

The Paper for the 2012 World Conference of the Public Choice Society

The Title of Paper: The “Accident” Theory of the State: Conceptualization based on the Kantian critical philosophy

Name: Yoshifumi Ueda

Affiliation: Hiroshima University, the Graduate School of Social Sciences

Address: 1-2-1, Kagamiyama, Higashi-Hiroshima City, Japan

Tel: (082)-854-1360; **E-mail:** yosifm@hiroshima-u.ac.jp

Abstract: According to “the bargaining-power theory of the state” (Ueda, 2009; 2011b), the early state came into existence as the necessary result of such an innovative metal-revolution that the chieftains of preceding communities were self-interestedly motivated to make the transaction of external trade more profitable by resorting to a military force, because the net-benefit to those chieftains of a military force could be sure to be increased by innovative bronze weapons, taken over by iron ones later on. Though this synthetic proposition was derived on the basis of the recent empirical studies of both ancient history and evolution anthropology, in this paper I show that if subsumed *mutatis mutandis* under the Kantian categories of transcendental philosophy, the above proposition is applicable to other types of the state coming on the later historical stages, such as the aristocracy, the ancient monarchy, the feudal system, the modern absolute monarchy, and the nation state. In order to subsume those types of the state under the Kantian categories, first of all, the “state as an accident (der Zufall)” is explicitly distinguished from the “society itself as the substance (die Substanz)” which is the entity having been existing to ensure the survivability of human individuals by way of not only coordinating economic and defense activities but also maintaining social and cultural systems designed so as to maintain those coordinating activities. Secondly, any type of the state is formulated as a result of the transformation (Geschehen) from a preceding form of the society, and the transformation is formulated as adaption to a change (Wechsel) in external factors, i.e., the cause. Therefore, a new type of the state is recognized as a change (die Veraendlung) in the essential elements of the preceding type of the society. Based on the main propositions of this paper, I criticize the traditional theories of the state. To prove the main propositions, some expected hypotheses are inferred and then deducted by the analysis of a two-stage game comprised of both the “network game with hierarchies” to make the model of an economic community and the “two-stage bargaining game” to formulate the process of bargaining in a foreign trade.

**The “Accident” Theory of the State:
Conceptualization based on the Kantian critical philosophy**

Yoshifumi Ueda*
Hiroshima University

Abstract

According to “the bargaining-power theory of the state” (Ueda, 2009; 2011b), the early states came into existence as the necessary result of such an innovative metal-revolution that the chieftains of preceding communities were self-interestedly motivated to make the transaction of then-prevailing external trades more profitable by resorting to a military force the expected net-benefit to those chieftains of which could be sure to be increased by innovative bronze weapons taken over by iron ones later on. In this paper I show that if subsumed *mutatis mutandis* under the Kantian categories of transcendental philosophy, the above proposition is applicable to other types of the state coming on the later historical stages.

Key Words: State, Bargaining Power, Game Model, Kantian Categories

JEL Codes: D74, C72, C71

*Correspond to: Yoshifumi Ueda, The Faculty of Economics and the Graduate School of Social Sciences, Hiroshima University, 1-2-1, Kagamiyama, Higashi-Hiroshima City, Japan (zip: 739-8525).

E-mail: yosifm@hiroshima-u.ac.jp.

1. Introduction

Even band-communities had a military force aiming at aggressive and defensive war as well as a coordinated system of economic activities (Chagnon 1974)¹. The state is also a societal organization with a power to enforce called the "sovereignty" which plays crucial role in determining the actual level of the security of life and property, as well as with a function to coordinate economic activities. Furthermore, it is the well-known evidence that even the primitive band-communities, as well as clan communities just preceding the early states, were networked via inter-community trades (Ridley, 1997; Polanyi, 1977; Sahlins, 1972). These empirical facts contradict the conventional concepts of the state based on the "legitimized monopoly in violence"(Weber, 1911; 1924; Bellner, 1983), on the "existence of a power to enforce"(Lowie, 1962), on the "opening-up of external trade to kin-based communities characterized with intra-community trade" (Engels, 1884; 1878), on the sovereignty or common defense (Plato; Aristotle; Bodin, 1576; Hobbes, 1651; Spinoza, 1677; Rousseau, 1762; Hegel 1824/1825; Hardin, 1995), even though they are different in evaluating the plus-sum aspect of the sovereignty. On the other hand, those facts may seem to support the main proposition of the modern evolution anthropology which argues for the "principle of continuity" and against a structural change,² even if the state should be recognized as a societal organization to be essentially distinguished from the preceding communities. Now is the time for asking fundamental questions not only on the concept of the state but also on the causality of a state. What is the difference between the state and the preceding communities, i.e., the one between the "politically-organized society" and the "kin-based society" in terms of Morgan? Under what conditions and by what motives the state is brought into existence? Why can a state be stable only during a certain period of time?

Those questions are what I try to answer in this paper. For this purpose, I criticize the previous study of the state for disregarding the Kantian categories, in particular, the relation-categories such as a distinction between the "society as the

¹ As to the classical work on the use of military forces for defenses and attacks to protect the members of kin-based communities, see Mommsen (1854), Morgan (1877) and Finley (1978). As to the modern anthropological study on both military forces and external trades of pre-state societies, see Polanyi (1977, 1966), Sahlins (1972) and Ridley (1997).

² As to the principle of continuity, see Lowie (1962) in which the evolutionary anthropological approach to the state originates.

substance” and the “state as the accident,” and the causality and the interdependence. Then, I seek the way to answer those questions on the state in the Kantian categorical approach, and subsume the main synthetic but not analytic propositions under the categorical frameworks of the critical philosophy by formulating the empirical representations of the state in accordance with the categorized propositions and then constructing a game theoretic model formalizing the process of building a state in irrigation society, with a view to applying to other types of states.³ Finally, by the analysis of the base model, some hypotheses inferred from the main synthetic propositions are deductively proved. They are tested by reference to historical evidence.

The *first synthetic proposition* of this paper is that the state is an “accidental form (der Zufall)” of the “society as the substance (die Substanz)” whose essential nature is to ensure the survivability of human individuals by way of not only coordinating economic and defense activities but also maintaining social and cultural systems designed so as to maintain those coordinating activities. The society as the substance is recognized as a societal organ into which a group of human individual organs is organized so as to help satisfy the biological conditions for an individual organ to survive which, though each individual organ has innate instinctive and emotional genes’ programs to drive itself to satisfy those conditions, it is hard to satisfy only by one individual. They are protection from external threats (maintaining of a cell membrane), metabolism, and prosperity of posterity (preservation of genes), and the essential elements of the society as the substance, i.e., the security of life and property, the coordination of economic activities, and the maintaining of institutional systems, correspond to those biological conditions in turn. The “justification” for exercising the power of a state is the politics jargon to mean the satisfying those three conditions, though another popular term “legitimatization” is misunderstanding. On the other hand, the “authorization” means approving the entitlement of a person in power to exercise the power of a state. Legality, legitimacy, charisma are some necessary conditions for gaining the authority.

Here, it should be noted that the social contract theory of the state, which originates in the political philosophers of 17th century represented by Hobbes(1651), Locke(1690), and Spinoza(1677) and whose modern versions are represented by

³ This application was supported by Spinoza saying in his letter to Tirunhaus dated January 1675, the concept of a “subject” should include the “causal relations” which are found in the origin of the subject.

Nozick (1974) and Buchanan (1975), has been concerned with the justification of the power but not with the causality of the state, and that the conquest theory of the state represented by Khaldun (1377), Hume (1752) and Oppenheimer (1926) and the circumstance theory of Carneiro (1972) are both concerned only with the process of forming a state but not the causality.

The *second synthetic hypothesis* is that any type of the state is formulated as a result of the transformation (Geschen) from a preceding form of the society, and the transformation is formulated as adaption to a change (Wechsel) in external factors, i.e., the cause. Therefore, a type of the state is recognized as a change (die Veraendelung) in the essential elements of the preceding type of the society. The driving factors which bring about the transformation are the innate instinctive and emotional programs motivating a group of entrepreneurial individuals in the preceding society to take this opportunity to adapt to a change in the circumstances with the aim of increasing the net-benefits to themselves of the transformation.

The corollary of those two synthetic propositions is that the early state came into existence as the necessary result of such an innovative metal-revolution that the chieftains of preceding kin-based communities were self-interestedly motivated to make the transaction of then-prevailing external trades more profitable by the use of military force the net-benefits to those chieftains of which could be sure to be increased by innovative bronze weapons, taken over by iron ones later on.

Before explaining the outline of this paper, the philosophical criteria to judge the truth and objectivity of scientific propositions should be mentioned, even though it may seem to be a digression. This is because as well as those criteria have yet to be established in the traditional social sciences, an immature state of the methodology of cognition is one of the reasons why arguments on the state are in a state of disorder until now. I note here that the cognitive frameworks of the critical philosophy pioneered by Kant (1787, 1780) can be considered as the criteria for judging the truth and objectivity. Coincidentally or not, the recent studies of evolution biology and neuroscience⁴ have been providing empirical evidences to lend support to the main propositions of the Kant's critical philosophy, even if the former may not be explicitly conscious of such a relation with the latter. According to some of its main propositions, the cognition of *Homo sapiens*, classified into he sensations and cognitive intuitions of sensitivity, the categorical cognition of

⁴ As an application to decision-making of the recent studies of both neuroscience and biology, see Ueda (2010).

understanding and the generalized inference of reason, is the processes of subsuming various types of representations under *a priori* sensibility frameworks classified into both external sensation and time-space frameworks for sensibility, categorical framework for understanding and inference framework for reasoning based on categorical propositions. These frameworks can be considered as some of the “neural modules” which is biologically comprised of neural networks.⁵

Firstly, the “relation” categories, consisting of the substance category, the causality category and the reciprocity category, especially plays a key role in examining the main notions of the state, since they provide the processes of cognition with *a priori* frameworks for explicating what a state is, under what conditions and by what motives it is brought into existence, and finally how it is related to other societies existing in the same period of time.

According to the first of the relation categories, called the “substance” (die Substanz) and the “accident” (der Zufall), the society is a substance in the sense that *Homo sapiens* have been organizing a type of society in order to adapt to the surroundings with which they were confronted, and that the purposes for organizing various types of the society in common are to provide for both necessities and peace as securely as possible, which are indispensable for the survival of both an individual organ and its descendents but cannot be securely provided by oneself. Therefore, it is necessary that the innate programs-for-survival tend to drive a group of individual organs to coordinate their activities of production including external trade, and to organize a part of those individual members into a military force. On the other hand, the state is an “accident,” i.e., a type of the society, in the sense that it comes into being as a result of the transformation of other types of the society preceding to it by changing and/or replacing some elements characterizing those preceding types for the sake of adapting themselves to a new circumstance. Therefore, “What is the new circumstance” is a key question to subsume the state under the “substance and accident” category. One corollary of the main synthetic propositions of this paper is that the new surrounding of the early state is the innovation of metal tools the application to a military force of which became profitable in foreign trade.

The second of the relation categories, called the “causality” (die Kausalitaet) or “the cause and the effect” (die Ursache und die Wirkung), provides the cognitive

⁵ As to the original work of the “module theory,” see Chomsky (1965). As to the biological study to support it, see Premack and Premack (2003).

frameworks according to which a happening (ein Geschehen), which means a change in the essential elements characterizing one type of the substance, is recognized as a combination between one representation and other ones prior to it, in such a way in which the preceding ones are recognized to have had an influence on the succeeding one. The synthetic proposition derived from this cognitive framework is that a type of the state comes into existence as a “result” caused by a change in circumstances but not a “creature,” that is, a transformation of the preceding type of the society by adapting to the change in circumstances, and that driving factors of the transformation are the innate genes’ programs which motivates a group of entrepreneurial members to adapt to the new circumstances. The corollary of the proposition is that the most crucial of the preceding influential causes of the early state is the innovation of metal tools, and that the chieftains of the preceding kin-based society were motivated to adapt to the circumstances by an increase in the net-benefits of resorting to military force equipped with metal weapons. I note here that the innate programs-for-survival, called the “motives” in the fields of economics, keep on working as one of the essential elements of the society regardless of a difference in its types, and therefore that the motives, *selfish or not*, should be regarded as one of the “necessary conditions” for a state to come into being as a result, but not the “cause.”

The third of the relation categories, i.e., the “interdependence” or the “reciprocity” (die Wechselwirkung), provides the cognitive frameworks according to which plural representations existing at the same period of time are recognized as interrelated ones or as ones influencing one another. This category should be applied to explicating the relativity of the sovereignty of a state in inter-state surroundings without any common enforcer. Needless to say, the sovereignty means how much independent of any pressure from within and without the decision-making of a state can be.

Secondly, the “modality” category (die Modalität) provides the cognitive framework to evaluate how often a synthetic but not analytic proposition is likely to be realizable, classified into the “possibility modality,” the “existence modality,” and the “necessity modality.” Taking it into consideration that since some earliest-states came into being several thousand years ago, the states have been sure to be here and there, only the “necessity modality” should be taken up and be applied to the causality category.

Thirdly, the “quality” category (die Qualität) provides the intuitively cognitive framework according to which the degree of sovereignty is evaluated.

Finally, the “quantity” category (die Quantität) provides the intuitively cognitive framework according to which the size of a state represented by population and/or territory is evaluated in a single measure.

According to the cognitive frameworks mentioned above, the main synthetic propositions of this paper are summarized in what follows below.

Firstly, a type of the state comes into existence as a “result” caused by a change in circumstances, that is, a transformation of the preceding type of the society by adapting to the change in circumstances, and the driving factors of the transformation are the innate programmed instinctive and emotional systems which motive a group of entrepreneurial individuals to adapt to the new circumstances. The corollary of the proposition is that the most crucial of the preceding influential causes of the early state⁶ is the innovation of metal tools, and that the chieftains of the preceding kin-based society were motivated to adapt to the circumstances by an increase in the net-benefits of resorting to military force equipped with metal weapons. Such causality is *mutatis mutandis* applicable to other types of the states appearing in the later stages.

Secondly, any type of the state is considered as a type of “the society as a substance” and therefore must be able to satisfy the essential functions of the society as the substance. The justification of the power of a state and the contract theory should be reexamined from this view point.

Thirdly, the sovereignty is relatively but not absolutely determined, and is dependent on the quantity category such as the size of a state surrogating economic and military power.

The procedure of the proof of those synthetic propositions is as follows: Firstly, they are formalized under the categorical frameworks corresponding to them.

⁶ The concept of the “early state” is not necessarily the same as that of its godparent, Service (1975; 1971) and Classen and Skalnik (1978, 1981) which defined it as a society considered to have existed between chieftdom and full-grown states with regional territory. In this paper, according to the definition of Weber (1924) and the representation described by both Finley (1978) and Plutarch (1914-54), the early state is defined as a social organization ruled by *basileus* in the ancient Greek or *rex* in the ancient Rome who tried to have a monopoly in foreign trade by organizing immediate subordinates into a private military team living together inside a castle type of residence, surrounded by residential areas of small merchants and handicraftsmen, with farm lands spreading further away from them. In return for accepting the rule of the early king, those engaged in direct production were provided with protection from external threats, based on reciprocal contracts. The early state should be distinguished from the polis.

Secondly, in accordance with the representations (images) of those propositions subsumed under the corresponding categories, the intuitive perceptions or phenomenon of observable states are synthesized and unified in accordance with the above categorized propositions. Finally, the synthesized unification is combined into an abstract image (a representation of phenomenon), called “formal model” in economics. That is, an abstracted basic model is set up so that its basic assumptions and conditions are in accordance with the representations or images of the synthetic propositions subsumed under the categories. In this paper the basic model is formalized as a game-theoretic model. Finally, some expected hypotheses inferred from the basic model are deductively proved by the analysis of the basic model. According to the criteria for judging the truth and objectivity of the critical philosophy, the coincidence of the inferred hypotheses with those analytically-deducted results prove the objective truth of the synthetic propositions, called “the possibility of experiences” (die Möglichkeit of Erfahrung). The truth is corroborated by reference to historical data (the hypotheses are tested).

In what follows, this paper is organized as follows: In the second section, an outline of the main logic and the historical backgrounds are explained. In the third section, the traditional theories of the state are reexamined. In the fourth section, the base model of an irrigation society is set up and the process of bargaining in foreign trade is formalized in the analytical framework of a two-stage bargaining game on the basis of the discussions of the second and third section, and the main propositions are derived. In the fifth section, the main propositions are showed to be applicable to the aristocracy of a *polis* type and to the ancient monarchy of an empire type. In the sixth section, it is shown that the main propositions are applicable to the feudal system of the middle ages and to the absolute monarchy of the early-modern times. In the seventh section, those propositions are showed to be applicable to the nation state under the constitutional monarchy. In the eighth section, those propositions are showed to be applicable to the republic nation states under the bourgeois democracy and to those under the mass-democracy. The last section concludes this paper.

2. An Outline of the Main Logic and the Historic Backgrounds

In this section, the main logic of the bargaining theory is outlined and some historic evidences to support it are shown.

(2.1) An Outline of the Logic

The foreign trade in the Bronze Age put the chieftains of kin-based communities – *clan* in English, *genos* in Greek, *gens* in Latin, *wuji* in Japanese—under a new external circumstance in the sense that the net-benefits of the use of a military force in the transaction of foreign trade were increased by the innovation of metal tools. If they could adapt to such a new circumstance, they could gain and secure big benefits from the foreign trade the transaction in which, however, had to be carried out under anarchic conditions without any common enforcer. On the contrary, if they had failed to adapt, they might have been colonized in the worst case. Such an opportunity and a peril drove those chieftains to strengthen the “bargaining power” in the transaction of the foreign trade. The level of the bargaining power ranges from the highest level of a power to conquer to the lowest one of being subjugated. It is an economic terminology to be used as surrogate for, or an equivalent to, the sovereign power in terms of politics. The pursuit of the bargaining power drove those chieftains to transform the traditional communities into an “enlarged” social organization armed with a military force. An increase in the size of society was necessary to integrate bargaining process and to bring about scale merits in the production of the means of payment, as well as to maintain such a strong military force as to be able to put the bargaining process at least on an equal footing. Depending on the relative strength of an early king’s military forth over neighbor chieftains, an increase in the society size was achieved through an enlargement of early king’s autarky or by way of forming a “star type of networked coalition” consisting of neighbor chieftains with an early king locating at the center of the network. The military forth was of a private nature, not only in the sense that the aim of maintaining it is to pursue the benefits of the chieftains but also in the sense that the cost was financed by their own economic and human resources. Therefore, when the net benefits to a chieftain of an increase in the bargaining power under an early state became sufficiently large, it was brought into existence as a result of the innovation of metal tools which motivated selfish chieftains to launch into a venturesome enterprise, i.e., the transformation of the existing communities into an early state. Ueda (2010, 2009, 2008, 2007) called such a proposition the “bargaining power theory” of the state. Though some classical works⁷ may be able to inspire us to hypothetically infer this proposition, I induced it on the empirical bases of recent

⁷ For example, see Ortega (1921, 1930), Mommsen (1854), Engels (1888, 1878) and Plato (1941).

historical and anthropological studies on the external trade of ancient irrigation societies⁸ under or in accordance with the categorical and intuitive frameworks of the critical philosophy. The main synthetic proposition is proved deductively by the analysis of a basic model formulating both irrigation societies formalized by the hierarchical coalition game of Gamage (2004) and the process of an external trade between a chieftain and a foreign counterpart⁹ formalized by the two-stage bargaining model of Querido (2007).

The categorical approach to the early state is applicable to other types of states *mutatis mutandis*, if we make an explicit distinction among four categories on the states—the accident category, the causality, the classification into the accident category and the causality applying to other types of the state) and the stability of a state (the reciprocity).¹⁰ Failure to distinguish these four has brought serious confusions into arguments on the state. In order to overcome the failure, each of those should be argued under its corresponding cognitive framework of the critical philosophy, summarized in what follows below.

Firstly, the early state is conceptualized as follows: it is such a new type of social organization as to be able to bring the “sovereign power” but not a mere violence into birth by combining economic and personnel resources with a military forth from the historical stage onward. The main purposes of the sovereign power are the same as those of military forth mobilized by the preceding kin-based communities in the sense that it is aimed at protecting against threats from within and without.¹¹ However, the early state—and therefore, other types of states appearing after it—is explicitly distinguished from those kin-based communities by the economic background of the former that though reciprocal exchanges characterizing the

⁸ As to the historical and archaeological study on external trades between ancient Japan and China, see Asai (2008), Matuki (2007) and Murakami (2007). Furthermore, see Okada (2008, 2004) regarding documental records written by some ancient Chinese dynasties. As to the historical studies on the irrigation systems in Japan, see Tude (2005, 1989). As to the classical work on the ancient irrigation societies, see Wittfogel (1957) and Nakashima (1973).

⁹ As a special case of the base model, it is applied to Basileus of the ancient Greek. See Wilson (1978) and Ridley (1997) as to reviews on anthropological studies on military actions organized by kin-based communities.

¹⁰ This topic was distinguished by Hegel (1824/1825), when he argued for the organic theory of the state.

¹¹ As a pioneering empirical study on such an organized war among primitive kin-based communities, see Chagnon (1974). Furthermore, see Wilson (1978) and Ridley (1997) as to the review of anthropological studies on military actions mobilized by primitive kin-based communities

kin-based communities were left to personal contracts between chieftains and follower-members, the external trades of the early state were carried out under the historical conditions of the Bronze Age. Since the conventional concepts of the state did disregard the relation between a state as the accident and the society as the substance, they could not distinguish the state from the preceding kin-based communities.

Secondly, the causality to explain what caused the early state come into existence is derived from the innovation of metal tools as a new impact influencing on the preceding communities, under the condition that the members of those communities are always motivated or driven by the innate genes' programs-for-survival. Since the sovereign power of a state contributes to making property rights more effective, those subjects who can gain the largest net-benefits from bringing a power into birth are motivated to build a state even at their own cost. Though such a hero type of political entrepreneurs should be distinguished from a non-political mediocrity by differences in the emotional and instinctual neural systems, historical evidences show that such a hero type of political entrepreneurs came on each epoch-making historical stage. It seems to be because of this selfish aspect of the motives for the power that the conquest theory and the circumstance theory apt to focus on violence as the origin of the state.

Thirdly, according to the concept of the power defined by political philosophers represented by d'Entreves (1967), Arendt (1958) and Lutz (2006), the state is classified by who are the origins of the power of a state, i.e., by who bears the cost of military force combined with both economic and personnel resources. It is because the power originates in the early king that the early state is called the "early kingship." This approach to classification of the state is applicable to other types of the states, *mutatis mutandis*. However, the justification of exercising the power depends on whether the power is exercised so as to meet the three conditions of the society as the substance mentioned already.

Fourthly, the stability of a state is, strictly speaking, the stability of a political system. The "political system" determines who are entrusted to exercise the power by the original holder of power. In the early state an early king and coalitional members not only bring the power of a state into birth but also exercise the power. In order to maintain the stability of the early kingship, therefore, the exercise of the power must meet the three conditions of the society as the substance. The participants' constraints of both coalition members and other follower-members are an economic term to express the metabolic condition. The ruler-ruled relations were

formed by reciprocal contracts on a voluntary basis, and could be maintained to the extent that those contracts were fulfilled by both parties. It seems to be because of this stability aspect of the state that the “contractual theory” of the state is apt to consider a state to originate in contracts. The stability of a state is subsumed under the accident-substance category and the interdependence category. The actual degree of the sovereignty is an approximate measures. As long as the bargaining power can play the role of a surrogate for the sovereignty, the main determinants of the bargaining power may be recognized as the main factors influencing on the stability.

If the above inferences from the main propositions of the early are considered as the essential categorical frameworks, the categorical approach to the early state of this paper is applicable to other types of the state, and the traditional theories of the state can be reexamined from those four points of view.

(2.2) The Historical Backgrounds of an Early State

The chieftains of traditional irrigation communities, who had theretofore formed intra-community economic networks among those communities, were faced with the foreign trade which could bring new necessities, bronze and iron goods later on (heretofore, represented by iron resources¹²) into the Far East Asia of those days. These necessities were vital for increasing both economic productivity and military power. Although at first they might have passively joined in this new trading network, they could take advantage of the chieftainship to have exclusive benefits from this external trade. However, whilst the intra-community trades could be under a repeated-game setting, firstly because residential areas are stuck to those near irrigated lands and secondly because the relative military forth equipped with stone-made weapons was not so distinguished as to be able to overwhelm others without too risky cost, they had to play with those new foreign counterparts in a finite-stage game setting under the condition that it is worth to resort to a military force equipped with iron weapons in terms of the net-benefits. Then, the validity of contracts concluded in each transaction in the foreign trade was doomed to reflect the relative strength of a power to enforce those contracts.¹³ In this paper the power to enforce is called the “bargaining power” above defined which is a surrogate for

¹² As to the origins of bronze iron tools, see Muhly (1995).

¹³ As to the pioneering work of such a difference between intra-community (domestic) trade and inter-community (external) trade, see Polanyi (1977, 1963).

the sovereign power¹⁴ in politics. It could be increased not only by integrating the process of transaction and the production of the means of payment but also by strengthening military forth. It was for the sake of increasing his payoff that those chieftains pursued these two ways to increase the bargaining power. The pursuit after the bargaining power motivated him to coordinate the traditional communities into an enlarged social organ with a regular military force justified by the society . As a result, those preceding communities were transformed into an early state, and the chieftains became an early king, called *basileus* in Greek, *rex* in Latin and *Ou* in Japanese. This causality to explain the origins of the early state is consistent with the “organic theory” of the state,¹⁵ in the sense that both propositions imply that the more organically the members of a society are coordinated, the bigger common interest such as the sovereign power can be achieved.

On the other hand, the cost to a chieftain of forming an early state is for the most part comprised of the cost to maintain his private army, to produce a means of payment for imported necessities and to manage an irrigation system. They were financed by earnings from the external trades and by farm rents. It might have been conventional to view the farm rents as an appropriation or benevolence without equivalent compensation, except for the benefits of protection from external threats. However, as to the farm rent of irrigation society, it should be recognized as farmers’ payment not only for benefits obtained from joining in an irrigation system but also for loan of seeds, on the basis of a reciprocal-exchange contract. This point of view on the farm rent is justified on a rational basis, if we take it into consideration that the irrigation system is of a club nature and, furthermore, that the chieftain of an irrigation society had a technological monopoly in the safekeeping and species-improvement of seeds as well as in the construction and maintenance of a large-scaled irrigation system. On the farmers’ side, they could flee to some traditional ways of life such as primitive field-farming or small-scaled

¹⁴ As to the original concept of the sovereignty, see Bodin (1576).

¹⁵ By “organic model,” I refer to the theories of the state argued by Plato, Aristotle, Cicero , Hegel (1824/1825), and Hardin (1995). They are common in arguing that a positive common interest can be given birth only by coordinating the members of a political organization into a networked division of labor functioning like an organic body. It is, in particular, Hegel that brought forth a consistent logic to reconcile the selfish motives to the achievement of a common interest. Strictly speaking, however, political philosophers considered as the contract theorist, such as Hobbes (1651), Spinoza (1677), Locke (1690) and Hume (1752), also argued for common interests such as a defense against foreign threat. In this respect, they may be subsumed under the organic theorist.

irrigation farming. Owing to these options the farmers had, the chieftain was required to meet their participants' constraints. Accordingly, the process of forming an irrigation society is formalized on a voluntary reciprocity basis. That is why the irrigation society can be formalized by a "networked-coalition game with hierarchies" and why its stability in the sense of the core is derived.

Though the historical backgrounds explained in the above reflect the characteristics of irrigation societies in the Japanese Archipelago, they are also applicable to other irrigation societies such as ancient Egypt, Mesopotamia, and China, India, Inca and Sri Lanka,¹⁶ *mutatis mutandis*. Furthermore, they are also applicable to the process of building the "early state" in ancient Greece,¹⁷ if it is taken into allowance, firstly, that the economic bases of the community are comprised of both dry field farming and cattle-breeding,¹⁸ and secondly that the early king is a "chief among the equals" whose election is much more influenced by *anassein iphi* (rule by forth)¹⁹ than in those irrigation societies, since the chieftains of irrigation societies could also control follower-members by way of social functions such as the management of irrigation system.

3. The Reexamination of the State

From the historical backgrounds explained in the previous section, the four propositions on the early state are derived, and summarized in what follows.

Firstly, the early state is defined as a social organization with the power of a state backed up by a military force, which was brought into birth under historically a new external circumstance, i.e., the innovation of metal tools by the application to military force of which the chieftains of the traditional kin-based communities could gain such a huge profit as to give them an incentive to take on the cost of transforming the existing communities into a new enlarged social organization equipped with a regular military force. The bargaining-power in the transaction of then-prevailing foreign trade could be increased by resorting to the military force which could contribute to enforcing the contracts of trade concluded under anarchic

¹⁶ Gunawardana, R.A.L.H. (1981) as to the early state in Sri Lanka. On the other hand, Maya did not reach the state in the sense that it had been in the Stone Age.

¹⁷ See Weber (1924) and Finley (1978) as to the early kings of the ancient Greek.

¹⁸ The production process is much less characterized with team production than that of irrigation society which is crucially dependent on an irrigation network.

¹⁹ Plutarch (1914-54) gives us the image of the early king by way of the mythological stories on Theseus and Romulus.

conditions.

Secondly, the driving factors of bringing it into birth is the selfish motives of those chieftains for gaining bigger profits from the foreign trade by resorting to a military force. The regular army of the early state was of a private nature in the sense that it was aimed at increasing the selfish benefits from a monopoly in foreign trade, as well as that at first it was financed by chieftain's private resources.

Thirdly, the power of a state originated in a chieftain-turned early king who could bring the power into birth by combining a military force with economic and personnel resources, subject to the participants' constraints of other members. It is because the power of an early state originated in the early king that the early state should be called the "early kingship."

Fourthly, the stability of an early kingship is dependent on how satisfactorily and extendedly the participants' constraints of both coalition members and follower-members can be met. In this sense, in order to maintain the stability of the political system of the early state, the power of a state had to be justified by the implicit consents of those other members, although the power of a state originated in an early king.

The categorical approach to the early state is applicable to other types of the states appearing in the later historical stages,²⁰ if relevant inferences are derived from the main synthetic propositions on the basis of Kant (1780; 1787). Those inferences from the main propositions on the early state, called the "causal factors" by Spinoza, are follows.

Firstly, the "power" is a political concept defined as the ability to enforce one's will on others, and the early state was also invented to bring into birth an "equivalent" to the power in the sense that since the power of the early state was brought into birth by the selfish motives to have an advantageous position in foreign trade, the equivalent is called the "bargaining power" in economics terms. This term is such an inclusive concept as to include two extreme cases, i.e., conquest and surrender. It should be re-emphasized that since an organized physical force for war is observed also in primitive kin-based communities, mere the existence of an organized military force is not sufficient to conceptualize the state.

Secondly, the preceding communities were faced with the innovation of metal goods, the application to military force of which gave sufficient incentives for taking

²⁰ It can also apply to federalism since the applicability can be confirmed by Hamilton's essays in the *Federalists*. As to the rational bases of federalism, see Riker (1962) and Alesina and Spolaore (2005).

on the cost of transforming the preceding communities into a new social organization aimed at increasing the bargaining power in the transaction of foreign trades under anarchy.

Thirdly, the entrepreneurial agents who if they can adapt to the new external-trade circumstances, gain the big benefit of an increase in the bargaining power are motivated to take the risk of, or to participate in, building a new social organization. The power of a state originates in those subjects who contribute to the birth of it by bearing the cost on their own.

Fourthly, as long as those entrepreneurial agents want to continue to gain the large benefits from adapting to the new external circumstance, they have to meet the participants' constraints of not only their coalition members but also follower-members such as economic agents engaged in production of the means of payment.

Though some political philosophers²¹ could contribute to classifying the state by who are the origins of the power of a state, they did make clear neither how and why the power of a state originates in those who contributed to bringing it into birth, nor under what historical conditions the power was brought into birth.

Generally speaking, under anarchy modeled by the analytical framework of a finite-stage game, threat by force as *ultra ratio* plays a crucial role to conclude and keep contracts in advantageous condition. Therefore, the bargaining power in foreign trade carried out under such anarchy may be assumed to be determined by the strength of a power relative to foreign counterparts. However, the actual efficaciousness of the power, in particular, the sovereign power against external societies, is dependent on what a strong military forth the state has in reality. Such a role of military forth in determining the actual degree of the sovereignty might have led some schools of social sciences except the "contract theory" to focus only on violence in conceptualizing the state²² or in justifying the exercise of power.²³

However, if we take it into consideration that many of primitive kin-based

²¹ See d'Entreves (1967), Arendt (1958) and Lutz (2006).

²² The definition of the state based on violence is represented by Engels (1884), Weber (1911) and North (1981).

²³ Beginning with Augustinus, Machiavelli (1532), Engels (1884), Veblen (1889), Weber (1911), Oppenheimer (1927),²³ Caneiro (1970), Service (1971), North (1981), Olson (1993, 2000), McGuire and Olson (1996), Findly (1996), Kurrild-Klitgaard and Sevendsen (2003), and the conflict game models represented by Skerpardas (1992), Hirshleifer (2001), Moselle and Polak (2001) and Grossman (2003) are all subsumed under this category.

communities had also an organized force for wars, it is obvious that, in order to distinguish the state from those kin-based communities, we are required to find out other crucial factors than mere the existence of such an organized force. That crucial factor is the emergence of a new external circumstance with which the chieftains were faced in the historical stage at which external trades had been prevailing under the condition of there being no common enforcer.

On the other hand, it should be noted that any state has its own preceding societies in which economic networks including external trades peculiar to those societies had been spontaneously grown, and that when faced with new external circumstances, some risk-taking entrepreneurial groups were driven to organize those existing societies into a new enlarged social organization with the aim of taking the opportunity of those new external circumstances by generating a power to enforce. Such a historical process leads us to view the state in a “dynamic perspective” in accordance with Ortega (1930), and to start formalizing the process of building a state from characterizing its preceding societies but not from an abstract society consisting of an ahistorical group of atomistic individuals. Such an ahistoric assumption, on which the traditional “contract theory” of the state²⁴ is based, makes it hard to solve the problem of collective action²⁵, but the categorical approach or the bargaining-power theory can solve it.

On the contrary, the evolution anthropology, originating in Lowie (1962), argues for the “principle of continuity” and denies a structural change in the transformation of the preceding society into a state, though they emphasize the aspect of enforcement as a factor distinguishing the state from the pre-states. It means that they focus on “the society as the substance” only. Any type of the society is organized with a view to securing the survivability of its members.

4. Irrigation Society with Canal system: The Base Model of the First Stage

²⁴ By the contract theory, I refer to Hobbes (1651), Spinoza (1677), Locke (1690), Rousseau (1762) and the modern followers of some of them, represented by d’Entreves (1967), Rawls (1971, 2001), Nozick (1974), Buchanan (1975) and Lutz (2006).

²⁵The classical works represented by Ostrom (1965), Wagner (1965) and Flohlich *et al.* assumed the existence of political entrepreneurship for collective action in order to solve the problem of collective failure. Olson (1965) emphasized the lack of a motive for political entrepreneurship itself except for the case that the selective-incentives scheme can be applied to political organization. Olson (1993, 2000) solved the problem of collective failure in politics by the “rational bandits” model.

For the purpose of proving the main propositions of the bargaining-power theory by the back-ward induction of a two-stage game, in this section the first stage is formalized as the process of networking an irrigation society and of producing and allocating its outputs. The second stage models the process of the bargaining in a foreign trade. In the first subsection the base model of an irrigation society is set up in the analytical framework of a networked-coalition game. In the second subsection, the price of iron is defined. In the third subsection, the stability of the networked irrigation society is proved in the sense of the core. In the fourth subsection, the model of the bargaining process at the second stage is presented and some optimal results of this stage are derived. In the fifth subsection the main results of the two stage game are derived. In the sixth subsection, the main propositions on the early state are derived.

(4.1) Irrigation Society Networked with Canal System²⁶

Irrigation system is an economic infrastructure indispensable for any irrigation society, but in order to set up and operate an irrigation system, the members of an irrigation society have to be coordinated into a networked coalition. On the other hand, various kinds of metal tools, represented by iron *en masse*, are vital for increasing both economic productivity and military power, but have to be procured by way of external trades with foreign counterparts.

In order to construct a formal model abstracting the essential characteristics of irrigation societies, suppose that a river is flowing down from mountain areas in its riverhead region, and that a canal system for irrigation is set up by taking irrigation-water from one point of the river called *sluice gate*. To be concrete, this canal system is assumed as the following: A trunk canal²⁷ is constructed which can irrigate prospective n paddy fields, numbered $1, 2, \dots, n$ in the order of distance from the sluice gate. I note here that “ n ” is a generic but not fixed number. This trunk canal can “technically” irrigate any number of paddy fields. Each paddy field is cultivated by one farmer. Paddy fields are developed and located one by one in line along the trunk canal.²⁸ In order to take irrigation-water to each paddy field

²⁶ Refer to Nakashima (1973) and Wittfogel (1957) regarding the details of ancient irrigation systems. As to the irrigation society of Japan, see Tude (1989).

²⁷ Whilst the trunk-canal system fits well to a multi-layered hierarchical society, a reservoir canal system to a star network. The base model is also applicable to the latter system, *mutatis mutandis*.

²⁸ Though paddy fields are assumed to be located in line, the model can be extended to the more complex irrigation systems in which each paddy field has its own sub-

from the trunk canal, each farmer has to construct one branch-canal so as to be connected with it. A chieftain locating in the sluice gate coordinates those expected farmers to construct and maintain the trunk canal at the farmers' expense on an equality basis. The total cost of the trunk is denoted by K . The farmers bear not only the equal share in the cost of K as the entrance fee on a club good, but also the marginal cost of joining in the irrigation system. The latter cost is denoted by $C_i \equiv C(i)$, for i farmer, which is comprised of the cost to construct the i branch canal locating in the i ordered distance from the sluice gate called the *zero* site where the chieftain locates, and of the cost to communicate and transport between the i site and the *zero* site. In this section, the number i is treated as a natural number standing for i farmer, for $\forall i \in \{1, 2, \dots, n\}$. A set of a chieftain and n farmers is denoted by $N = \{0, 1, 2, \dots, n\}$, whose first element stands for the chieftain.

The absolute number of the elements of N is defined by $|N| \equiv n$. If a new farmer joins in this irrigation system, his paddy field must locate in the next to the most-distanced site in the existing irrigation system. The cost function of the i branch canal, $C_i = C(i)$, is assumed to be an increasing function of the distance from the sluice gate, with an increasing rate. That is, the more distanced, the more rapidly it increases. These assumptions are formulated by the relations (1) below.

$$(1) \quad C_i = C(i), \\ 0 = C(0) < C(1) < C(2) < \dots < C(n), \text{ and} \\ C(i+1) - C(i) < C(i+2) - C(i+1), \quad \forall i \in \{0, 1, 2, \dots, n\}.$$

The above assumptions on $C(i)$ are justified by assuming the technological characteristic that the further away from the sluice gate a farmer is, the costlier for him to communicate with the chieftain locating at zero site, to bring back loaned seeds and to transport a part of annual harvests for the payment of charges on the loaned seeds and on the consumption of irrigation-water.

The trunk canal is constructed by using the iron whose volume and technology are denoted by M *en masse*. They replace conventional tools, i.e., stoneware. Therefore, given a scale of irrigation system, K is assumed as a decreasing function of M ,²⁹ defined by (2).

$$(2) \quad K = K(M); \quad K(0) > 0, \quad K'(M) < 0, \text{ and } K''(M) > 0.$$

irrigation systems.

²⁹ Iron tools for construction of canals and for farming were usually lent to farmers by chieftains.

The above assumptions on K are justified, because, if the more of iron-tools replace the existing less-effective tools to construct one set of canal system, the less costly it can become, subject to the “as-usual” assumption on the second derivative.

If the trunk canal is constructed by the cooperative work of s farmers coordinated by the chieftain, the farmer i bears the cost amounting to $K(M)/s, \forall i \in \{1,2,\dots,s\}$. Such a way of burden-sharing may appear to mean slave labor, but in a contractual term, it stands for *entrance fee* or *basic charge* for irrigation system.³⁰ Crops are harvested after each farmer is engaged in a farming work whose energy expenditure is denoted by e . It is assumed as a constant for all farmers. This assumption is for simplicity but justified by the historical condition under which farmers on those days could not have so much option for leisure. The harvest on each cropland is assumed as an increasing function of the iron (iron tools), and defined by $f(M), f(0) > 0, f'(M) > 0$, and $f''(M) < 0$. This function is assumed to be the same for all croplands. Difference in the fertility of each crop land is reflected in the increasing marginal cost of the branch canal. A fixed percentage of $f(M)$, denoted by $\alpha, 0 \leq \alpha \leq 1$, is paid to the chieftain as *variable charges* for the consumption of irrigation-water and for the loaned seeds. It is considered as the contractual representation of so-called annual tributes.³¹

(4.2) Payment for Iron: The Terms of Trade

Iron, M , has to be procured by way of the foreign trade in which the chieftain (the *zero* player, hereafter) can take advantage of chieftainship to have a monopoly. According to the historical background, the foreign trade is classified into two cases. In the first case, the external trade is carried on under the existing community system, and the price of iron is a given P per unit of M because it is set by a foreign counterpart and he is a price-taker. In the second case, the foreign trade is carried on under an early state comprised of s members excluding the chieftain, and

³⁰ For example, according to the data documented by the centralized monarchy system in the 7th to 8th century, each farmer was liable to do “sixty day work” per year under the supervision of a local chief. This work is considered to be allocated to construction and maintenance of irrigation systems of the local community. On the other hand, each branch canal is considered to have been maintained by the farmer engaged in farming along it.

³¹ According to the above document, the tribute from annual harvest, called S_0 , was about 3 to 5 per cent of the total harvest. Seeds were loaned at about fifty percent of interests. Payment in other products than paddy crops, called C_{yo} and Y_0 , can be also subsumed in $\alpha f(M)$ for simplicity.

the price is a decreasing function of s , defined by $\phi(s)$, per unit of M . The size of a social organization, denoted by s , is taken as a surrogate for the consolidation effects on the bargaining process in the transaction of the foreign trade, the scale effects on producing the means of payment for M , and the military force.

The cost to govern the early state is denoted by $G(s)$ *en masse* that covers the cost of maintaining a regular military force and that of other administrative work.³² In order to indicate the positive effects of forming a state on the bargaining power in the foreign trade, approximated by $\phi(s)$, it is assumed that $\phi_1 \equiv \partial\phi(s)/\partial s < 0$. On the other hand, in order to emphasize the costly nature of maintaining the state aiming at an increase in the bargaining power, it is assumed that $G' \equiv \partial G(s)/\partial s > 0$ and $G'' \equiv \partial^2 G(s)/\partial s^2 > 0$. Then, the total cost to the zero player of acquiring M , denoted by Ψ , is defined below.

$\Psi \equiv \Psi(M : P) = P \cdot M$, for the existing community with a given P , and

$\Psi \equiv \Psi(M, s) = \phi(s)M - G(s)$, for an early state with $|S|$ size of society.

In the fifth section, it is justified that $P > \phi(s)$.

(4.3) Hierarchical Network and Stability

If, in order for a group of members to produce a cooperative output, they have to be *ex ante* coordinated into a networked team, it is called the “networked coalition with hierarchies”.³³ In this sub-section, the cooperative process of the irrigation society set up in the subsection (4.1) is formalized in the analytical framework of the networked coalition game with hierarchies.

The process of forming a hierarchical coalition begins with a two-player coalition and ends with a hierarchical coalition of $|N|$ size, for simplicity, under the condition that the superadditivity is satisfied until the coalition size gets to $n \equiv |N|$.

The zero-player coordinates other members into a hierarchical network and is at the top of any hierarchy if he is superior to others in managing the irrigation system.

³² According to the ancient centralized dynasty system called the *Rituryo* system, the regular force was comprised of about 200 thousands military services and the cost of maintaining it was financed by the dynasty government. They were exempt from both payment in cloth called *Cho* and 60 days work for construction called *Zoyo*. These exemptions are considered as a payment to the military servicemen. The cost of constructing roads and metropolis was also financed by taxes.

³³ As to the details of the concept, see Demange (2004) and Bala and Goyal (2000).

Suppose a generic stage of the process, denoted by a networked coalition, $S = \{0, 1, 2, \dots, s\}, 1 \leq s \leq n$. Then, the zero-player offers those s members those contracts according to which if they join in an irrigation system, they are assured of payoffs satisfying the participants' constraints. The clauses of the contracts are classified into two types as follows: The first is comprised of the production technology or the means of production he offers and of the cost burden of each member. The second is of interest rates on loaned seeds and of charges on consumption of irrigation-water. The former is represented by $K(M), f(M), C(i)$ and $|S|$. The latter is by α . By abstracting the common factors, those contract clauses can be condensed into a three-element set³⁴, $\{\alpha, M, s\}$. For S , there exist various combinations of α with M . The combination is denoted by $a(S) = \{\alpha, M : \exists S\}$. Denoting a set of $a(S)$ by $A(S)$, $A(S) = \{a(S) : \forall S \subseteq N\}$. For mathematical simplicity, $A(S)$ is assumed to be compact. In order for those s farmers to accept a contract offer, $a(S)$, it has to meet the participants' constraints and must be feasible. The participants' constraints are defined by the opportunity cost, zero-normalized. The feasibility condition of $a(S)$ is satisfied, if the total payoffs are nonnegative.

If each of those s farmers accepts a contract offer $a(S)$ and it is feasible, the payoff of the zero-player, $\pi_0 = \pi_0(a(S))$, and that of i farmer, $\pi_i = \pi_i(a(S))$, $\forall i \in S \setminus \{0\}$, is defined by (3) and (4), respectively.

$$(3) \pi_0(a(S)) = |S|\alpha f(M) - \Psi.$$

$$(4) \pi_i(a(S)) = (1 - \alpha)f(M) - K(M)/|S| - C(i) - e, \quad i \in S \setminus \{0\}.$$

The above payoff functions are defined over the compact set $A(S)$ and are *continuous* over $A(S)$. If, furthermore, S is extended to the domain of positive real number, the payoff functions are also continuous over $A(S)$.

Since the opportunity cost of each player was zero-normalized, the participant's constraints of the zero-player and those of the farmer i are defined by (3)' and by (4)' in turn. Under the assumption of transferable utility, the feasibility of $a(S)$ is defined by (5).

³⁴ Even if α is assumed as a given parameter, it does not influence the main conclusions of this paper, but in what follows, the general formalization is adopted.

$$(3)' \pi_0(a(S)) \geq 0.$$

$$(4)' \pi_i(a(S)) \geq 0.$$

$$(5) |S| \cdot f(M) - \Psi(M) - K(M) - \sum_{i=1}^s C(i) - |S| \cdot e \equiv v(S) \geq 0.$$

$v(s)$ of the right side of (5) means the value of the s -player cooperative game. In what follows, the constant parameter, e , is omitted without loss of generality.

Since the main assumptions of the above set-up, i.e., the superadditivity, the compactness and the continuous utility functions, meet the conditions of the hierarchical coalition game of Gemange (2004), we can prove the stability of the irrigation society along the same mathematical algorithm.³⁵

The stability nature of the irrigation society is summarized as *Proposition 1*. See Ueda (2011b) as to the general proof.

Proposition 1: The irrigation society is stable in the sense that neither a player nor a coalition has an incentive to deviate from it, under the assumption that the superadditivity prevails and that utility functions are continuous over a compact set of variables. Furthermore, if chieftainship cannot be taken over by any other member, then this stable system is unique. If the superadditivity stops at some size, then the society is divided into more than one irrigation systems called “heterarchy” each of which meets the superadditivity.

Proposition 1 not only proves that under the superadditive condition an irrigation

³⁵ The mathematical algorithm begins with the process of the zero-player’s maximizing his payoff in forming a two-player network subject to the participant’s constraint of a farmer. Next, in the same way, he forms a three-player network subject to the participants’ constraints of two farmers, and etc. The payoff allocations of $(n + 1)$ players’ network satisfying such a procedure not only meet the participants’ constraints of those networked members, but also do not give an incentive to make any coalition deviating from it. Furthermore, it is unique, if the zero-player is assumed as only one coordinator. The basic model can be extended to more complex types of irrigation system, as far as the main assumptions are maintained. For example, each branch canal can have its own hierarchical irrigation systems by extending smaller branch canals from it and connecting them. For another example, a canal system with reservoir is also formalized by a similar model, *mutatis mutandis*. The reservoir system is applicable to Sri Lanka.

society with hierarchies is stable in the sense that no member of the irrigation society has an incentive for deviating from it, but also implies that it is not right to trace the origins of the state back to the fissiparous tendencies of preceding societies, which some evolution archaeologists consider had caused those preceding societies to break up and to form a state on a contractual basis.³⁶

(4.4) The Bargaining Process: A Two-stage Bargaining Game at the Second Stage

The intra-community economic network of an irrigation society is formed in a repeated-game setting. This is because any irrigation society is not free from adherence to farmland and because stone-tools have to be used, the power to enforce is not so different not only among a community's members but also among those communities. Therefore, economic networks among them are spontaneously grown on a voluntary basis and the chieftains of those communities are neither motivated nor capable to unite them into a state maintaining a regular military force equipped with metal tools.

On the other hand, foreign trades with foreign counterparts are carried out in the setting of a non-repeated game, in particular, if those foreigners have other options for business connections and are free to shift their options backed up by having a recourse to a military force. In this section an external trade of iron, carried on without any common enforcer, is formulated by a two-stage bargaining game between a chieftain called "buyer" and a foreign counterpart called "seller." It is only through this foreign trade that the buyer can obtain iron. It may be too costly to have so strong a regular army as to conquer the seller, but he wants to keep the external trade in as advantageous a condition as possible.

At the first phase³⁷ of the two-stage bargaining game, the seller offers a supply price, denoted by P , per one unit of the volume of iron, M . At the second phase, the buyer decides whether to accept or reject it. If he accepts, the contract is concluded and the buyer obtains M at the price of P . On the contrary, if he rejects the offer,³⁸ the bargaining process enters into conflict, and is settled so as to reflect the

³⁶ For example, see Classen and Saknik (1978, 1981), and Carneiro (1970).

³⁷ The term "stage" is replaced with the term "phase" in order to avoid the confusion between the "whole game" comprised of the stage of production and the stage of external trade stage and the "two-stage bargaining game" comprised of the stage of offering and the stage of conflict.

³⁸ Under the structure of non-repeated game, the players are exposed to various kinds of risks such as exorbitant overcharge, stealing off proceeds and plundering during transport. When the buyer values the offered price, as a matter of course he takes those risks into allowance.

relative strength of a power to enforce. Both military power and economic power, and how much effectively the social organization is coordinated into an organic body are crucial factors determining the relative degree of the enforcement power. In each homeland, military forth such as a standing army may be or not may be waiting behind agents charged with bargaining on the spot. It is those agents including attendants and transporters that are involved with conflict on the spot. In any way, the extra cost to the buyer (seller) of exercising a military forth on the spot is assumed as a given parameter, denoted by $V_0(V)$. However, how the conflict is settled depends on whether the buyer is the chieftain of a preceding community or the king of an early state.

If the buyer is the chieftain of the preceding community which is categorized as a social organization without a regular force, the probability of his winning in conflict is assumed to be a constant denoted by λ_0 . On the other hand, if the buyer is the “king” of an early state, categorized as a social organization with a regular army, the probability of winning in conflict is considered to be more flexible and assumed to depend on the relative strength of the sovereign power he can exercise. The relation among the bargaining power (sovereign power), military power, economic power and their organic combination are formalized by the “Conflict Success Function”³⁹ (CSF, hereafter).

In the previous section, the number of the players, denoted by s , was defined as a natural number in order to make the explanation suitable to the analytical framework of a cooperative game. However, since differential calculus is required in this section, the space of s is extended to a real number,⁴⁰ if necessary. The superadditivity is not assumed in what follows, because the main topic is optimal decision on $\{\alpha, s, M\}$ by maximizing π_0 , *s.t.*, $\pi_i \geq 0, \forall i \in \{1, 2, \dots, s\}, s \leq n$.

(4.4.1) Foreign Trade without a Regular Military Force equipped with Iron Weapons

If the offered price P is accepted by the chieftain, the seller’s payoff function, $\pi = \pi(P)$, and the chieftain’s one, $\pi_0 = \pi_0(P)$, are defined by (9) and (9)’ respectively, with α, s and M being given at the second stage of the backward induction

³⁹ The “conflict success function” used in this paper is defined in the next subsection.

⁴⁰ This extension is not a contradiction, since the differential calculus in this section is required to derive the optimal number of the players or society’s members of a cooperative game played in the first stage. If the optimal value is not a natural number, the first decimal place is rounded off to the nearest natural number.

process. To be simple, in what follows, it is assumed that the seller supplies iron at no cost.

$$(9) \pi(P) = P.$$

$$(9)' \pi_0(P) = s\alpha f(M) - PM.$$

All of α , s and M in (9)' are determined at the first stage in the backward induction process, and therefore, are recognized to be given at the second state.

On the contrary, if the price is rejected by the buyer, the bargaining process proceeds to the second phase. The condition (10) is necessary for the buyer to choose "rejection" of the offered price P .

$$(10) s\alpha f(M) - V_0 > s\alpha f(M) - PM.$$

The above condition (10) means that M can be taken away by exercising a force at the cost of V_0 on the spot.

In the conflict, the chieftain and the seller expend V_0 and V , respectively. The chieftain wins the conflict with the probability of λ_0 assumed as a given parameter. If he wins the conflict, his payoff amounts to the value of $\{s\alpha f(M) - V_0\}$. If he loses, he has to pay $P \cdot M$ in return for M , and thus his payoff is reduced to the value of $\{s\alpha f(M) - PM - V_0\}$. Then, the expected payoff of the seller, $\pi(\alpha, s, M : \lambda_0)$, and that of the chieftain = buyer, $\pi_0(\alpha, s, M : \lambda_0)$, are defined by (11) and (11)' respectively.

$$(11) \pi(\alpha, s, M : \lambda_0) = (1 - \lambda_0)(PM - V) + \lambda_0(-V).$$

$$(11)' \pi_0(\alpha, s, M : \lambda_0) = \lambda_0 \{s\alpha f(M) - V_0\} + (1 - \lambda_0)\{s\alpha f(M) - PM - V_0\}.$$

Denote by $P^* \equiv P^*(\alpha, s, M : \lambda_0)$ the maximum of P which the chieftain can accept at the first phase. Then, P^* is determined so as to solve the equation, $\pi_0(P) = \pi_0(\alpha, s, M : \lambda)$. By solving it and arranging the result, (12) is derived.

$$(12) P^* \equiv P^*(\alpha, s, M : \lambda_0) = V_0 / (\lambda_0 M).$$

Substitute (12) into $\pi(P)$ and $\pi(\alpha, s, M : \lambda)$ and compare the results. Then the optimality of $P^*(\alpha, s, M : \lambda_0)$ for the seller is confirmed by the inequality (*) below.

$$(*) \pi(P^*) - \pi(\lambda_0) = V_0 / \lambda_0 > 0.$$

It is obvious from (12) that the higher λ_0 is, the lower is P^* , *ceteris paribus*.

This *ceteris paribus* causality between λ_0 and P^* motivates the chieftain to raise the probability of winning, anyhow. Then, if the probability of winning is considered to be increased by maintaining a regular army equipped with iron weapons, the chieftain is driven to transform the existing community into a new social organization with a regular army, which is usually an enlarged social organization to achieve an increase in the bargaining power. As a result, an early state comes into being.

(4.4.2) Foreign Trade with Regular Military Force equipped Iron Weapons

Suppose that the zero-player transforms the existing communities into an early state with regular army equipped with iron weapons. The cost of governing the early state is denoted by G . It is assumed as an increasing function of the state's members, denoted by s , with a slope becoming steeper in accordance with the "as usual" assumption on the cost function of economics. That is, $G = G(s)$, $G' > 0$ and $G'' > 0$.

On the other hand, it is assumed that the probability of winning in conflict is determined by CSF⁴¹, defined by (13).

$$(13) \lambda(s; \theta) = \frac{F(s)\theta_0}{F(s)\theta_0 + \theta_1 V} = \frac{F(s)}{F(s) + \theta V}.$$

It is defined as an increasing function of the "increasing function of s , denoted by $F(s)$," with θ_0 and θ_1 given. The function, $F(s)$, approximates how much effectively a set of the s members of the state are coordinated into a team type of organic body,⁴² where $F'(s) > 0$ and $F''(s) < 0$. These assumptions on $F(s)$ are justified, if we take it into consideration, firstly that the personnel and logistic capacity of a regular army must be backed up by both economic and human power, approximated by the size of society, secondly that the effects of those powers are subject to a gradually-increasing pattern, and finally that how effectively those physical factors can function depends on "how well-organized they are," which is represented by the functional form of $F(s)$.

⁴¹ As to the original concept of the conflict success function, see Skaperdes (1992). The CSF is also taken as a proxy function measuring a sovereign power.

⁴² In other word, $F(s)$ stands for the "coordinated or organized degree of the members of a society argued by Hegel (1924/25), or a combination of the "exchange power" and the "coordination power" by Hardin (1995).

$\theta \equiv \theta_1 / \theta_0$ on the right of (13), where θ_1 and θ_0 stands for the military technology of the seller and that of the buyer, respectively. It is easy to derive the signs of the first and second derivatives of $\lambda(s : \theta)$; $\partial \lambda / \partial s > 0$, $\partial^2 \lambda / \partial s^2 < 0$; $\partial \lambda / \partial \theta < 0$, and $\partial^2 \lambda / \partial \theta^2 > 0$.

Here, we can re-define λ_0 as $\lambda_0 = \inf_{s \in S} \lambda(s : \theta)$. That is, λ_0 is achieved when s takes the minimum threshold value with θ being given.

By contrast, if there exists some s satisfying $\lambda^* \equiv \text{Sup}_{s \in S} \lambda(s : \theta) \doteq 1$, the buyer can acquire M only at the cost of V_0 . Then, the buyer's payoff, denoted by $\pi_0(\lambda^*)$, approximates to $\{s\alpha f(M) - V_0 - G(s)\}$. Such an extreme case may fit well with the "predatory theory" of the state.

Then, if the offered price, P , is accepted by the buyer, the payoff of the seller, $\pi(P : \alpha, s, M)$, and that of the buyer, $\pi_0(P : \alpha, s, M)$, are defined by (14) and (14)' respectively.

$$(14) \quad \pi(P : \alpha, s, M) = PM.$$

$$(14)' \quad \pi_0(P : \alpha, s, M) = s\alpha f(M) - PM - G(s).$$

On the contrary, if the buyer rejects the offer, the bargaining process falls into conflict and proceeds to the second phase of the bargaining process. It is noted here that in order to be rejected, the following inequality condition,⁴³ $s\alpha f(M) - V_0 - G(s) > s\alpha f(M) - PM - G(s)$, is required but it is essentially the same as (10). Then, the expected payoff of the seller, $\pi(\lambda(\alpha, s, M : \theta))$, and that of the zero player, $\pi_0(\lambda(\alpha, s, M : \theta))$, are defined by (15) and (15)' respectively.

$$(15) \quad \pi(\lambda(\alpha, s, M : \theta)) = (1 - \lambda(\alpha, s, M : \theta))(PM - V) + \lambda(\alpha, s, M : \theta)(-V).$$

$$(15)' \quad \pi_0(\lambda(\alpha, s, M : \theta)) = \lambda(\alpha, s, M : \theta)\{s\alpha f(M) - G(s) - V_0\} \\ + (1 - \lambda(\alpha, s, M : \theta))\{s\alpha f(M) - G(s) - V_0 - PM\}.$$

⁴³ In the case of the early state, it is realistic to assume that $V_0 = V_0(s)$, with $\partial V_0 / \partial s < 0$. Such an assumption is justified, if we take it consideration that the cost of guards can be reduced provided a regular army is standing behind. Since the assumption and the consideration can strengthen the causal logic below, it is for simplicity that the assumption of a constant V_0 is maintained in what follows..

Denoting by $P^{**} \equiv P^{**}(\lambda(\alpha, s, M : \theta))$ the maximum of the offered price which the buyer can accept, it satisfies the equation, $\pi_0(P : \alpha, s, M) = \pi_0(\lambda(\alpha, s, M : \theta))$, or (14)' = (15)', and it is derived as (16) in the end.

$$(16) P^{**} \equiv P^{**}(\lambda(\alpha, s, M : \theta)) = V_0 / \{\lambda(\alpha, s, M : \theta) \cdot M\}.$$

Whether P^{**} is optimal for the seller is examined by substituting (16) into (14) and (15) and then by comparing the results. The optimality is proved by deriving the following equality; $\pi(\lambda(\alpha, s, M : \theta)) - \pi(P : \alpha, s, M, \lambda_0) = V + V_0 > 0$, for $P = P^{**}$.

When the buyer accepts P^{**} , then, his payoff function is defined by (17) which is derived from substituting (16) into (14)' or (15)'.

$$(17) \pi_0(P^{**} : \alpha, s, M) = s\alpha f(M) - V_0 / \lambda(\alpha, s, M : \theta) - G(s) \\ = \pi_0(\lambda(\alpha, s, M : \theta)), \text{ for } P = P^{**}.$$

(4.5) Production and Distributions: the First Stage of the Backward Induction

The first stage of the backward induction is also classified into two cases: In the first case, the zero-player is the chieftain and determines the optimal-value set of $\{\alpha, s, M\}$ which maximizes $\pi_0(P)$ defined by (9)' or $\pi_0(\alpha, s, M : \lambda_0)$ defined by (11)' subject to the constraint that $P = P^*(\alpha, s, M : \lambda_0) = V_0 / (\lambda_0 \cdot M)$, with λ_0 given and that the participants' constraints of the farmers are satisfied. In what follows, the derived optimal-set is denoted by $\{\alpha(\lambda_0), s(\lambda_0), M(\lambda_0)\}$, and the value of $P^*(\alpha, s, M : \theta)$ substituted by those optimal variables is denoted by $P^*(\alpha(\lambda_0), s(\lambda_0), M(\lambda_0) : \lambda_0) \equiv P^*(\lambda_0)$.

In the second case, the zero-player derives the optimal-value set of $\{\alpha, s, M\}$ by maximizing $\pi_0(P : \alpha, s, M)$ for $P = P^{**}(\lambda(\alpha, s, M : \theta))$ defined by (14)' or $\pi_0(\lambda(\alpha, s, M : \theta))$ for $\phi(s) = P^{**}(\lambda(\alpha, s, M : \theta))$ defined by (15)', both of which result with (17) subjected to the constraint that $P^{**}(\alpha, s, M : \theta) = V_0 / \{\lambda(s : \theta)M\}$, and that the participants' constraints of the farmers are met.

In what follows, the derived optimal-value set is denoted by $\{\alpha(\theta), s(\theta), M(\theta)\}$ and the value of $P^{**}(\lambda(\alpha, s, M : \theta))$ substituted by those optimal values is denoted

by $P^{**}(\alpha(\theta), s(\theta), M(\theta) : \theta) \equiv P^{**}(\theta)$.

It seems apparent from (16) that if M were set at a given value, then $\partial P^{**} / \partial s < 0$, in any case. This “ceteris paribus causality” between P^{**} and s may lead the zero-player to conjecture that he can make the better terms of trade by an increase in the bargaining power, which requires the transformation of the existing community into an enlarged political organization. His conjecture turns out right, as proved in *Appendix 2*. That is, even if not only the direct but the indirect effects of s on M are taken into consideration in making the decision at the first stage, the positive effects of an increase in the size of society on the better terms of trade are preserved. The main results of the backward induction are summarized in *Proposition 2*. See Ueda (2011b) as to the mathematical proofs.

Proposition 2

Under the condition that $C'(s) \geq K(M)/s^2$, (i), (ii) and (iii) hold.

- (i) $\partial M / \partial s > 0, \partial \alpha / \partial s < 0$.
- (ii) $\partial s(\theta) / \partial \theta > 0, \partial \alpha(\theta) / \partial \theta < 0, \partial M(\theta) / \partial \theta \geq 0$.
- (iii) $dP^{**} / ds < 0, P^*(\lambda_0)M(\lambda_0) > P^{**}(\theta)M(\theta)$.

The condition of *Proposition 2* means that an increase in the marginal cost of the irrigation system locating at the marginal site (s^{th} site), denoted by $C'(s)$, is larger than a decrease in the average cost $K(M)/s$, measured by $K(M)/s^2$, which is brought about by incorporating one more farmer into the irrigation system. This condition is justifiable as far as the cost of constructing a branch canal is in the increase at an increasing rate due to, for example, a drastic increase in the transportation cost.

The first inequality of (i) implies that the size of society is positively related to the demand for iron at the optimal. The second one means that the society size is negatively related to the zero-player’s sharing in annual crops at the optimal.

The three inequalities of (ii) show the effects of a change in the relative military power of the foreign counterpart on the optimal values; $\alpha(\theta), s(\theta)$, and $M(\theta)$. The implication of each sign are obvious.

The first part of (iii) implies that if the existing communities are transformed into an enlarged social organization with a regular army, the terms of trade are made more advantageous to the zero-player. The second part of (iii) means that in spite of more iron being imported, the total payment for them is smaller under the early state than under the preceding community. This means that the negative

effects on the price of imported iron could offset the positive effects on the volume of imported iron.

(4.6) The Rational Foundations of the Early State in Irrigation Society

According to *Proposition 1*, the chieftain of the preceding kin-based community is not motivated to have a military force aimed at keeping domestic economic networks in order. According to *Proposition 2*, however, even if it is maintained at private cost, he may be motivated to have a regular military force provided that he can get more profits from the transaction in the foreign trade by resorting to the military force the net-benefits of whose use could increase by the innovation of metal goods. The increase in the net-benefits is accompanied with an increase in the size of society which represents the quantity aspect of not only territory but also economic and military power. Now, we have arrived at the final stage where we have to examine the truth and objectivity of the main synthetic propositions of this paper.

According to the criteria for judging the objective truth of a synthetic but not analytic proposition, the synthetic propositions, which are derived from unifying or combining the syntheses of various kinds of intuitions and/or categories so as to be subsumed under (or in accordance with) the cognitive frameworks of the pure categories, are “objectively true.” The so-called “test of hypothesis” is already implied in these criteria for judging objectivity and truth, because phenomenon or empirical images are abstracted when they are subsumed under the categorical frameworks. (Such an objectivity and truth of synthetic proposition was called “the possibility of the experience” by Kant.) According to the empirical study of neuroscience, such Kantian criteria for judging the objective truth of a synthetic proposition are supported by the “neural modules” theory, which implies that although when all of those modules are set up has yet to be explicated, Homo sapiens have innately common cognitive-frameworks. This is why regardless of nationality or gender, we think we are persuaded and convinced providing that an opinion or a view is explained along some logical framework. The causality is an example for such a common cognitive framework, let alone sensibility and emotional programs.

However, modern sciences call for revealing explicitly the test of hypothesis as qualification as truth. According to the procedure of the test, first of all some expected hypothesis should be deductively derived from the analysis of the main proposition. Next, the hypothesis has to be corroborated by experimentation or by

reference to relevant empirical data serving as evidence, called the “test.” In what follows in this section, the expected hypotheses are deduced by analyzing the base model. Historical evidence is referred to in the next two sections.

Before deducting the expected hypotheses from the analysis, the three main propositions of this paper are summarized below:

Firstly, the new circumstances in which metal tools were innovated and its application to military force could increase the expected net-benefits of resorting to the military force in the transaction of then-prevailing foreign trade drove the chieftains of the preceding communities to take this opportunity to have much more advantageous position in the foreign trade, i.e., to increase bargaining power in the transaction of the foreign trade by resorting to the military force strengthened by equipping with metal weapons, under the necessary condition that those chieftains had been steadily throughout motivated by self-interests. Thus, an early state came into existence as a result of the innovation of metal tools under the condition that the driving engine-factors had been the selfish motives of those chieftains. The self-interest motive is the economic version of the innate genes’ programs-for-survival

Secondly, therefore, the early state is an “accidental situation” of the society as a substance and should be recognized not as a creature but as the transformation of the preceding kin-based communities into a new social organ which came into being as an effect of adaption to a change in the essential elements of the preceding type of the society. That change, called a new environment or surrounding, was the innovation of metal tools.

Thirdly, the bargaining power of an early state, the increase of which is the direct goal of those chieftains, is a surrogate for the sovereignty and is interdependent on one another’s sovereignty. The quantity category such as the relative economic and military power represented by the relative size of society and the quality category such as the organic degree of a social organization and the relative military technology are the main determinants of the bargaining power.

The first expected hypothesis to be deduced from the analysis of the main propositions is as follows: that when faced with some new surrounding to be adapted to (i.e., faced with the innovation of metal goods), the chieftain prefers an “early state type” of the society to the existing type of the society called the “preceding kin-based community just prior to it,” under the condition that as the main driving-engine factor to bring about a change in the essential elements of the preceding community, he can take this opportunity to satisfy the

innately-programmed self-interested motives by increasing his payoff.

The second expected hypothesis to be deduced from the analysis of the main propositions is as follows: the pursuit of the direct goal of his self-interested motives, i.e., the pursuit of an increase in the bargaining power in the foreign trade, led to the establishment of an early state as the end result, whether conscious of its historical implications or not. This second hypothesis is simultaneously proved in the process of the deductive proof of the first one.

The third expected hypothesis to be deduced from the analysis of the main propositions is as follows: that an increase in the bargaining power is consistent with an increase in the payoff of the chieftain, and therefore, that with given external factors such as opponents' military technology, the actual degree of the sovereignty of an early state is determined so as to meet the condition of maximizing his payoff. The deductive proof of this hypothesis is also addressed in the process of the proof of the first one. Thus, we can concentrate on the deductive proof of the first hypothesis in what follows.

For those purposes just mentioned in the above, it is enough to prove that the inequality (18) or its rewritten form (19) holds. Either way, it leads to the conclusion that the chieftain in the Metal Age prefers an early state to the existing community.

$$(18) \quad \pi_0(P^{**}(\theta)) = s(\theta)\alpha(\theta)f(M(\theta)) - \frac{V_0}{\lambda(\alpha(\theta), s(\theta), M(\theta); \theta)} - G(s(\theta))$$

$$> \pi_0(P^*(\lambda_0)) = s(\lambda_0)\alpha(\lambda_0)f(M(\lambda_0)) - \frac{V_0}{\lambda_0}.$$

$$(19) \quad s(\theta)\alpha(\theta)f(M(\theta)) - s(\lambda_0)\alpha(\lambda_0)f(M(\lambda_0))$$

$$> G(s(\theta)) - \left\{ \frac{V_0}{\lambda_0} - \frac{V_0}{\lambda(\alpha(\theta), s(\theta), M(\theta); \theta)} \right\}.$$

The inequality (19) holds, if a combination of the following three conditions, (i), (ii) and (iii), are satisfied: the condition (i) that an increase in the share of annual harvests, i.e., the left side of (19), is large enough, the condition (ii) that $G(s(\theta))$ is small enough, and finally the condition (iii) that $\lambda(\alpha(\theta), s(\theta), M(\theta); \theta)$ is large enough relatively to a given parameter, λ_0 .

Whilst the left side of (19) means an increase in the "tributes" to the king obtained by transforming the existing community into an early state, the right side means an increase in its net cost. Therefore, both of (18) and (19) mean that in order for the chieftain to prefer an early state to the existing community, the net benefits

to the chieftain of transforming into an early state must be positive. Let's examine the conditions for the positivity of the net-benefits in what follows.

The second term on the right side of (19) is positive because $\lambda_0 < \lambda(\alpha(\theta), s(\theta), M(\theta) : \theta)$. If this positive second term is so large as to cause the inequality (19) to hold always, then the first hypothesis turns out to be proved.

However, when we take it into consideration that $G((\theta))$ is usually large, a change for the better in the terms of trade, defined by $\{V_0 / \lambda_0 - V_0 / \lambda(s(\theta) : \theta)\}$ on the right side of (19), must be so sufficiently large as to satisfy the inequality (19). When the left side of (19) is large enough, the dependence of the required positivity of the chieftain's net-payoff on the better terms of trade is weakened. However, though both the inequality, $s(\theta) > s(\lambda_0)$, and the first part of (i) of Proposition 2 can contribute to the positivity of the left side of (19), the positivity itself of the left side cannot be proved, because how α changes is not obvious. On the other hand, as the left side becomes smaller, the right side must become smaller, too, in order for the net-payoff to continue to be positive.

The general conclusion is as follows: If there exist a range of the set whose elements consist of an increase in the chieftain's share in the annual harvests, the cost of governance and an increase in the bargaining power and which can meet the inequality condition (19), the chieftain prefers the early state to the preceding community. However, if not, for example, if an increase in the chieftain's share in annual harvests is not so large and/or the cost of governance, $G(s)$, is not so small enough as to always assure the inequality (19), an increase in the bargaining power, approximated by $\{1/\lambda_0 - 1/\lambda(\alpha(\theta), s(\theta), M(\theta) : \theta)\}$, must be large enough in order for the chieftain to prefer the early state to the preceding community and therefore, for him to be motivated to transform the latter into the former as the end result.

On the other hand, the actual degree of the bargaining power or that of the sovereignty is relatively determined in the sense that it is determined so as to meet the optimality condition of the maximization of his payoff. However, thanks to his direct motives for its increase, the bargaining power is sure to be higher in the early state than in the preceding community.

5. Application to the Aristocracy and the Ancient Monarchy

In this section, the main propositions derived in the previous sections are showed to be applicable to the polis state of an aristocratic type and to the ancient monarchy of an empire type. The aristocracy took over the early kingship in the

ancient Athens and reconciled with the early kingship in the ancient Roma and in the ancient Japan, and the ancient monarchy of an empire type was established via a federal system comprised of the early states with a hegemonic early king as the center of the federal coalition and in the end resulted in the centralization of the power into a military entrepreneur of an emperor type who could conquer new territories of his own. Though the aristocracy is a political system in the Bronze Age and on the other hand, the ancient monarchy of an empire type is the one in the Iron Age, each type of the state is the transformation of the preceding type of the state into itself caused by the appearance of much riskier surroundings or more profitable opportunity with which it was hard for one state to deal.

(5.1) The Ancient Aristocracy: the “Elected Rex = Basileus” System

After the aristocracy took over the early kingship, the original meaning of *rex* in Latin and *basileus* in Greek were degraded to the status of a military commander entrusted with administrative work and management of religious institutions, elected or appointed by the chieftain-turned aristocrats and approved by other citizen members. Since they were usually a lifetime officer and had some discretionary power due to the informational incompleteness of his military actions, they are often misunderstood as the same as the king in the Modern Age. However, they came on the historical stage when the aristocracy had been overwhelming. It came into being as the result of an endeavor to satisfy two requisites as follows: the first one that the early states had to organize themselves into an enlarged political union for the sake of attaining the larger economies of scale in the process of adapting to new external circumstances common to all of them such as Persian, Carthaginian and Chinese threat, and the second one that the effective use of military force depends crucially on personal competence for military leadership. That is, some part of the functions taken by the kings of the early states were separated from them and entrusted to a new *rex* or a new *basileus* when the personal competence became crucial. The cost to the new *rex* or *basileus* of organizing a military army was financed by customs duty imposed on imported goods, though the core personnel of army corps were comprised of the aristocrat family members and other citizens engaging in commerce or independent farming. In this subsection, I show the main propositions of this paper are applicable to this new *rex* system or the new *basileus* system under the aristocracy (hereafter, the *rex system*, for short).

I begin with the situation where h early states similar to each other as to the number of members and the level of productivity are separately engaged in an external trade with a new dominant foreign counterpart common to all of them. The early king i pays P for one unit of iron he acquires, denoted by M_i . He provides M_i for s_i farmers in exchange for sharing harvests the percentage of which is fixed at a given α . The probability of winning in the conflict which may arise in the process of the transaction is denoted by λ_0 , assumed as a given parameter. The optimal value of M_i and of s_i are determined by the same mathematical procedure as the previous sections, denoted by $M_i(\lambda_0) \equiv M_0$ and by $s_i(\lambda_0) \equiv s_0$, for $i = 1, 2, \dots, h$. Here, the new notations, $M(\lambda_0)$ and $s(\lambda_0)$, are defined as the following: $M(\lambda_0) = h \cdot M_0$ and $s(\lambda_0) = h \cdot s_0$. Then, the optimal payoff of the early king i is defined by (20).

$$(20) \pi_i(s_0, M_0 : \lambda_0) = s_0 \alpha f(M_0) - \frac{V_0}{\lambda_0}, \quad i = 1, 2, \dots, h.$$

In the above, I note that $P^*(\lambda_0) \cdot M_0 = V_0 / \lambda_0$.

Next, suppose that a rex is appointed by the early kings and entrusted with arranging the foreign trades and foreign affairs in which those early kings have been separately involved. The rex serves not only as a negotiator in the foreign trade but also as a military officer in charge of keeping the foreign trade in order. He bears the cost of doing these regular tasks, denoted by $G_0(M)$. It is assumed that $G_0' > 0$ and $G_0'' > 0$. He finances the cost to do those tasks by imposing customs duty on imported goods, represented by iron. The customs duty is denoted by β per unit of iron.

On the other hand, those h early kings make a political coalition comprised of $h \times s_0$ members excluding those h early kings under the assumption that each member state has the same member-size. When they join in the enlarged political union, each early king incurs the cost of governing this enlarged union in proportion to the member size of the early state he has been governing. It is denoted by $G_i(s_0) \equiv G(s_0)$, for $i = 1, 2, \dots, h$. He has to pay the customs duty in addition to the price of iron, in proportion to the share of iron they obtain, $1/h$.

Then, the payoff function of the rex, denoted by $\Pi_0 \equiv \Pi_0(\beta, M : \theta, h s_0)$ and that of the early king i , denoted by $\Pi_i(\beta, M : \theta, s h_0)$, $i \in \{1, 2, \dots, h\}$, are defined by (21) and (22), in turn.

$$(21) \Pi_0(\beta, M : \theta, hs_0) = \beta \frac{V_0}{\lambda(hs_0 : \theta)} - G_0(M).$$

$$(22) \Pi_i(\beta, M : \theta, hs_0) = s_0 \alpha f(M/h) - (1 + \beta) \frac{V_0}{h\lambda(hs_0 : \theta)} - G(s_0), \quad i = 1, 2, \dots, h.$$

Note that the total value of imported iron is $PM = V_0 / \lambda(hs_0 : \theta)$, and that those h early kings share the imported iron, M , on an equal basis.

Under the prototypical new rex system, the early kings maximize their payoffs subject to the participant constraint of the new rex. The process of the optimal decision made by those early kings is formalized by the Lagrangian (23) and its constraints (23)'.

$$(23) \varphi(\beta, M : \theta, hs_0) = \sum_{i=1}^h [\alpha s_0 f(M/h) - G(s_0) - \frac{1}{h} (1 + \beta) \frac{V_0}{\lambda(hs_0 : \theta)}] + \mu [\frac{\beta V_0}{\lambda(hs_0 : \theta)} - G_0(M)].$$

$$(23)' \mu \geq 0, \Pi_0(\beta, M : \theta, hs_0) \geq 0, \quad 0 < \beta < 1, \text{ and } 0 < M.$$

Firstly, the necessary condition for β is shown by (24).

$$(24) \frac{\partial \varphi}{\partial \beta} = - \sum_{i=1}^h [\frac{V_0}{h\lambda(hs_0 : \theta)}] + \mu \frac{1}{h} \frac{V_0}{\lambda(hs_0 : \theta)} = 0.$$

Taken it into consideration that $1/h$ is the share of iron allocating to the early king i , it is obvious that $\mu = h > 0$.

Secondly, taking the positivity of μ into allowance, the necessary condition for μ is shown by (25).

$$(25) \frac{\partial \varphi}{\partial \mu} = \frac{\beta V_0}{\lambda(hs_0 : \theta)} - G_0(M) = 0.$$

By arranging (25), the optimal value of β , denoted by $\beta(\theta)$, is derived from (26) below.

$$(26) \beta(\theta) = G_0(M) \frac{\lambda(hs_0 : \theta)}{V_0}.$$

Taking it into consideration that $G_0(M)$ is assumed as an increasing function of M and its coefficient in (26) is given, the necessary condition (26) implies that β has to move in the same direction as M . That is, if the early kings want to acquire more iron, they have to pay more to the new rex in the form of an increase in the customs duty.

Thirdly, from the necessary condition for M with the given relation, $\mu = h$,

Eq.(27) is obtained.

$$(27) \quad G_0'(M(\theta)) = \frac{1}{h} \alpha s_0 f'(M(\theta)/h).$$

In the above, the optimal value of M , denoted by $M(\theta)$, is determined so as to equate the marginal cost to the new rex of acquiring $M(\theta)$ to a marginal increase in the annual tributes to each early king which is brought about by a marginal increase in the acquisition of iron. .

Each early king prefers joining in this enlarged political coalition, if the inequality condition (28) is satisfied.

$$(28) \quad \Pi_i(\beta(\theta), M(\theta) : hs_0, \theta) > \Pi_i(s_0, M_0 : \lambda_0), \quad i \in \{1, 2, \dots, h\}.$$

From the arrangement of the inequality (28), the inequality (28)' is derived.

$$(28)' \quad \alpha s_0 [f(M(\theta)/h) - f(M_0)] - G(s_0) > - \left[\frac{V_0}{\lambda_0} - \frac{1}{h} \{1 + \beta(\theta)\} \frac{V_0}{\lambda(hs_0 : \theta)} \right].$$

The left side of (28)' is the net increase in the tributes to the early king i of joining in the enlarged political coalition. On the other hand, the right side is the decrease in the amount of payment for the imported iron. If the winning probability, $\lambda(hs_0 : \theta)$, can be made so large as to make the right side of (28)' negative and therefore to satisfy the inequality (28)' by forming the enlarged political union, then, each early king prefers joining in it.

On condition that $G(s_0)$ is not so small, however, $\lambda(hs_0 : \theta)$ has to be large enough in order for the relation (28)' to hold, leading to the same conclusion as the first hypothesis to be inferred and deducted from the first of the main propositions of this paper.

As to the influence of the new *rex*, it is inferred from (27) that the more competent the *rex* is, measured by $G_0'(M)$, the higher is the optimal volume of imported iron. This explains why the rex had to be entrusted with those special tasks common to all of the early kings. This conclusion is reinforced, if the technical effects of iron on the agricultural productivity, measured by $f'(M(\theta)/h)$, are more increased.

(5.2) The Ancient Monarchy of an Empire Type

If the political processes of building the ancient monarchy of an empire type are ignored, then, in spite of its opposite image the ancient monarchy of an empire type is subsumed under the constitutional monarchy in the sense that a lifelong ruler is

entrusted with the exercise of power subjected to the participants' constraints of the state's members. Making use of the notations of the previous subsection, *mutatis mutandis*, the decision-making of the ancient monarch is formalized by the Lagrangian (29) and its constraints (29)'.

$$(29) \quad L(\beta, M : \theta, hs_0) = \beta \frac{V_0}{\lambda(hs_0 : \theta)} - G_0(M) \\ + \eta [\alpha s_0 f(M/h) - G(s_0) - \frac{1}{h} (1 + \beta) \frac{V_0}{\lambda(hs_0 : \theta)} - A].$$

$$(29)' \quad \eta \geq 0; \quad \pi_i(\beta, M : \theta, s_0) \geq A; \quad 0 < \beta; \quad 0 < M.$$

In the above formulation, the rex in the previous subsection was replaced with the ancient monarch. He is required to meet the participants' constraints of the kings of the early state, which are denoted by A . In return for resigning the early kingship they had in the preceding early state, those kings secure themselves of the privileged status of an aristocrat in the ancient monarchy, even if their social status changed to the bureaucrats of the monarchy system.

First of all, from the necessary condition for β , we can derive the result that $\eta = h > 0$.

Next, by inserting that result to the necessary condition for M , (30) is obtained.

$$(30) \quad G_0'(M) = \alpha s_0 f'(M/h).$$

Finally, from the necessary condition for η , (31) and its rewritten version (31)' are derived.

$$(31) \quad \alpha s_0 f(M/h) = G(s_0) + \frac{1}{h} (1 + \beta) \frac{V_0}{\lambda(hs_0 : \theta)} + A.$$

$$(31)' \quad \beta = \frac{h[\alpha s_0 f(M/h) - G(s_0) - A]}{V_0 / \lambda(hs_0 : \theta)} - 1.$$

From comparing (30) with (27), it follows that the optimal value of M under the monarchy is larger than under the rex system. Therefore, economic welfare in terms of production level is better under the monarchy than under the rex system.

On the other hand, though (31)' and (26) are relevant to the customs duties, it is uncertain whether those duties are heavier in the monarchy or not. This is because the participants' constraints are changed from those of the rex to those of the former early kings.

Since A is the participants' constraints of those h kings, it may be assumed that

$$A \geq s_0 \alpha f(M_0) - V_0 / \lambda_0 \equiv \pi_i(s_0, M_0 : \lambda_0), \quad i \in \{1, 2, \dots, h\}.$$

In the above, the identity part on the right side is defined by (20), *mutatis mutandis*. The above inequality at least assures the former early king of the payoff obtained in the preceding early state.

Comparing the above inequality with (31), it is shown that the hypothesis inferred from the analysis of the above model of the ancient monarchy leads to the same conclusion as the first of the main propositions of this paper.

Furthermore, from the comparative analysis of (31) as to A , we can derive the results as follows: $\partial M / \partial A > 0$, and $\partial \beta / \partial A < 0$. The former means that the more restricted the participant's constraints of the former early kings denoted by A are (that is, the larger is A), then the higher the level of economic activity is in terms of M . The latter means that the larger is A , the lower the customs' duty is. By inserting these results into the payoff function of the ancient monarch, it turns out that the larger A is, the smaller his payoff is. Under the constitutional monarchy subjected to participant's constraint, therefore, the payoff of the ancient monarch may fall to the minimum level. The lower level of his payoff promotes political stability.⁴⁴

A tyrant may take over the monarch, when a ruler gripping the power of a state takes little care of the participants' constraints on the monarch. Then, the tyrant's optimal decision is defined by (32).

$$(32) \text{ Max. } \pi_0(\beta, M : \theta, s(\lambda)) = \beta \frac{V_0}{\lambda(s(\lambda) : \theta)} - G_0(M),$$

$$\text{s.t., } 0 < \beta, \text{ and } 0 < M.$$

From the first necessary condition of (32), it is obvious that $\partial \pi_0 / \partial \beta > 0$, and $\partial \pi_0 / \partial M < 0$. That is, the signs of those two derivatives demonstrate that the tyrant is intent both on maximizing tax revenues and on minimizing his duties.

6. Application to the Feudal System of the Middle Ages and the Absolute Monarchy of the Early-Modern Times

In this section it is shown that the main propositions are applicable to the feudal system of the middle age and to the absolute monarchy of the early-modern times in the analytical framework of the fifth section, *mutatis mutandis*.

⁴⁴ The mathematical procedure in subsection (2.4) is applicable to the proof of the political stability of the ancient monarchy.

(6.1) The Analytical Framework: A Three-Stage Game

The analytical model of each section in what follows is comprised of a three-stage game. At the first stage, the members of a society form a network as the *ex ante* condition for achieving any common interest. This process is formalized by a variant of the “link and claim” game. At the second stage, “persons in power” determine the optimal levels of economic variables so as to maximize their payoffs subject to the participation conditions of other members, under the assumption that at the next (third) stage, a necessary good crucial to production is acquired through an external trade carried out under anarchy without any common enforcer. The necessary good is represented by “iron” *en masse*. At the third stage, the necessary good is acquired via bargaining with a trading counterpart foreign to the society in the sense that the trade is carried out under anarchy mentioned above. The price of the necessary good is determined via the bargaining between the persons in power and a foreign counterpart. The bargaining-power is formalized by the conflict success function which is also used as a surrogate for the sovereign power of a state. The whole process of the transaction in foreign trade is formalized by a two-stage bargaining game.⁴⁵

(6.2) A Simple Model of the Feudalism

Suppose that there are $h + 1$ feudal societies. They are numbered $0, 1, 2, \dots, h$, in turn and a set denoted by H the elements of which are those feudal societies, is defined as $H = \{0, 1, 2, \dots, h\}$. Each one of those feudal societies is comprised of a lord, peasants and touring tradesmen. The lord keeps markets in order and provides peasants with necessities for agricultural production, represented by iron *en masse*. The revenues of the lord come from commodity taxes and farm rents. In what follows, we abbreviate the process of distribution and assume that the lord directly purchases the iron from the tradesmen and lends it to the peasants. Therefore, the lord pays the net price of iron to the tradesmen. If the feudal lord i pays to the touring tradesmen, the net price is denoted by P_i , defined as the market price excluding the commodity tax. He pays P_i per unit of the iron M_i he procures via

⁴⁵ Both of the second and the third stage were already examined. In this section, the main results and propositions derived from it are quoted without re-examination to save space.

markets opened inside of his territory. He provides M_i for s_i members of his society in exchange for sharing their agricultural products, the percentage of which is denoted by α_i . The probability of winning when the process of bargaining with the touring tradesmen falls into conflict is denoted by λ_0 , assumed as a given parameter.⁴⁶ The optimal value of M_i and of s_i are determined so as to maximize the payoff of the lord subjected to the participation conditions of the peasants.

If all of the feudal societies are assumed to be the same in terms of size, production and the share of harvests, the optimal value of s_i and that of M_i are denoted by $s_i(\lambda_0) \equiv s_0$, $M_i(\lambda_0) \equiv M_0$, and $\alpha_i = \alpha$, $\forall i \in H$. Here, new notations, if required, $M(\lambda_0)$ and $s(\lambda_0)$, are defined as the following; $M(\lambda_0) := h \cdot M_0$ and $s(\lambda_0) := h \cdot s_0$.

The optimal price is determined in accordance with the logic of the bargaining-power theory of the state. It is derived as the equation, as following:

$P^*(\lambda_0) = V_0 / [\lambda_0 M_0]$ in which λ_0 and V_0 are respectively a given probability of the lord's winning in conflict and the cost to cope with the conflict on the spot. They should be taken into allowance when the process of bargaining with the tradesmen falls into conflict. Then, the payoff of the lord i is defined by (1).

$$(6-1) \pi_i(\lambda_0) \equiv \pi_i(s_0, M_0 : \alpha, \lambda_0) = \alpha s_0 f(M_0) - V_0 / \lambda_0, \quad i \in H.$$

In the above (6-1), s_0 , M_0 and $f(M_0)$ denote the optimal size of the i feudal society in terms of peasants' number, the one of procured iron, and that of annual production as a function of iron, in turn.

Here, suppose that at some point of time in history the lord numbered *zero* allies with the tradesmen of a new type who venture on an ocean voyage business. He can maintain a much stronger standing army at his own cost, because the cost can be financed by customs duties on the ocean voyage business. In the political arena within those feudal societies, the stronger army he can maintain, the wider territories and the more people he can rule over via conquest or bargaining under threat, and *vice versa*. In the business arena of the ocean voyage trade, the more powerful army he can maintain, the more iron he can obtain at the lower price. The optimal price of iron is derived as $P^{**}(\lambda(\theta)) = V_0 / [\lambda(s(\theta) : \theta) \cdot M(\theta)]$, where θ stands

⁴⁶ The touring tradesmen are actually armed and have options to flee from the territory.

for the relative advantage of the foreign trading-counterpart in military technology, and $\lambda(s(\theta)) \equiv \lambda(\theta)$ is the probability of the lord's winning when the process of a bargaining in the ocean voyage business falls into conflict. The probability, $\lambda(s(\theta))$, is assumed to be determined in accordance with the idea of the conflict success function (C.1) below.

$$(C.1) \lambda(s(\theta)) = F(s)/[F(s) + \theta]; \quad F'(s) > 0, F''(s) < 0.$$

In the above, $F(s)$ is an increasing function of s , approximates the military power of the *zero* lord, and s denotes the size of the members of a society over whom the *zero* lord can control and stands for a combination of economic power with man power. On the other hand, the functional form of $F(\bullet)$ expresses how much organic way the political system is set up. The more organic the political system is, the steeper the slope of the function is and the more slowly the slope declines. For example, it may be assumed that $F(s) = s^\Delta$ where Δ , $0 < \Delta < 1$, is a parameter-surrogate for expressing the potentiality of an absolute king in power to be entitled to be justified.

Then, the payoff of the zero-lord at the optimality $\pi_0(\theta)$, is defined by (2).

$$(6-2) \pi_0(\theta) \equiv \pi_0(s(\theta), M(\theta); \alpha, \theta) = \alpha s(\theta) f(M(\theta)) - V_0 / \lambda(s(\theta)) - G(s(\theta)).$$

In the above, $G(s)$ is the cost of governance including the standing army, assumed to be a progressively-increasing function of the size of the society.

The optimal size of a territory ruled over by the *zero* lord is determined, in general, so as to equate an increase in the scale merits achievable by the ocean business to an increase in the cost of incorporating other feudal societies into his realm. What follows begins with the case where an increase in the scale merits continues to exceed the marginal cost. When the *zero* lord rules over those feudal societies in any way and organizes them into a state, called the "nation state" under the absolute monarchy, he becomes an absolute monarch of the early-modern times.⁴⁷

⁴⁷ Strictly speaking, the absolute monarchy of the early-modern times is not subsumed under the nation state in the sense that the power does not originate in the people comprised mainly of the bourgeois, the land-owner farmers and the manager class. On the other hand, as long as the power is formally shared with a part of the people, i.e., in particular with the bourgeois and the land owner-farmers, the constitutional monarchy with the authority despite the power appearing to

(6.3) A Simple Model of the Absolute Monarchy in the Early-Modern Times

In this section the base model of an absolute monarchy is set up, and its political stability is examined. Strictly speaking, the absolute monarchs are classified into European type and Japanese type. The former is entitled to be justified by religious authority but the latter is by the descendant family of the ancient monarch (the early king) in spite of possessing also the authority of some traditional religions. Furthermore, both types of the authority also share the total net payoffs with the members of a state. In what follows, it is assumed that the share of the both types of authority changes in the same direction as that of the absolute monarch, in other word, that the lion's share of the absolute monarch represents not only his share but the authority's one.

(6.3.1) The Basic Assumptions and the Base Model

Now, a king, grown out from the *ex*-feudal lords, exercises the power of a state over the ocean voyage business and reigns over the kingdom. Those *ex*-feudal lords are demoted to the status of a subject in spite of being called an aristocrat. Though the process of transforming from those feudal societies into the absolute monarchy is full of historical episodes such as cunning bargaining and war for conquest, it is sure that an overwhelming military power financed by revenues from oversea business, the king could shorten the process and that the absolute advantage in both economic and military power became possible by taxing the ocean voyage business. In order to emphasize only financial source, we abbreviate the process of coping with the ocean voyage business and assume that the king is directly engaged in the ocean voyage business, and that the sources of his revenues are commodity taxes on the imported iron. The tax rate is denoted by β that is a strategic variable determined by the king. He also serves as a military officer in charge of keeping foreign relations in order, and he has to bear the cost of doing those tasks, denoted by $G_0(M)$, with $G_0' > 0$ and $G_0'' > 0$.

On the other hand, the administration of an *ex*-feudal society, i.e., a province now, is left to those aristocrat-subjects. Each subject obtains iron by paying $(1 + \beta)P$ per

originate in the monarch is subsumed under the constitutional monarchy of the modern-times. Usually, the territory of the absolute monarchy overlaps that of the latter type of the monarchy.

unit to the king and bears the cost of administrating the province of the kingdom. The cost is in proportion to the territory size of each ex-feudal society approximated by the number of the peasants the rule over whom is left to each aristocrat-subject. Since the ex-feudal society was assumed to be the same, the cost of administration is denoted by $G_i(s_0) = G(s_0)$, with $G' > 0$, for $i = 1, 2, \dots, h$.

The Conflict Success Function in this case, $\lambda(s_0 h : \theta)$, is defined by (C.2).

$$(C.2) \quad \lambda(hs_0 : \theta) = \frac{F(s_0 h)\Delta}{F(s_0 h)\Delta + \theta}$$

The parameter on the right side of (C.2), Δ , is a surrogate for the potentiality of the absolute king to be entitled to be justified by the religious or the other traditional authority. It is assumed to be an increasing function of the reservation utilities of the subjects. That is, $\Delta = \Delta(A)$, $\Delta' > 0$, where A is a given reservation utility of an aristocrat-subject. This assumption is justified if taking it into consideration that the higher are those reservation utilities, the smaller the king's share is and then, that the king is recognized to be a reliable king in the sense that he is contributive to furthering the well being of all members of the state. In this sense, $\Delta = \Delta(A)$ may be taken to stand for the disincentives for rent-seeking activities including a revolt for taking power, i.e., the political stability of the absolute monarchy.

Then, under the condition of θ , h , α and s_0 being given, the payoff function of the king, denoted by $\Pi_0 \equiv \Pi_0(\beta, M : \theta, hs_0)$ and that of the subject i , denoted by $\Pi_i(\beta, M : \theta, s_0 h)$, are defined by (6-3) and (6-4), respectively.

$$(6-3) \quad \Pi_0(\beta, M : \theta, hs_0) = \beta \frac{V_0}{\lambda(hs_0 : \theta)} - G_0(M).$$

$$(6-4) \quad \Pi_i(\beta, M : \theta, hs_0) = s_0 \alpha f(M/h) - (1 + \beta) \frac{V_0}{h\lambda(hs_0 : \theta)} - G(s_0), \quad i = 1, 2, \dots, h.$$

In the above (6-3) and (6-4), it should be noted that the total value of imported iron is $V_0 / \lambda(hs_0 : \theta)$, and that it is allocated to those h aristocrat-subjects on an equal basis.

The political system defined in this section is the absolute monarchy in which the hereditary king not only takes the power of a state but also exercises its power subject to the participation conditions of the state's members who are the ex-feudal

lords but his subjects now. The decision making of such a king is formalized by the *Lagrangian* (6-5) and its constraints (6-5)' below.

$$(6-5) \quad L(\beta, M : \theta, hs_0) = \beta \frac{V_0}{\lambda(hs_0 : \theta)} - G_0(M) \\ + \eta [\alpha s_0 f(M/h) - G(s_0) - \frac{1}{h}(1 + \beta) \frac{V_0}{\lambda(hs_0 : \theta)} - A].$$

$$(6-5)' \quad \eta \geq 0; \pi_i(\beta, M : \theta, s_0) \geq A, \text{ for } \forall i \in H \setminus \{0\}; 0 < \beta; 0 < M.$$

In the above (6-5)' the parameter, A , stands for the reservation utility of the ex-feudal lord i , which is approximated by the payoff of the feudal lord, $\pi_i(\lambda_0)$.

First of all, from the necessary condition for β , we can derive the result that $\eta = h > 0$. Next, by inserting this result to the necessary condition for M , (6) is obtained.

$$(6-6) \quad G_0'(M) = \alpha s_0 f'(M/h).$$

Finally, from the necessary condition for η , (6-7) and its rewritten version (6-7)' are derived.

$$(6-7) \quad \alpha s_0 f(M/h) - G(s_0) - \frac{1}{h}(1 + \beta) \frac{V_0}{\lambda(hs_0 : \theta)} = A.$$

$$(6-7)' \quad \beta = \frac{h[\alpha s_0 f(M/h) - G(s_0) - A]}{V_0 / \lambda(hs_0 : \theta)} - 1.$$

Since A is the participant's constraint of the ex-feudal lord, the inequality (6-8) holds if it is taken into account that $\eta = h > 0$.

$$(6-8) \quad A = s_0 \alpha f(M_0) - V_0 / \lambda_0 \equiv \pi_i(\lambda_0), i \in \{1, 2, \dots, h\}.$$

The right hand identity in (6-8) was defined by (6-1). The equality (6-8) assures each subject of at least the payoff obtained under the ex-feudal society. From comparing (6-7) with (6-8), it is derived that those ex-lords support the absolute monarchy based on the calculus of their well being. Moreover, since the sovereignty is represented by $\lambda(hs_0 : \theta)$, the comparison of (6-7) with (6-8) derives the result that the higher sovereignty, i.e., the common interest of the common wealth, is supported by those ex-feudal lords. This result is consistent with the logic of Hobbes arguing for the justification of the absolute monarchy.

From the comparative analysis of (7) as to A , we can derive that $\partial M / \partial A > 0$, and $\partial \beta / \partial A < 0$. The former means that the higher the reservation utility of the ex-lord is, the higher the economic activity of the kingdom is in terms of M , and the latter means that under the same condition the customs duty is lowered.

By inserting those results into the payoff function of the king, it turns out that the larger A is, the smaller his payoff is. Under the absolute monarchy, therefore, the payoff of the monarch may fall to the minimum level provided that the ex-lords can increase their reservation utility, A , to as high a level as possible. How much constrained the absolute monarch is depends on how much of the power he can exercise. And the power of a state he can exercise is limited by the reservation utility of the subjects, approximated by A .

Though it may be a digression, I note that the absolute monarchy should be distinguished from tyranny. It is the historical evidence that the tyranny came out from democracy. But the behavior of a tyrant can be formalized as a special case of the absolute monarch in the sense that the participation conditions are taken little of. Then, the tyrant's optimal decision is defined by (6-9).

$$(6-9) \text{ Max. } \Pi_0(\beta, M : hs_0, \theta) = \beta \frac{V_0}{\lambda(hs_0 : \theta)} - G_0(M),$$

st., $0 < \beta$, and $0 < M$.

From the first necessary condition of (6-9), it is obvious that $\partial \pi_0 / \partial \beta > 0$, and $\partial \pi_0 / \partial M < 0$. That is, the signs of those two derivatives show that the tyrant is intent both on maximizing tax revenues and on minimizing the productive duties of a ruler.

(6.3.2) The Problem of Political Stability

The process of establishing an absolute monarchy can be formalized by applying the analytical framework of the "link and claim" network-game. The players consist of a monarch, called the *zero* layer, and the h subjects, called the player i , $i \in H \setminus \{0\}$. Cooperative outcomes can be generated, only when those players can make a team-type of coalition in which they are directly or indirectly connected via forming links among them. Those links are formed by way of playing a non-cooperative game, as follows: The strategies of each player are defined by a set consisting of $(h+1)$ elements. The j element in the strategy set of i player,

denoted by $a_{i,j}$, $i \neq j$, is the payoff which, if the player i intends to form a link with the player j , the latter assures the former of claiming for. The notation " x " denotes "no intention to form a link." By definition, x is sure to be put on the $(i+1)th$ element of the strategy set of the i player.

Since the absolute monarch does not so deadly do a fatal blow as to extinguish the ex-feudal lords due to the cost-benefit consideration in spite of his relative advantage in military power, the process of establishing the absolute monarchy, at some stage, proceeds to bargaining for contracts, if any, after conquest over some weak rivals. This bargaining process is formalized in the analytical framework of a "link and claim" game defined below.

The strategies of the zero-player, w_0 , and those of the i player, w_i , are denoted below.

$$w_0 = (x, a_{01}, \dots, a_{0h}),$$

$$w_i = (a_{i0}, a_{i1}, \dots, a_{i,i-1}, x, a_{i,i+1}, \dots, a_{i,h}), \text{ for } i = 1, 2, \dots, h.$$

Only when $a_{ij} \cdot a_{ji} > 0$, a link between i and j can be formed. Then, the payoff of the j player, denoted by $\pi_j(w_0, w_1, \dots, w_h)$, is defined below.

$$\pi_j(w_0, w_1, \dots, w_h) = \sum_{i \in H \setminus \{j\}, a_{ij} \cdot a_{ji} > 0}^h \{a_{ji} - c(\Delta)\}$$

In the above, $c(\Delta)$ on the right side is the cost to communicate in forming a link and to maintain it after formation. It is assumed to be a decreasing function of the potentiality of the absolute monarch in power to be entitled to be justified by religious or other authorities, denoted by Δ . This assumption is justified, since under the justified monarchy, interrelationship among the subjects are believed to continue for a long period and the society is believed to be kept in order.

The game is feasible, if the following inequality is satisfied.

$$\sum_{j=0}^h \pi_j(w_0, \dots, w_h) \leq v(H).$$

$v(H)$ on the right side of the above inequality is the value of a coalition of H .

As a special case, take up the optimal payoffs derived in the subsection (6.3.1), and denote them by $\Pi_0(A), \Pi_1(A), \dots$, and $\Pi_h(A)$, in turn, and set each of them as each player's total claim of the "link and claim" game of this subsection. Denoting

the Nash payoff of the “link and claim” game of this subsection by $\pi_i(A), i \in H$, they are defined in turn, as below.

$$\pi_0(A) \equiv \Pi_0(A) - c(\Delta), \pi_1(A) \equiv \Pi_1(A) - c(\Delta), \dots, \pi_h(A) \equiv \Pi_h(A) - c(\Delta).$$

Furthermore, suppose that those payoffs are obtained by way of forming a star-network with the absolute king as the center player of the star-network. Then, the strategy profile is defined by (6-10) and (6-10)′.

$$(6-10) \ w_0(A) = \{x, 1/\pi_0(A), \dots, 1/\pi_0(A)\}.$$

$$(6-10)' \ w_i(A) = (\pi_i(A), x, \dots, x), \text{ for } i = 1, 2, \dots, h.$$

The network formed by the strategy profile defined by (6-10) and (6-10)′ is a star-network the center of which is occupied by the absolute monarch, representing both him and the authority entitling him the justification. If, thanks to the potentiality of the authority to entitle the absolute monarch in power to be justified, it is hard for any other player to replace the status of the monarch, the payoffs allocated in accordance with those strategies are in the core and the strategy profile is the Nash equilibrium of the “link and claim” game.⁴⁸ Moreover, it is obvious that those payoffs are increased as that potentiality of the authority is higher due to a decline in the cost of forming and maintaining each network.

However, the Nash equilibrium is not necessarily the strong Nash equilibrium of the “link and claim” game, if some other player can take over the status of the absolute king. If and only if the zero-player’s claims on other players are reduced to the minimum, a star-network is formed by the strong Nash equilibrium, the strategy profile of which is defined by (6-11) and (6-11)′.

$$(6-11) \ w_0(\hat{A}) = (x, 0, \dots, 0).$$

$$(6-11)' \ w_i(\hat{A}) = (\pi_i(\hat{A}), x, \dots, x), \text{ for } i = 1, 2, \dots, h.$$

$\pi_i(\hat{A})$ in the right side of (6-11)′ denotes the payoff obtained under the condition that the reservation utility of the subjects, A , is increased to the maximum, \hat{A} .

Then, the above strategy profile defined by (6-11) and (6-11)′ is the strong Nash

⁴⁸ It can be proved along the mathematical procedure of Theorem 3.3 in Slikker and van den Nouweland (2001).

equilibrium of the “link and claim” game, and at the same time, those payoffs are in the core.⁴⁹

If π_0 is large and corresponding to it, $\pi_i, i \in H \setminus \{0\}$, is small, then, some subjects may be motivated to take over the status of the center-player if possible. In order to formalize such a process leading to a political instability, we have to take into allowance the definition of Δ , $\Delta \equiv \Delta(A), d\Delta/dA > 0$. If Δ takes sufficiently a low value due to A being forced to a low level, some rent-seeking activity is encouraged. The lower it is, the larger are the incentives for taking over the status of the absolute king. The sovereignty, represented by $\lambda(hs_0 : \theta)$ defined by (C.2), is also weakened. This logic supports the organic theory of the state as a corollary.

The hereditary system under the absolute monarchy was designed so as to be consistent with the aim of avoiding such a political instability. However, the problem of how to explain the political stability on a rational basis should remain to be solved, so far as we stick to the argument that the monarchy is based on a monopoly in military power.

7. Application to the Nation State under the Constitutional Monarchy

In this section, the constitutional monarchy under the modern bourgeois democracy is formalized and examined in the analytical framework set up in the previous subsection, and its rational foundations are derived.

(7.1) The Basic Assumptions and the Base Model

Under the archetypical constitutional monarchy under the bourgeoisie democracy, the people are represented by the bourgeois and the landowner-farmers, both of whom could play the role of a new economic power to finance the cost of governance. Based on this economic power, they share the power of a state with the constitutional monarch who has also his own economic power regardless of its origins. The constitutional monarchy is classified into two archetypal cases, called the “parliamentary cabinet system” and the “imperial cabinet system.” Though the latter type is represented by the imperial system of Japan or the kingdom of Prussia established in the 19th century, the former in such a pure form does not find its historical equivalence. The British system was an in-between in the sense that the parliament had the right to legislation but the cabinet was appointed by the

⁴⁹ As to the proof, refer to Theorem 4.1. and 4.2. of Slikker and Noeweland (2001).

monarch. In this section, the former archetype is examined as a reference point, and it is assumed that under that political system “those people in power” maximize their payoffs subject to the participant’s constraint of the constitutional monarch.⁵⁰

Now, they procure iron on their own, and leave the tasks of keeping formal external relations in peace to the constitutional monarch. Under this type of a political system, the monarch does not actually exercise the power of a state but entitles those ruling groups in power to be justifiable. Then, the decision-making of the ruling side is formalized by the *Lagrangian* (7-1) and its constraints (7-1)’ with $\theta, h,$ and s_0 being given.

$$(7-1) \quad \varphi(\beta, M : \theta, hs_0) = \sum_{i=1}^h [\alpha s_0 f(M/h) - G(s_0) - \frac{1}{h}(1+\beta) \frac{V_0}{\lambda(hs_0 : \theta)}] \\ + \mu [\frac{\beta V_0}{\lambda(hs_0 : \theta)} - G_0(M)].$$

$$(7-1)' \quad \mu \geq 0, \quad \Pi_0(\beta, M : \theta, hs_0) \geq 0, \quad 0 < \beta, \quad \text{and} \quad 0 < M.$$

Firstly, from the necessary condition for β , (7-2) is derived.

$$(7-2) \quad \frac{\partial \varphi}{\partial \beta} = - \sum_{i=1}^h [\frac{V_0}{h \lambda(hs_0 : \theta)}] + \mu \frac{1}{h} \frac{V_0}{\lambda(hs_0 : \theta)} = 0.$$

Taken it into consideration that $1/h$ is a share in the imported iron allocated to the i subject, it is derived that $\mu = h > 0$.

Secondly, from the necessary condition of μ and from the positivity of μ , (7-3) is derived.

$$(7-3) \quad \frac{\partial \varphi}{\partial \mu} = \frac{\beta V_0}{\lambda(hs_0 : \theta)} - G_0(M) = 0.$$

By arranging (7-3), the optimal value of β , denoted by $\beta(\theta)$, is determined by (7-4).

⁵⁰ Even in the Japanese type of the constitutional monarchy which was designed so that the power of a state originates only in the monarch, his exercise of the power was formally limited to decision-making required for some national emergencies. Decision-making on governance in other fields was required for the consents of ministers in charge, and they were responsible for any result of the exercise of power. The main difference between the Japanese type and the British type is that whilst the latter allowed the legislation body to make laws, the former allowed it only to approve laws proposed by the cabinet appointed by the monarch. The base model set up in this subsection is more applicable to the British type where the power-sharing tilts more toward the people in power than the Japanese type.

$$(7-4) \beta(\theta) = G_0(M) \frac{\lambda(hs_0 : \theta)}{V_0}.$$

Taking it into consideration that $G_0(M)$ is assumed as an increasing function of M and its coefficient in (7-4) is given, (7-4) implies that β has to move in the same direction as M . That is, if the subjects want to acquire more iron, they have to pay more to the monarch in the form of an increase in the custom duty.

Furthermore, (7-4) implies that the optimal payoff of the monarch, denoted by $\Pi_0(\theta)$, is zero. That is, $\Pi_0(\theta) = 0 = \Pi_0(\hat{A})$, the payoff of the absolute monarch defined in the previous section, which is obtained under the condition that the reservation utility of the ex-feudal lord is the maximum.

Thirdly, from the necessary condition of M with $\mu = h$, (7-5) is derived.

$$(7-5) G_0'(M(\theta)) = \frac{1}{h} \alpha s_0 f'(M(\theta)/h).$$

In the above, the optimal value of M , denoted by $M(\theta)$, is determined so as to equate the marginal cost to the monarch of doing the task of keeping external relations in peace, indirectly required for acquiring $M(\theta)$, to a marginal increase in the benefits to each subject of procuring $M(\theta)$.

As to the influence of a monarch's personal competence if any, it is inferred from (7-5) that the more competent he is, measured by $G_0'(M)$, the higher is the optimal volume of the acquired iron.

In the case of the second archetype of the constitutional monarchy, the analytical results derived from the analysis of the absolute monarchy in the previous section are applicable to it, *mutatis mutandis*.

(7.2) The Rational Foundations of the Constitutional Monarchy

A star-network should be formed in order to establish the constitutional monarchy, too, if some common interest should be achieved under the constitutional monarchy. The process of forming the star-network indispensable for the common interest is described along the mathematical procedure of the "link and claims" game defined in the subsection (6.3.2).

In the case of the constitutional monarchy, however, the strong Nash equilibrium strategies, defined by (6-11) and (6-11)', are realizable more easily than the absolute monarchy. That is, the constitutional monarchy is stable in the sense that any member is more unmotivated to destroy this political system. Since, under the

star-network of this type with the constitutional monarch as the center player whose payoff is the minimum level, it is assured that thanks to the highest potentiality of the constitutional monarch to entitle the persons in power to be justifiable, the common interests for the people are increased to the maximum not only in terms of an increase in the payoffs of the people but also in terms of an enhancement in the sovereignty.

The monarch's role as the center of the star-network may be fulfilled by not so talented persons. This means that if qualification as the king is a talent or it is used as the criteria for electing the monarch, some talented person who purports to fulfill the role may try to take over the monarch's status. In particular, in the case of the second archetypical monarchy where the constitutional monarch appears to have more of the power of a state, such an election system is sure to invite a political instability. The hereditary system was established to avoid such instability, and contributes to leading to the situations in which the constitutional monarchy is put in a repeated-game setting, i.e., a lasting institutional setting. This is why the constitutional monarchy is compatible with the bourgeois democracy in spite of those members being a self-interest seeker.

(7.3) Compatibility with the Modern Mass-Democracy

In order to examine the compatibility of the constitutional monarchy with the modern mass-democracy, first of all we have to set up a politico-economic model of the constitutional monarchy under the modern mass-democracy. For this purpose, we have to take at least four points into consideration, as follows:

Firstly, the constitutional monarch has retreated from the main economic arenas which are now occupied with the stock-holders, managing workers and manual laborers of large joint-stock companies in spite of the traditional property owners being not yet small in number. Then, if, as well as the monarch is out of the power of a state, he is freed from the role of a coordinator in the material life of a politico-economy, it raises the question of what can justify the monarchy under the modern mass-democracy.

Secondly, though the above-said retreat of the monarch from the material life is accompanied with the development of capitalism, the capitalism is an unprecedented socio-economic system in the sense that the new ruling groups in capitalism are freed from the traditional duties to care the ruled people outside of a factory or after the expiration of an employment contract. Whilst such freedom from

the duties is justified by the sacredness of “free contracts by free individuals,” the new ruled-people freed from being cared outside of an employment contract are driven to seek the sustenance of their livelihood in the outside of factory. Rent-seeking activities have been an easy way in which many people without a will and an economic means to self-stand on their own foot seek a solution to the problem of survival.

Thirdly, the modern mass-democracy characterized with equal political rights is historically an unprecedented category of political system in the sense that it is set up on the basis of an idea as follows: the power of a state originates in the people consisting of not only those people with some means to share the cost of maintaining a state under that political system but also those without any. According to this idea, even if the exercise of the power is left to political representatives and government bureaucrats, the origin of the power is purported to be the people in general.

However, the power of a state, in particular, the sovereignty, cannot be engendered without coordinating the former type of the people having means and/or incentives for burden-sharing into a team-like organization. Such coordination is a necessary condition to achieve common interests on a nationwide scale, because “an organized crowd” remains disorderly. Therefore, as far as such an organized crowd forms a majority in social decision-making and is allowed to seek a short-sighted self-interest, it is possible for the modern mass-democracy to lead any state ruled under that system into its ruin. Therefore, in order to formalize the politico-economic model of the constitutional monarchy under the modern mass-democracy, we have to begin with re-examining categorically the concept of the state and that of its qualified members. For this purpose, we have to trace back to the relation between the state as the accident and the society as the substance again.

Finally, the modern mass-democracy should be also re-examined from the viewpoint of psychology. This is because, since one of the most vital criteria for justifying the democracy is the equality, the equality itself must be justified. As Plato/Aristotle via Hegel/Nietzsche/Ortega to the modern neuroscience pointed out, it has been common sense that egalitarian claims are a political expression of jealousy, envy, and *ressentiment en masse*. They are the weakest point of human nature but nevertheless, they are the human emotions innately-programmed in the genes of Homo sapiens. When a person falls to a misfortune, these emotional programs, in order to restore homeostasis, drive the unfortunate person to seek the cause in other things except for himself or herself. Thus, *ressentiment*

automatically comes out. Therefore it is a natural inclination for many people to be driven by those emotions, even if all of them know that those emotions should be considered as the ugliest and weakest point of human nature. The French revolution took off the cover which had been capping the box of Pandora confining those emotions, and Karl Marks gave the “scientifically appearing but false” economic theorizing to justify that the poor classes should be driven by those emotions.

Though the constitutional monarch has been losing the power of a state under the modern democracy, the monarch family with an authority can still contribute to promoting common interests indirectly through reducing the cost of coordination for common interests and eventually having the potentiality of authorizing the person in power. If the constitutional monarchy is maintained so as to remedy the above-mentioned weak points of the mass-democracy, the constitutional monarchy is compatible with the modern mass-democracy.

8. Application to the Republic Nation States

In this section it is shown that the main propositions are applicable to the republic nation states under the bourgeois democracy and those under the mass democracy. For simplicity, the latter type of the republic states is characterized with stronger pressure for redistributive policies. The bourgeois in this section specialize in business activities, that is, they procure the means of production (iron) and employ labors but entrust the governance of a state to other members by paying taxes. On the other hand, the labors earn wages paid by the bourgeois and welfare benefits from the government. The governing agents spend the taxes to the welfare benefits and to the hard and soft infrastructure contributing to strengthening the economic and military power in the long run, subject to the constraint that budget deficit must not surpass a given ceiling the absolute amount of which is denoted by A .

(8.1) The Basic Assumptions

The bourgeois engage in the business where iron, M , and labors, s , the total cost of which is denoted by $C(s)$, are employed. The bourgeois payoff, π , is defined by (8-1).

$$(8-1) \pi(s, M : \alpha, \beta, \theta) = (1 - \alpha)f(s, M) - PM - C(s).$$

$$\partial f / \partial s \equiv f_1 > 0, \quad \partial^2 f / \partial s^2 \equiv f_{11} < 0;$$

$$\partial f / \partial M \equiv f_2 > 0, \quad \partial^2 f / \partial M^2 \equiv f_{22} < 0; \quad C' > 0, \text{ and } C'' > 0.$$

In the above, α is a share for tax payment. P is determined by the two-phase bargaining process in the same way as the previous sections. That is, before the nation state is formed, $PM = V_0 / \lambda_0$. On the other, $PM = V_0 / \lambda(s : \theta)$. θ is the relative advantage of technologies contributing to strengthening the economic and military power of this nation state to a foreign counterpart.

The conflict success function is defined by (8-2).

$$(8-2) \lambda(s : \theta) = \theta F(s) / \{\theta F(s) + 1\}; \quad F' > 0, F'' < 0.$$

On the other hand, the payoff of the governing agent is defined by (8-3).

$$(8-3) u(s, M; \alpha, \beta, \theta) = \alpha f(s, M) - \beta C(s) - G(\theta) \geq -A; \quad G' > 0, G'' > 0.$$

In the above, $G(\theta)$ means the government expenditures on the hard and soft infrastructure contributing to an increase in θ . β is a coefficient determining the welfare expenditure which is assumed to be in proportion to wage incomes taken as a surrogate for the standard of living.

Then, the *Lagrangian* of the bourgeois is defined by (8-4).

$$(8-4) L(s, M : \mu, \alpha, \beta, \theta) = (1 - \alpha)f(s, M) - PM - C(s) \\ + \mu\{\alpha f(s, M) - \beta C(s) - G(\theta) + A\}, \\ \text{s.t., } 0 < s; 0 < M; 0 < \mu, \quad \text{and} \\ \alpha f(s, M) - \beta C(s) - G(\theta) + A = 0, \quad \text{given } (\alpha, \beta, \theta).$$

(8-2) The Rational Foundations of the Republic Nation State

In order for the main proposition to hold true, it is necessary for the inequality (8-5) to be met at the optimal.

$$(8-5) \pi(s, M : \alpha, \beta, \theta) > \pi(s, M : \alpha, \beta, \lambda_0).$$

After arrangement, (8-5) is written to (8-6).

$$(8-6) \quad (1-\alpha)f(s^{**}, M^{**}) - (1-\alpha)f(s^*, M^*) > \{C(s^{**}) - C(s^*)\} \\ + V_0 \{1/\lambda(s^{**}; \theta) - 1/\lambda_0\}.$$

In the above, two asterisks denote the optimal values under the nation state and one asterisk does those before it is formed.

Due to the assumption of an incentive of the bourgeois for increasing the bargaining power in the foreign trade, $s^{**} > s^*$. On the other hand, from the analysis of the constraint on the government agent, we can derive the relation (8-7).

$$(8-7) \quad \alpha f_2 \cdot \partial M / \partial s = \beta C'(s) - \alpha f_1.$$

From the above, it is inferred that the larger β becomes *ceteris paribus*, the more likely the positivity of $\partial M / \partial s$ is. That is, as pressure by the labors claiming the welfare expenditure becomes stronger, M moves in the same direction as s .

Therefore, under the condition that the pressure for redistributions is strong enough, the left side of (8-6) is positive. Otherwise, the left hand may be negative. Since the first term of the right side of (8-6) is positive, in order for (8-6) to hold in any case, the second term of the right side must be small enough. That is, $\lambda(s^{**}, \theta)$ must be sufficiently larger than λ_0 .

9. The Main Conclusions

In this paper I proved the applicability of the main synthetic propositions derived from the early state to other types of the states in accordance with the criteria of the Kantian categorical framework for judging the truth and objectivity of the synthetic proposition. Those synthetic propositions are summarized below:

Firstly, the new circumstances in which metal tools were innovated and its application to military force could increase the expected net-benefits of resorting to military force in the transaction of then-prevailing foreign trade drove the chieftains of the preceding kin-based communities to take this opportunity to have much more advantageous position in the foreign trade, i.e., to increase a bargaining power in the transaction of the foreign trade by resorting to the military force strengthened by equipping with metal weapons, under the necessary condition that those chieftains had been steadily throughout motivated by self-interests. Thus, an

early state came into existence as a result of the innovation of metal tools under the condition that the driving engine-factors had been the selfish motives of those chieftains. The self-interest motive is the economic version of the innate programs-for-survival of Homo sapiens.

Secondly, therefore, the early state is an “accidental situation” of the society as the substance and should be recognized not as a creature but as the transformation of the preceding kin-based communities into a new social organ which came into being as the effect of adaption to a change in the essential elements characterizing the preceding kin-based community. The crucial change in the elements, so called a “new environment or surrounding,” was the innovation of metal tools.

Thirdly, the bargaining power, the increase of which is the direct goal of those chieftains, is a surrogate for the sovereignty and is interdependent on one another’s sovereignty. The quantity category such as the relative economic and military power represented by the relative size of society and the quality category such as the organic degree of a political system conceptualized by Hegel (1824/1825) and a difference in military technology are the main determinants of the bargaining power.

The main propositions can be summerrised as follows: when faced with some new surrounding to be adapted to (i.e., faced with the innovation of metal goods), the chieftain prefers an “early state type” of the society to the existing type of the society called the “preceding kin-based community just prior to it,” under the necessary condition that as the driving-engine factor to bring about the transformation into a new type of the society, he can take this opportunity to satisfy the innately-programmed self-interested motives by increasing his payoff. This was proved deductively by analyzing the basic model subsumed under the categorical frameworks of the transcendental philosophy and then, can be corroborated by referring to the political processes of building the early states in the ancient periods.

Based on the main propositions of the early state, the conventional theories of the state were critically examined, leading to the conclusion as follows: it is because they are not explicitly based on the categorical frameworks of the critical philosophy that those discussions on the state have been in a state of disorder up till now.

In other paper (Ueda, 2011a), I already showed the applicability of this paper to the bourgeois democracy, the modern constitutional monarchy and the feudal system. So, this second stage to the last of this paper is considered as a good opportunity to mention again about how to address the problems with the modern

mass-democracy from the view points of this paper. According to the category of the society as the “substance,” the sovereignty is one of the essential elements of the society as the substance and therefore, any type of the state cannot escape from an endeavor to keep the actual degree of the sovereignty at as a high level as possible. However, one of the key factors to determine the actual level of the sovereignty is the “organic degree” of a state.” This is the Hegelian term to express how organically the state is coordinated into one team-like organization. In order to strengthen the sovereignty, the qualified members of the state are required to contribute to strengthening it in any way through some means such as personnel contributions and/or financial ones. According to the categories on the state, those members contributory to the sovereignty should be considered as the “qualified” ones. According to this criterion for the qualification, the modern mass-democracy with universe suffrage may be a digression from the concept of the state in the sense that the qualified members have not been explicitly defined yet. From the viewpoint of the sovereignty, therefore, the modern mass-democracy may be considered as an abnormal situation of the society itself as the substance. That is why it is called the mass-tyrant. As Aristotle, Hegel, Nietzsche, Burke, and Tocqueville said, it is sure to be based on emotional bases such as envy, jealousy, or *ressentiment*. The French Revolution uncovered the Pandora’s box having been confining them and Karl Marx gave those *ressentiment*-driven people the economic reasoning why they *should* be driven by those emotions.

Finally, this paper is concluded by suggesting how to apply the approach of this paper to the modern political integration. The process of the United States of America being formed can be interpreted as a venturesome effort to keep external trades with Europe in as favorable condition as possible. If applied to the possibility of EU being transformed into a federal state, we should examine whether or not external threats common to the main member-countries becomes so serious that they prefer resigning at least some part of the sovereignty of a state to accepting some worse terms of trade for necessities vital for their survival (for example, energies). Such a worsening of the terms of trade is backed up by a difference in the military forth. The modern nation-state came into being as a result of the innovation of industrial technologies under the condition that the motives of industrial capitalists for securing markets in as favorable a condition as possible are the human driving-engine factor. It was a transformation into an enlarged social organ adaptable to the new surroundings. In order to promote the enlarged social

organ to be stabilized, at first the concept of the nation was created as an “imagined community” (Anderson, 1991). As far as there are some economic backgrounds which drive the main member-countries to stick to their own special interests, for instance, in foreign affairs such as African problems, the traditional system of a nation state is more suitable to dealing with those special interests and on condition that the surroundings are not drastically changed, it is impossible to integrate EU countries into a state.

References List

- Alesina, Albert and Spolaore, Enrico (2005). *The Size of Nations*, Cambridge, MA: The MIT Press.
- Anderson, Benedict (1991). *Imagined Communities: Reflection on the Origin and Spread of Nationalism*, London:Verso.
- Arendt, Hannah (1958). *The Human Condition*, Chicago: The University of Chicago Press.
- Aristotle (1946). *The Politics of Aristotle*, translated with an introduction notes and appendixes by Ernest Barker, Oxford: Oxford University Press, 1946.
- Asai, Souichiro (2008). *The History of Ancient Iron-Making* (in Japanese). Tokyo: Sairyuusya.
- Bala, Venkatesh and Sanjeev Goyal (2000). “A Noncooperative Model of Network Formation,” *Econometrica*, Vol.68, No.5, pp. 1181-1229.
- Bellner, Ernest (1983). *Nations and Nationalism*, Oxford: Basil Blackwell.
- Bentham, Jeremy (1948). *A Fragment of Government and An Introduction to the Principle of Morals and Legislation*, edited with an introduction by W. Harrison, Oxford: Blackwell.
- Bodin, Jean (1576). *The Six Books of a Commonwealth*, edited with an Introduction by Kenneth D. McRae, Cambridge, M.A.: Harvard University Press, 1962.
- Buchanan, James M.(1975). *The Limits of Liberty: Between Anarchy and Leviathan*, Chicago: The University of Chicago Press.
- Burke, Edmund (1790). *Reflections on the Revolution in France, and on the Proceedings in Certain Societies in London Relative to that Event*.
- Carneiro, Robert L. (1970). “A Theory of the Origin of the State,” *Science* 169 (3947), pp.733-738.
- Chagnon, Napoleon A. (1974). *Studying the Yanomamö*, New York: Holt, Rinehart

and Winston.

Chomsky, Noam (1965). *Aspects of a Theory of Syntax*, Cambridge, MA: MIT Press.

Claessen, H. and P. Skalik (1978). *The Early State*, Hague: Mouton Publishers.

_____ (1981). *The Study of the State*, Hague: Mouton Publishers.

Demange, Gabrielle (2004). "On Group Stability in Hierarchies and Networks," *Journal of Political Economy* 112 (4), 754-778.

d'Entreves, Alexander P. (1967). *The Notion of the State: an Introduction to Political Theory*, Oxford: Oxford University Press.

Engels, Friedrich (1878). *Herrn Eugen Dührings Umwälzung der Wissenschaft*.

_____ (1884). *Der Ursprung der Familie, des Privateigentums und des Staats—Im Anschluss an Lewis H. Morgan's Forschungen*.

Findley, Ronald (1996). "Towards a Model of Territorial Expansion and the Limits of Empire," In M.R.Garfinkel and S. skaperdas (Eds.), *The Political Economy Conflict and Appropriation*, 41-56. Cambridge: Cambridge University Press.

Finley, M.I. (1978). *The World of Odysseus*, Pelican Books, the second edition.

Friedman, David (1997). "A Theory of the Size and Shape of Nations," *Journal of Political Economy*. Vol. 85(1), pp.59-77.

Frohlich, N., Oppenheimer, J.A, and Young, O.R. (1971), *Political Leadership and Collective Goods*, New Jersey: Princeton University Press.

Grossman, Herschel I. (2003). "Choosing between Peace and War," working Paper.

Gunawardna, R.A.L.H. (1981). "Social Function and Political Power: A Case Study of State Formation in Irrigation Society," in *The Study of the State*, edited by Claessen, H. J. M. and P. Stalnik, Hague: Mouton Publishers, pp.132-153.

Hamilton, Alexander, Jay, John, and Madison, James (1878). *The Federalists*, New York: G.P. Putnum's Sons.

Hardin, Russel (1995). *One for All: The Logic of Group Conflict*, New Jersey: Princeton University Press.

Hegel, G.W.F.(1824/1825). *Philosophie des Rechts nach der Vorlesungsnachschrift K.G.v.Gresheims 1824/1825*, hersusge. V. K.H. Ilting.

Hirose, Kazuo (2003). *The Early State with Keyhole-shaped Japanese Burial Mound of the Tumulus Period* (in Japanese), Tokyo: Kadokawa-Shoten.

Hirshleifer, Jack (2001). *The Dark Side of the Force: Economic Foundations of Conflict Theory*, Cambridge, UK: Cambridge University Press.

Hobbes, Thomas (1651). *Leviathan*, edited by Michael Oakeshott, Oxford: Basic Blackwell, 1946.

- Hume, David (1752). *Political Discourses*, edited by T. H. Green and T. H. Grose in *Essays, Moral, Political and Literary*, Vol.1 & Vol.2.
- Kant, Immanuel (1980). *Immanuel Kant's Logik: Ein Handbuch zu Vorlesungen*. Herausgegeben von Gottlob Benjamin Jaesche, Koenigsberg, Friedrich Niclovius.
- _____ (1787). *Kritik der Reinen Vernunft 2nd Auflage*. *Kants Werke, Akademie Textausgabe*, III, Water de Cruyter & Co.
- Khaldun, Ibn (1377). *The Mugaddimah: An Introduction to History*, reprinted by New York: Pantheon, 1958.
- Kurrild-Klitgaard, Peter and Sevendsen, Gert T. (2003). "Rational Bandits: Plunder, Public Goods and the Vikings," *Public Choice* 117, pp.255-272.
- Locke, John (1690). *Two Treatises of Government*, edited by Thomas P. Pearson, New York: Macmillan Publishing Co., 1985.
- Lowie, Robert H. (1962). *The Origin of the State*, New York: Russell and Russell.
- Lutz, Donald S. (2006). *Principle of Constitutional Design*, New York: Cambridge University Press.
- Machiavelli, Niccollo (1532). *Il Principe (The Prince)*, translated by Quentin Skinner and Russel Price, Oxford: Oxford University Press, 1988.
- Matuki, Takehiko (2007). *The War and the Early State Building of the Japanese Archipelagos* (in Japanese), Tokyo: The University of Tokyo Press.
- _____ and Udagawa, Takehisa (1999). *War System and Foreign Strategy*, (in Japanese), Tokyo: Toyo-shorin.
- McGuire, Martine C. and Mancur Olson (1996). "The Economics of Autocracy and Majority Rule: The Individual Hand and the Use of Force," *Journal of Economic Literature* 34, pp.72-96.
- Mommsen, Theodor (1854). *Römische Geschichte*, Berlin: Weidmannsche Buchhandlung.
- Morgan, Lewis H. (1877). *Ancient Society or Research in the Lines of Human Progress from Savagery throughout Barbarism to Civilization*, New York: Henry Holt and Company.
- Moselle, Boaz and Benjamin Polak (2001). "A Model of a Predatory State," *Journal of Law, Economics, and Organization*, vol. 17,(1), pp. 1-33.
- Muhly, James D. (1995). "Mining and Metalwork in Ancient Western Asia," in *Civilizations of the Ancient Near East*, Vol. 3, ed. by Jack M. Sasson, New York: Charles Scribner's Sons, 1995.
- Murakami, Yasumichi (2007). *The Process of Building the Ancient State and*

- Iron-Making* (in Japanese), Tokyo: Aoki-Shoten.
- Nakashima, Kennichi (1973). *The Development and Decline of the Ancient Mesopotamian Civilization* (in Japanese), Tokyo: Koukura-Shobo.
- North, Douglass C. (1981). *Structure and Change in Economic History*, New York: W.W. Norton and Company.
- Nozick, Robert (1974). *Anarchy, State, and Utopia*, New York: Basic Books.
- Okada, Hidehiro (2008). *The Birth of the Japanese History* (in Japanese), Tokyo: Chikuma-Shobo.
- _____ (2004). *The History of Chinese Civilization* (in Japanese), Tokyo: Kodansya.
- Olson, Mancur (2000). *Power and Prosperity*, New York: Basic Books.
- _____ (1993). "Dictatorship, Democracy, and Development," *American Political Science Review*, vol.87, No.3, pp.567-576.
- _____ (1965) *The Logic of Collective Action: Public goods and the Theory of Groups*, Cambridge, MA: Harvard University Press.
- Oppenheimer, Franz (1926). *The State: Its History and Development viewed Sociologically*, New York: Vanguard Press.
- Ortega y Gasset, Jose (1930). *La Rebelion de las Masas*, in *Obras completas de Jose Ortega y Gasset, Revista de Occidente, (4). Madrid, 1946-1962*.
- _____ (1921). *Espanã invertebrada, ibid., (3)*.
- Ostrom, E.(1965), '*Public Entrepreneurship: A Case Study in Ground Water Basin Management*,' Dissertation Paper, University of California-Los Angeles.
- Plato (1941). *The Republic of Plato*, translated into English with Introduction and Notes, by Francis M. Cornford, Oxford: Oxford University Press, 1941.
- Plutarch (1914-54). *Plutarch's Lives with an English translation* by B. Perlin. 11 volumes, Loeb Classical Library.
- Polanyi, Karl (1977). *The Livelihood of Man*, New York: Academic Press.
- _____ (1963). "Ports of Trade in Early Society," *The Journal of Economic History*, vol.23, no.1, pp.30-45.
- Premack, David and Ann Premack (2003). *Original Intelligence: Unlocking the Mystery of Who We Are*, New York: McGraw-Hill.
- Querido, Chyanda M. (2007). "A Game-Theoretic Approach of War," Paper presented at the First World Conference of Public Choice Society, held at Amsterdam University.
- Rawls, John (2001). *Justice as Fairness: A restatement*, Cambridge, MA: Harvard University Press.

- _____ (1971). *A Theory of Justice*, Cambridge, MA: Harvard University Press.
- Ridley, Matt (1997). *The Origins of Virtue*, London: Penguin Books.
- Riker, William H. (1964). *Federalism: Origin, Operation, Significance*, Boston, MA: Little, Brown and Company.
- _____ (1962). *The Theory of Political Coalition*, Westport.: Greenwood Press.
- Rousseau, Jean-Jacques (1762). *The Social Contract*, translated with a historical and critical introduction and notes by Henry J. Tozer, London: George Allen & Unwin, 1920.
- Sahlins, Marshall (1972). *Stone Age Economics*, Chicago: Aldine Publishing Co.
- Service, Elman R. (1975). *Origins of the State and Civilization*, New York: Norton.
- _____ (1971). *Primitive Social Organization: An Evolutionary Perspective*, the second edition, New York: Random House.
- Skaperdas, Stergios (1992). "Cooperation, Conflict, and Power in the Absence of Property Rights," *American Economic Review* 82 (4).
- Slikker, Marco and Anne van den Nouweland (2001). "A One-Stage Model of Link Formation and Payoff Division," *Games and Economic Behavior* 34, pp. 153-175.
- Spinoza, Baruch (1677). *Tractus Politicus*.
- Tsude, Hiroshi (2005). *A Keyhole-Shaped Japanese Burial Mound of the Tumulus Period and Society* (in Japanese), Tokyo: Hanawa-Shobo.
- _____ (1989). *The Process of Forming the Agricultural Society of Japan*, Tokyo: Iwanami
- Ueda, Yoshifumi (2007). "The Bargaining Power Model of the State," paper presented for the 2007 Conference of EAEPE, held in Port in December 2007.
- _____ (2008). "The Economic Theory of the States," paper presented for the 2008 National Conference of the Japan Public Choice Society, Kansai University, July 5-6, 2008.
- _____ (2009). "The Bargaining Power Theory of the State: A synthesis of Economic Approaches to the Origins of the State," *The Hiroshima Economic Review*, Vol.32, No.3, pp.93-111.
- _____ (2010a). "Passion within Reason: A Hierarchical Model of Decision-Making," paper presented for the 2010 Annual Conference of European Public Choice Society, held in Izmir in April, 2010.
- _____ (2010b). "The State as a Prerequisite of Economic Policy," paper presented for the 2010 Annual Conference of JEPA, in Kyoto.

- _____ (2011a). "The Rational Model of the Constitutional Monarchy," paper presented for the 2011 Annual Conference of European Public Society, University of Rennes, April 2011.
- _____ (2011b). "The Rational Origins of the State: The Accident Theory of an Early-State's Building," *The Hiroshima Economic Review*, 35 (2), pp.37-71.
- Veblen, Thostein (1889). *The Theory of the Leisure Class*.
- Wagner, R.E.(1965), 'Pressure Groups and Political Entrepreneur: A Review Article,' Papers on Non-Market Decision Making, Vol:1, 161-70.
- Weber, Max (1911). *Politics als Beruf*, Berlin: Duncker & Humblot.
- _____ (1924). *Agrarerhältnisse im Altertum*, Tübingen: Verlag von J.C.B. M.
- Wilson, Edward O. (1978). *On Human Nature*, Cambridge, M.A: Harvard University Press.
- Wittfogel, Karl A. (1957). *Oriental Despotism: A Comparative Study of Total Power*, Yale University Press.
- Ziegler, K.(1974). *Cicero, Staatstheoretische Schriften*, Berlin, translated in Japanese by Michio Oka, Tokyo: Iwanami, 1999.