\left[\alpha + (1-\alpha)P_i\right] R_i = 1$. It follows that the equilibrium interest rates are necessarily $R_1 = R_2 = 2/(1+\alpha)$. Since the contestants are price takers they do not take into account how their effort choice affects the interest rates. As a consequence they maximize their expected utilities taking the interest rate as given. Their expected utility when $R_i = 2/(1+\alpha)$ is

$$E\left[u_i(x_i, x_{-i})\right] = P_i(x_i, x_{-i})\left[1 - \frac{2}{(1+\alpha)}x_i\right]$$

which is concave when win probabilities are of the ratio form. Imposing symmetry of equilibrium efforts and after few manipulations the first order condition becomes

$$\frac{\partial}{\partial x_i}\left[P_i(x,x)\right] = 1 + \frac{2(1-\alpha)}{(1+\alpha)} x \frac{\partial}{\partial x_i}\left[P_i(x,x)\right] \tag{5}$$

Let us compare condition 4 with condition 5. Since the win probabilities are concave in their own effort the LHSs are decreasing in x. Since the LHSs are decreasing and for all $x > 0$ the RHS of condition 4 is always smaller than the one of condition 5 then the equilibrium efforts are necessarily such that $\hat{x} > x$.

□

We have seen that in our setting price making behavior exacerbates competition, making equilibrium efforts higher than under price taking behavior. It is easy to verify that under price making behavior the contestants are worse off than under price taking behavior. Since in both cases their equilibrium efforts are symmetric then by anonymity their win probabilities are $P_i = 1/2$.

We have already seen that if the investor allocates funds in all three assets then the equilibrium interest rates are such that $R_i = [\alpha + (1-\alpha)P_i]^{-1}$, hence $R_i = 2/(1+\alpha)$ both under price making and price taking behavior. Given this the equilibrium expected utility of contestant i is

$$E\left[u_i(x,x)\right] = \frac{1}{2}(1 - \frac{2}{(1+\alpha)}x)$$

where x is the equilibrium effort. Notice that the expected utility is decreasing in x. Since x is higher under price making behavior than under price taking behavior we can conclude that price making makes the contestants worse off. This result is in contrast with the one of the classical duopsony model, where price making makes the firms better off. The reason of this opposite result is that in our setting the supply function $R_i = [\alpha + (1-\alpha)P_i]^{-1}$ is downward sloping, while in the classical duopsony it is assumed to be upward sloping.

2 Risk averse contestants

Suppose the contestants are risk averse, with utility

$$u_i(x_i, x_{-i}) = \begin{cases} v\left(1 - R_i(x_i, x_{-i})x_i\right) & if w = i \\ v\left(-\alpha R_i(x_i, x_{-i})x_i\right) & otherwise \end{cases}$$

where for all $z > 0$ it holds $v'(z) \geq 0$ and $v''(z) \leq 0$. In order to simplify the problem assume the loser completely defaults of his debt, $\alpha = 0$, as in the winner pay contests in Skaperdas and Gan (1995). Recall from equation 2 that, given the optimal behavior of the investor, if $\alpha = 0$ the interest rate of contestant i is the inverse of his win probability, $R_i(x_i, x_{-i}) = P_i(x_i, x_{-i})^{-1}$. The expected utility of contestant i is then

$$E\left[u_i(x_i, x_{-i})\right] = P_i(x_i, x_{-i})v\left(1 - \frac{x_i}{P_i(x_i, x_{-i})}\right)$$

which is assumed to be concave. Manipulating the first order condition brings to

$$\frac{\partial}{\partial x_i}\left[P_i(x_i, x_{-i})\right] = \frac{v'\left(f(x_i, x_{-i})\right)}{v\left(f(x_i, x_{-i})\right) + v'\left(f(x_i, x_{-i})\right)\left(1 - f(x_i, x_{-i})\right)} \qquad (6)$$

where $f(x_i, x_{-i}) = 1 - x_i/P_i(x_i, x_{-i})$. If the contestants are risk neutral the first order condition is simply $P'(x,y) = 1$ as in an all pay contest. Assume there exists a symmetric equilibrium $x_1 = x_2 = x$ and that contestants play it. Imposing symmetry of the equilibrium efforts the first order condition becomes

$$\frac{\partial}{\partial x_i}\left[P_i(x,x)\right] = \frac{v'\left(1-2x\right)}{v\left(1-2x\right)+v'\left(1-2x\right)2x} \tag{7}$$

Since the win probabilities are concave in own effort the LHS of equation 7 is decreasing in x. The RHS of equation 7 is larger than 1 if and only if

$$v'\left(1-2x\right)\left(1-2x\right) > v\left(1-2x\right) \tag{8}$$

Recall that when the contestants are risk neutral the RHS of equation 7 is equal to 1. Since the LHS of equation 7 is decreasing in x then a RHS higher (lower) than 1 implies an equilibrium effort x lower (higher) than when the contestants are risk neutral. It follows that equation 8 is the necessary and sufficient condition for efforts in a symmetric equilibrium being lower with risk averse contestants than with risk neutral ones. Moreover if the inequality is reversed then the opposite holds. Condition 8 allows us to identify the following simple but powerful sufficient conditions.

Proposition 3 *Suppose contestants are price makers and risk averse, $\alpha = 0$ and their win probabilities fulfill anonymity and are concave in own effort. If $v(z)$ is positive, increasing and concave then risk aversion induces higher efforts than risk neutrality in a symmetric equilibrium.*

Proof: Since $1 - 2x$ in equation 8 is a point in the domain of the utility function, if $v'(z)z > v(z)$ holds for all values in the domain then it holds also for such point. The same reasoning applies to $v'(z)z < v(z)$. It follows that if for all $z > 0$ the utility of the risk averse contestants is such that $v'(z)z > v(z)$ then their efforts in a symmetric equilibrium are strictly lower than if they would be risk neutral. If for all $z > 0$ the utility of the risk averse contestants is such that $v'(z)z < v(z)$ then their efforts in a symmetric equilibrium are strictly higher than if they would be risk neutral. It is easy to verify that if for all z $v(z) \geq 0$, $v(z) \geq 0$ and $v''(z) \leq 0$ then it is always the case that $v'(z)z < v(z)$. It follows that risk aversion always increases the efforts in a symmetric equilibrium.

□

The following proposition compares the efforts in a symmetric equilibrium when contestants are price makers with the ones when they are price takers. As before we will keep assuming $\alpha = 0$ and win probabilities concave in own effort.

Proposition 4 *Suppose the contestants are risk averse and their win probabilities fulfill anonymity and are concave in own effort. For* $\alpha = 0$ *their efforts in a symmetric equilibrium are always higher when they are price makers than when they are price takers.*

Proof: The efforts of risk averse and price making contestants in a symmetric equilibrium $x_1 = x_2 = \hat{x}$ are identified by condition 7, which is

$$\frac{\partial}{\partial x_i}\left[P_i(x,x)\right] = \frac{v'\left(1-2x\right)}{v\left(1-2x\right)+v'\left(1-2x\right)2x}$$

Let us analyze the case of price taking contestants. If the contestants play a symmetric equilibrium then their equilibrium win probabilities are $P_i = 1/2$. We have already seen in the proof of proposition 1 that if the investor allocates funds in all three assets then their expected returns have to be equal, hence $P_i R_i = 1$. It follows that the equilibrium interest rates are necessarily $R_1 = R_2 = 2$. Since the contestants are price takers they do not take into account how their effort choice affects the interest rates. As a consequence they maximize their expected utilities taking the interest rate as given. Their expected utility when $R_i = 2$ is

$$E\left[u_i(x_i, x_{-i})\right] = P_i(x_i, x_{-i})v\left(1-2x_i\right)$$

which is assumed to be concave. Imposing symmetry of equilibrium efforts and the ratio form of the win probabilities after few manipulations the first order condition becomes

$$\frac{\partial}{\partial x_i}\left[P_i(x,x)\right] = \frac{v'\left(1-2x\right)}{v\left(1-2x\right)} \tag{9}$$

Condition 9 identifies the efforts of the risk averse price taking contestants $x_1 = x_2 = x$ in a symmetric equilibrium. The LHSs of conditions 7 and 9 are equal and decreasing in x since the win probabilities are concave. Since the RHS of condition 7 is always smaller than the RHS of condition 9 it follows that $x < \hat{x}$.

□

As we have seen with risk neutral contestants it is easy to verify that price making behavior makes the contestants worse off than price taking behavior, in contrast with the results of the classical duopsony model.

3 Discussion

Let us briefly discuss the main assumptions of this paper: risk neutrality and price taking behavior of the representative investor, price making behavior of the contestants. Moreover let us discuss the problem of existence, uniqueness and symmetry of the equilibrium.

3.1 Risk neutrality of the investor

Assuming the investor to be risk neutral is a standard assumption in finance. The results rely heavily on it and hence would be interesting to relax the assumption, but introducing risk aversion makes the problem hard to solve analytically. CRRA utility allows in some cases to get analytical expressions for the interest rates. Let us consider for example the case with $W = 1$, $\alpha = 0$ and the utility of the investor to be CRRA with parameter 1/2. The expected utility of the investor is then

$$E\left[u_3(y)\right] = \sum_{i=1}^{2} P_i(x)\left[R_i(x)y_i\right]^{1/2} + \left[1 - y_1 - y_2\right]^{1/2}$$

where y_3 is to be interpreted as consumption today and $y_1 + y_2$ as aggregate savings. Price maker contestants will anticipate the optimal behavior of the investor and hence the equilibrium interest rates to be

$$R_i(x) = \frac{x_i}{(1 - x_1 - x_2)P_i(x)^2}$$

We can identify two effects of efforts on the interest rate: an increase in own effort increases the interest rate because it increases aggregate demand, but on the other hand it decreases the interest rate because it lowers the probability of default. We showed in proposition 1 that with risk neutrality of the investor $R_i = P_i^{-1}$, hence the first effect is absent and the second effect is weaker. It is difficult to say anything more without further assumptions on the functional forms. It is important to notice that a strong negative effect of own effort on the interest rate is likely to bring to multiple equilibria, since high effort makes effort cheaper and hence creates a self-fulfilling mechanism.

3.2 Price taking and price making behavior

The contestants are assumed to be price makers. The reason is that they are big players and by announcing to the market the amount of issued debt they are conscious of influencing the interest rates. The case with price taking contestants is also contemplated and compared with the price making one. Price taking behavior of the contestants is reasonable when they lack the possibility of announcing their strategies to the financial market.

The investor is assumed to be price taker. The reason is that it is a representative investor, whose behavior to be interpreted as the aggregate behavior of many investors, each investing a very small sum. This is a standard assumption in finance and macroeconomics, but it seems interesting to analyze also the case in which there is a monopoly or an oligopoly in the supply of loanable funds.

For analyzing the case of price making investor, let us consider a monopoly in the supply of loanable funds. Suppose for simplicity that the investor and the contestants are risk neutral. The investor is price maker and the contestants price takers: allocating his funds the investor will anticipate optimal behavior of the contestants and hence the consequences on interest rates and win probabilities. We can imagine the investor being the first mover, announcing his strategy to the contestants. By backward induction the investor will anticipate the equilibrium interest rate of contestant i to be

$$R_i(x) = \frac{\partial/\partial x_i \left[P_i(x_i, x_{-i})\right]}{P_i(x_i, x_{-i}) + x_i \partial/\partial x_i \left[P_i(x_i, x_{-i})\right]} \tag{10}$$

which is always decreasing in x_i if the win probability is concave. Since market clearing requires $x_i = y_i$ the expected utility of the investor is

$$E\left[u_3(y)\right] = \sum_{i=1}^{2} P_i(y_i, y_{-i}) R_i(y_i, y_{-i}) y_i + (W - y_1 - y_2)$$

where $R_i(y_i, y_{-i})$ is as in equation 10 with $y_i = x_i$. Given the complexity of this function it is very difficult to say something about optimal behavior of the investor without making further assumptions on the functional forms. Let us make some further qualitative comments.

We know from equation 10 that when the market clears the interest rate R_i decreases with y_i. Since this effect is clear let us abstract temporarily from it, setting the interest rates constant: $R_1 = R_2 = 1$. In this way we will concentrate on the effects of the allocation of funds on the win probabilities and the quantities.

Suppose for simplicity that all funds are invested in the contest, $y_1 + y_2 = W$, and that the win probabilities are of the Tullock type, $P_i(x_i, x_{-i}) = x_i/(x_1 + x_2)$. The expected utility of the investor (anticipating the best replies of the contestants) is then $E[u_3(y)] = \left[(x_i)^2 + (W - x_i)^2\right]/W$ which is maximized at the two corners of the distribution of funds

$$\{x_1 = 0, x_2 = W\}$$

$$\{x_1 = W, x_2 = 0\}$$

Intuitively it is convenient for the investor to make one contestant to win with certainty in order to avoid loosing funds due to the limited liability of the loser.

We can conclude that when the investor is price maker there are two forces which push his allocation of funds in opposite directions. The interaction of the effects on probabilities and quantities may push the allocation towards corners, in order to minimize the losses due to the limited liability of the loser. On the other hand the effect on interest rates pushes the allocation towards a symmetric distribution of funds among the contestants, since an asymmetric distribution makes credit too cheap for the contestant which is the most likely to win. Saying something more precise about the optimal distribution of funds requires further assumptions on the functional forms.

3.3 Existence, uniqueness and symmetry of equilibrium

A serious problem which has not been tackled in this work is the existence and uniqueness of the equilibrium. Instead of facing this problem we have chosen to focus on the properties of the symmetric equilibrium, assuming it exists. Let us briefly discuss the different degrees of relevance of this problem in the various settings we explored. In some settings it seems safe to assume the equilibrium to be unique and symmetric, while other settings seem likely to generate multiplicity of equilibria and asymmetry.

With price making and risk neutral contestants by proposition 1 we know that the situation is equivalent to an all pay contest. Existence and uniqueness of pure Nash equilibrium in all pay contests has been studied in Szidarovszky and Okuguchi (2008). In this setting it is generally safe to assume existence, uniqueness and symmetry.

The situation with price making and risk averse contestants has not been studied before. Let us consider an example. With Tullock win probabilities and $\alpha = 0$ there exists a unique symmetric equilibrium. In this setting the first order condition of contestant i in equation 6 after few manipulations becomes

$$\frac{x_{-i}}{x_i} = \frac{v'(1 - x_1 - x_2)(x_1 + x_2)}{v(1 - x_1 - x_2)}$$

Notice that the RHS is the same for both contestants, hence the RHSs have to be equal for all i. It follows that there exists a unique equilibrium which is such that $x_1 = x_2$. This example shows that all equilibria studied in the propositions in first and second sections exist and are unique for the Tullock contest success function.

Unfortunately this result is far from general. With different functional forms this setting may generate multiple and asymmetric equilibria. The reason is that since the interest rate decreases with own effort there exists a self fulfilling mechanism such that higher effort brings to lower cost of effort, which then makes higher effort convenient.

Let us consider the case of price taking contestants. At first sight one may think that since contestants take the interest rate as given the results about existence and uniqueness of Nash equilibrium in winner pay contests in Yates (2011) apply here. This would be a mistake, since this setting is particularly likely to generate multiple and asymmetric equilibria. The reason is roughly the same self fulfilling mechanism described before, but here the effect is even stronger since the contestants do not realize that interest rates are affected by their efforts. Asymmetric interest rates then may sustain asymmetric equilibria. Let us see a simple example with binary effort. The effort space of each contestant is $x_i \in \{l, h\}$ where $\{l, h\} \in \mathbb{R}_+$ and $l < h$. The winning probabilities are of the Tullock type

$$P_i(x) = \frac{x_i}{x_1 + x_2}$$

The contestants are price takers, hence they maximize their expected utility taking the interest rate as given. Regarding symmetric equilibria, $x_1 = x_2 = h$ is an equilibrium if and only if

$$\frac{1}{2}(1 - 2h) \geq \frac{l}{l + h}\left[1 - 2l\right]$$

Instead $x_1 = x_2 = l$ is an equilibrium if and only if

$$\frac{1}{2}(1 - 2l) \geq \frac{h}{l + h}\left[1 - 2h\right]$$

Regarding asymmetric equilibria, $x_i = h$ and $x_{-i} = l$ is an equilibrium if and only if

$$\frac{h}{h+l}(1-h-l) \geq \frac{1}{2}\left[1-(h+l)\frac{l}{h}\right]$$

$$\frac{l}{h+l}(1-h-l) \geq \frac{1}{2}\left[1-(h+l)\frac{h}{l}\right]$$

both hold. Consider $h = 1/4$ and $l = 1/16$. With these parameter values there are three equilibria in pure strategies: the symmetric one in which both contestants exert high effort and the two asymmetric ones where one contestant exerts high and the other low effort. The reason for this high multiplicity of equilibria is that the interest rates adjust with the efforts of the contestants: if one exerts high (low) effort than the interest rate is low (high), making it convenient to actually exert such effort.

Let us compare these results with the situation in which the contestants are price makers. If the contestants are price makers they will anticipate the changes in the interest rate induced by their effort choices. It is easy to verify that the game is dominance solvable and it always has a unique symmetric equilibrium. The equilibrium is $x_1 = x_2 = l$ if

$$\frac{1}{2}(1-2l) \geq \frac{h}{h+l}(1-h-l)$$

and $x_1 = x_2 = h$ otherwise. Consider again the parameter values $h = 1/4$ and $l = 1/16$. In this case the unique equilibrium is $x_1 = x_2 = h$.

This simple example with binary efforts has shown how price taking behavior is likely to generate multiplicity and asymmetry of equilibria, while price making behavior is likely to induce uniqueness and symmetry. As a further development would be interesting to explore the continuous effort case, identifying under which conditions the multiplicity of equilibria arises.

4 Conclusion

In this work we have seen that if all players are risk neutral and contestants are price makers then the expected utility of the contestants is equivalent to the one of an all pay contest with cost x_i. This result is powerful since it simplifies the analysis of a relatively complex situation.

In this work we have seen also that price making behavior of the contestants always makes them worse off than when they are price takers. This result is in contrast with the classical duopsony model, where price making behavior makes the duopsonists better off. The reason of this discrepancy in the results is that in our setting the supply function is downward sloping.

In this work we have seen also that when contestants are price makers risk aversion always increases the equilibrium efforts, as in Skaperdas and Gan (1995) which analyzed the case with price taking contestants.

As further developments would be interesting to study the problem of uniqueness of equilibria. Moreover would be interesting to explore the settings with risk averse investor, price making investor and with an oligopoly of investors.

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