

# Sources of Incumbency Advantage and Disadvantage

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## Abstract

In this paper I first summarize the sources of incumbency advantage and disadvantage. Then by incorporating both effects I construct a simple unified voting model and use candidates' policy position and time allocation between electoral campaigning and legislative work as choice variables to determine their effects on the candidates' strategic decision-making, voters' behavior and electoral outcome. First, the two office-motivated candidates will choose the policy position equal to the median voters' ideal point. Second, the returns from campaigning will diminish over the career of the candidate. The longer-term incumbent will prefer to spend more time on legislative work and less time on electoral campaigning. Even when facing a competitive challenger, the senior incumbent gains little by electoral campaign spending. So, he will respond by spending more time on legislative work. Third, macroeconomic conditions are vital to voters' behavior and electoral outcome. Strong economic conditions favor the incumbent whereas weak economic conditions favor the challenger. And, if economic conditions are realized ex-ante, they will also affect the candidates' strategic decision-making on time allocation.

## 1 Introduction

In democratic states, electoral processes strongly influence public policies, and electoral results allocate the de jure political power among parties. Electoral results are linked to the voters' behavior. Voters are concerned about the policies that the government will implement, so that they will vote for the candidates whose policy promises are closest to their own preferences. Even though candidates face a trade-off between maximizing the probability of winning and minimizing the cost incurred when deviating from their ideal policy positions, the positions candidates choose still depend on their beliefs about their constituents' voting behavior. Yet aside from policy positions, what are the other determinants essential to voting behavior? Beginning in the mid-1960s, the effect of incumbency advantage on the electoral outcome grew substantially in the elections for the U.S. House of Representatives. Thus, since the mid-1970s, many Congressional scholars have devoted their efforts to studying the magnitude, sources of incumbency advantage and its effect on the voters' voting behavior and the electoral outcome (Ansolabehere and Snyder 2002). However, the effect of incumbency advantage did not dominate the 2010 U.S. midterm elections. In these elections, voters attributed the economic recession to the ineffectiveness of the Democratic Party, and opted to shake up the status quo by voting for the Republican Party. Therefore, in contrast to what had been observed more prominently, incumbents might face some disadvantages when running for reelection.

Nevertheless, existing electoral models place a larger emphasis on incumbency advantage and gloss over the importance of incumbency disadvantage. Most of the electoral models focus on the effect of position issues, valence issues<sup>1</sup> and incumbency advantage on voters' voting behavior and electoral outcome. The unbalanced weights on these two issues cannot explain all the real world cases, especially where the challengers overwhelmingly defeat incumbents during an election. For example, Linden (2004) uses a regression discontinuity design to estimate the effect of incumbency on candidates' electoral prospects in the Indian national parliamentary elections from 1951 to 1999. He finds that from 1991 to 1999 due to the higher number of political parties and the number of candidates contested in each constituency, political competition increased in the elections, and Indian incumbents were 14% less likely to win an election than non-incumbents. In addition, one of the most important disadvantages that the incumbent might face is endemic discontent resulting from unfavorable macroeconomic conditions. Molina (2001) explores 328 post-war democratic elections results in 34 countries up until 1995 and demonstrates that due to higher endemic discontent of constituents, electoral attrition of an incumbent party's share of votes and government turnover are more intense and frequent in non-industrialized countries (Latin American and the Caribbean area) than in industrialized countries.

The purpose of this paper is to construct an electoral model that incorporates both incumbency advantage and disadvantage in order to determine their effects on voters' choice and electoral outcome. Firstly, I will summarize different sources of incumbency advantage and disadvantage as identified in the existing literature. Secondly, I build a simple unified voting model with electoral competition between two candidates, using policy positions and candidates' time allocations as choice variables to analyze the decisions of the candidates. The baseline model states that depending on the incumbents valence advantages/disadvantages (*vis-à-vis* political opponents) and the magnitude of his incumbency advantage, the incumbent has strategic incentives to allocate his time differently between electoral campaigning and legislative work. Challengers' strategic entries in the election and time allocation are also affected by the valence issues. From the comparative statics, we can find that due to diminish returns of campaign spending, senior incumbent will prefer to spend more time on legislative work than electoral campaigning. For the same reason, the challenger is more willing to compete with a longer-term incumbent but with relatively weaker charisma. Aside from candidates' characteristics and campaign strategies, macroeconomic conditions shape voters' voting behavior through endemic content/discontent. Strong economic conditions favor the incumbent whereas weak economic conditions favor the challenger. However, if economic conditions can be realized *ex-ante*, the candidates will have enough time to respond and adjust their campaign strategies to intensify/weaken voters' endemic content/discontent. Thus, the effect of macroeconomic conditions on electoral outcomes will be lessened. Combining both incumbency advantage and incumbency disadvantage yields a more comprehensive picture to understand candidates' decision-making and voters' voting behavior.

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<sup>1</sup>Valence issues were first to be addressed and incorporated into spatial model by Stokes (1963). He defined valence issues as those that "merely involve the parties with some condition that is positively or negatively valued by the electorate" (Stokes 1963).

## 2 Literature Review

### 2.1 Incumbency Advantage

Much of the literature in the last thirty years has focused on providing explanations of incumbency advantage. These explanations point to three distinct, but related effects: the quality effect, the signaling effect and the structural effect. The quality effect is that incumbents have, on average, more expertise than challengers.<sup>2</sup> Higher quality of incumbents comes from the electoral advantage that incumbents accrue while they are in office. Compared to challengers, incumbents have more administrative resources (such as staff and office space) allowing them to do more casework for their constituents. Incumbents also have more experience running electoral campaigns. So, it is not surprising that they have better campaign strategies and know how to attract more positive attention. It is also easier for incumbents to access the personal information of constituents and make policy proposals that satisfy the needs of their constituents (Cox and Katz 1996; Ansolabehere et al. 2000). Moreover, a high quality incumbent deters a potentially stronger competitor from entering an election, known as the scare-off effect. As a result, strategic challengers would be less likely to contest an incumbent, which increases the likelihood of the incumbent's re-election (Cox and Katz 1996; Ashworth and Mesquita 2008).

The second incumbency advantage is the signaling effect. Incumbency becomes a label to voters, signaling incumbents' professional ability. Higher media exposure also makes voters more likely to recognize the name and party identification of incumbents and convinces voters of their better ability<sup>3</sup> (Ansolabehere and Snyder 2002; Ashworth and Mesquita 2008). For example, in India prior to 1991, incumbency was more valuable to voters than non-incumbency by virtue of the incumbent's past experience in office.<sup>4</sup>

The third advantage is the structural effect, which states that many things thought to affect re-election rates are unique to incumbents, the most important being patronage effects, seniority, and redistricting. Patronage effects occur when an incumbent rewards supporters by providing more public goods or the use of state resources to the district that favors the incumbent. Another example is when an incumbent does more casework for constituents in order to win their support in future elections (Miguel and Zaidi 2003). A seniority system implies that the length of service determines one's power in an organization. McKelvey and Reizman (1992) argue that power is tied to seniority in the U.S House of Representatives, creating an incentive for voters to select incumbents. Incumbents with seniority have more power to reward their supporters once re-elected. Thus, it is less likely that voters would vote for a new person who would be ranked the lowest in the legislature. Aside from patronage effects and seniority, Cox and Katz (1996) attribute the growing vote margin of U.S House of Representative incumbents from the

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<sup>2</sup>If the voters vote along a party line instead of for the candidate, the incumbent's quality is trivial to the voters and the effect of the incumbency advantage diminishes. Therefore, in countries with more polarized party platforms, with larger partisan tides, or with more unbalanced electorates in their partisan preferences, the quality effect of incumbency advantage is expected to be smaller (Ashworth and Mesquita 2008).

<sup>3</sup>Compared with challengers, incumbents have more opportunities to attend celebrity dinners, attract media attention, and garner photo opportunities with foreign leaders. Therefore, it is not surprising that they have higher media exposure than their opponents.

<sup>4</sup>In the national parliamentary elections before 1991, the Indian National Congress was a single political party dominating the Indian Congress. Incumbents who gained experience under the Congress system would interact with the same system once being re-elected, so their experience was valuable and became a signal to voters that they have stronger organizational ability and adaptability to the Congress (Linden 2004).

1960s -1980s as the outcome of redistricting. After redrawing district lines, new political niches open up, old party organizations become less relevant, and voters' loyalty to a particular party declines. Due to the conflicting voting cues between party and incumbency, the qualities and personal characteristics of the candidates become more important, incumbency advantage plays an important role during the election. Redistricting also benefits incumbents by dividing constituents according to the maximization of their vote shares and the decrease in the potential competition from challengers.

## **2.2 Incumbency Disadvantage**

While the effect of incumbency advantage is well documented in the literature, incumbency disadvantage has only recently appeared in the literature of election studies. In fact, these two effects can be thought to occur in every election and the electoral results depend on which effect dominates the other. There are three disadvantages that incumbents might face during an election: position disadvantage, policy pressure liability and endemic discontent. Position disadvantage is when the incumbent sometimes needs to announce his policy position ahead of challengers during the election, because the incumbent receives more attention by the media or he has to announce his policy position on certain issues in his past service in office. Anderson and Glomm (1992) present a simple spatial model where the candidate cares about not only the probability of winning the election, but also the cost of compromising his policy position to the voters' ideal point. They find that, if the candidates have the same expected charisma, the incumbent has a lower probability of winning the election than the challenger. This is because the challenger has higher flexibility in choosing his policy position and the freedom to locate his policy position between the incumbent and at least 50% of the voters to grab more vote shares.

The second disadvantage of incumbency is policy pressure liability. It occurs in cases in which both the incumbent and the society have strong preferences concerning certain referendums or policy proposals requested by organized groups of citizens, but their preferences are not aligned. Unlike the challengers, who are not required to respond and react to the proposals, the incumbent has to make a policy decision and at the same time faces a trade-off between their own policy preferences and their benefit from being re-elected. If the cost of implementing such a policy is more costly than the payoff from holding office, the incumbent will prefer to forgo re-election and choose not to fully satisfy the demands of voters. It is not surprising that the incumbent will subsequently lose support. Therefore, in such cases, a policy-motivated incumbent will be more disadvantaged than an office-motivated incumbent (Aragons and Snchez-Pags 2010).

The third disadvantage is endemic discontent.<sup>5</sup> When voters feel persistent deprivation, possibly stemming from economic inequality, social disorder or deficient public services, they are forced to live in a state of endemic discontent. Voters' endemic discontent is intensified during economic and political crises. Because they may tend to attribute such discontent to the incumbents and the ruling party and vote for the challengers in the opposing party, the ruling government is less likely to win election (Molina 2001). The current literature focuses on developing countries, because endemic discontent occurs more frequently in relatively poor, insecure, and public-good deficient countries. Take India for example; in the Indian state legislative elections from 1975 to 2003, incumbents were 15% less likely to win an election than non-incumbents, and this effect has increased to 22% after 1991. Uppal (2009) finds that

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<sup>5</sup>Note that endemic discontent is the mirror image of another possible effect of incumbency advantage, which is endemic content. When macroeconomic conditions are good, voters are satisfied with the performance of the incumbents and the ruling party, and vote for them. The ruling government is more likely to win election.

this disadvantage is driven by the endemic discontent of voters concerning the availability of public goods (such as health centers), the low rate of employment, and the low level of per capita income. Another factor that intensifies voters' endemic discontent is elite mobilization where the challengers and opposing party try to increase endemic discontent during an election campaign by using political slants against incumbents in order to increase voter turnout.

### 3 The Baseline Model

#### 3.1 Preferences of Candidates and Voters

Summarizing the previous literature, voters base their voting decision on position issues, valence issues, and endemic discontent/content. In the baseline model, two candidates A and B are running for office. A denotes the incumbent and B denotes the challenger. I assume both candidates are only office-motivated. Thus, I rule out the possibility of the disadvantage of the policy pressure liability that incumbent A might face. In this model, I only consider position disadvantage and endemic discontent. Voters base their voting decision on three factors: position issues, valence issues and endemic discontent/content.

Position issues involve advocacy of candidates' policy position from a set of alternatives. In this model, there is only one issue. Candidates choose their policy positions over the policy space  $X$  to maximize the share of votes. The policy position chosen by incumbent A is represented by  $X_A$  and the policy position chosen by challenger B is represented by  $X_B$ . There is large odd number of voters who have single-peaked preference over the policy space  $X$ , and the ideal policy position of the voter  $i$  is represented by  $X_i$ . There exists a median voter  $m$  for whom the ideal candidate would have the policy position  $X_m$ . The disutility of the median voter  $m$  to vote for the candidate  $r$ , denoted by  $U_m(X_r)$ , is the squared deviation of  $X_r$  from  $X_m$ , such that

$$U_m(X_r) = -[(X_r - X_m)]^2, r = A, B. \quad (3.1)$$

Voters evaluate candidates based not only on their chosen positions but also on non-policy attributes of candidates, which are called valence issues, denoted by  $Q$ . Valence issues differentiate the candidates by their competence or by some conditions that are positively or negatively valued by the electorate. In this model, two effects capture the candidates' competence. One is a charismatic effect, denoted by  $C$ , and the other one is incumbency advantage, denoted by  $Y$ . The charismatic effect,  $C$ , can be viewed as the candidate's charisma, which is his conferred talent, power and some inherent personal characteristics such as integrity, leadership, intelligence or physical appearance. Incumbency advantage,  $Y$ , is the extent that the incumbent has more resources, skills or experience than challengers to resolve an issue. The incumbency advantage comes from the quality effect, the signaling effect and the structural effect. Note that it is important to distinguish these two mutually exclusive concepts: the charismatic effect includes the candidate's inherent characteristics and such personal characteristics does not come from serving in the office whereas the incumbency advantage refers the resources, skills and the abilities that the incumbent accrues only from serving in the office.

For simplicity, both candidates' charisma and the incumbent's advantage are fixed attributes and are represented by a positive value. The incumbent's charisma is denoted by  $C_A$  and the challenger's charisma is denoted by  $C_B$ , whereas the incumbency advantage is denoted by  $Y_A$ . These non-policy fixed attributes are not something that can be altered or maximized by the candidates. But, while such

attributes become common knowledge by all the candidates once running for the election, they are not fully perceived by voters. The amount perceived by voters depends on the candidates' time allocation. As previously mentioned, this paper not only uses policy position but also uses the candidates' time allocation as a choice variable. The incumbent faces a trade-off between spending time on the electoral campaign and legislative work.<sup>6</sup> On the one hand, spending time on the electoral campaign can increase the candidate's visibility and allow voters to better understand their personality and characteristics, enhancing the candidate's charisma as perceived by voters (Ashworth and Mesquita, 2008). On the other hand, spending time on legislative work allows the incumbents to accumulate more resources, skills and abilities whereby they can solve issues and increase the degree of incumbency advantage as perceived by voters. Therefore, balancing the allocation between the two is vital to the incumbent. At the same time, even though the challenger, by definition, is not in office, he faces a trade-off between spending time on the electoral campaign and his original job. I assume the challenger has to pay the cost  $\tau$  for per unit of time spending on the electoral campaign.  $\tau$  is assumed to be a positive number that represents the opportunity cost of the earnings from the challenger's original job that he has to give up to run for the election. A  $\tau$  value close enough to zero but greater than zero implies the challenger may be currently unemployed, with no source of income. Yet he can spend time in looking for a new job, only if the probability to get a job is positive,  $\tau$  is positive. The lower the value of  $\tau$ , the greater the time the challenger will spend on the electoral campaign. And increasing the value of  $\tau$  results in a higher opportunity cost to active campaigning. Thus, candidate B's time allocation on the electoral campaign is based on the magnitude of  $\tau$  and the marginal benefit of expanding his charismatic effect, such as expanding his personal popularity, increasing his reputation and increasing the probability of winning the election.

Here, I normalize the total time allocated by the candidates to be 1. Let  $t_A^e$  and  $t_B^e$  be respectively the proportion of time incumbent A and challenger B spend on the electoral campaign. The proportion of time incumbent A spent on legislative work is given by the function,

$$t_A^l = 1 - t_A^e. \quad (3.2)$$

The more time the candidate spends on the electoral campaign, the higher the proportion of his charisma that can be perceived by voters. Yet, note that the proportion of charismatic effect perceived by voters not only depends on the time the candidate devotes to the electoral campaign but also his charisma advantages/disadvantages vis-à-vis his opponents. For example, when incumbent A competes with challenger B having a higher degree of charisma, voters' attention to A gets diluted. As such the charismatic effect of A perceived by voters decreases. That is, in this scenario, the proportion of charismatic effect of incumbent A perceived by voters is assumed to be increasing in incumbent A's charisma  $C_A$  and decreasing in challenger B's charisma  $C_B$ . Additionally, the greater proportion of time incumbent A spends on legislative work, the higher proportion of his incumbency advantage can be perceived by voters. Hence, the assessment of incumbent A's competence, the valence term  $Q$ , by the individual  $i$  is

$$Q_i(A) = \frac{(C_A * t_A^e)}{(C_A * t_A^e + C_B * t_B^e)} + Y(1 - t_A^e). \quad (3.3)$$

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<sup>6</sup>Legislative work includes public service and constituency work. The former focuses on certain economic sectors (police service, military etc.) or certain topics (education, environmental protection, town planning, etc.), whereas the latter focuses on individuals request.

And the assessment of challenger B's competence by individual  $i$  is

$$Q_i(B) = \frac{(C_B * t_B^e)}{(C_A * t_A^e + C_B * t_B^e)}. \quad (3.4)$$

The first term in equation (3.3) represents incumbent A's charismatic effect perceived by voters, and the second term is incumbent A's incumbency advantage perceived by voters. The first term in equation (3.4) represents challenger B's charismatic effect perceived by voters. Note that the challenger has to pay the cost  $\tau$  per unit of time spent on the electoral campaign, so by expanding the charismatic effect he has to pay total cost  $t_B^e * \tau$ . The first term in equations (3.3) and (3.4) are frequently used specifications in the theoretical literature. For example, Snyder (1989) and Baron (1994) use this specification in their theoretical studies.

As mentioned before, voters base their voting behaviors on endemic content/discontent in addition to position issues and valence issues. In the model, I use the voters' satisfaction level of the incumbent, denoted by  $\sigma$ , to capture the effect of macroeconomic conditions on voters' voting behavior. The voters' satisfaction level includes two components ( $\sigma = \delta + u$ ): an individual component  $\delta$  and an economic component  $u$ . When negative reflecting discontent,  $\delta$  derives from a deficient provision of public goods, social disorder and political persecution. When positive reflecting content,  $\delta$  comes from a sufficient provision of public goods, social serenity and political stability. For simplicity, I assume that society is relatively homogeneous. The extremes are much less numerous than the number of people who are nearer to the average income and living standard.  $\delta$  is assumed to be a random variable and distributed normally with mean 0 and variance 1. The cumulative distribution function is denoted by  $\Phi$ . The other component of voters' satisfaction level,  $u$ , derives from an economic shock. In modern societies, one's standard of living is affected by economic conditions and fluctuations. Accordingly, it becomes a straightforward indicator, by which voters evaluate the incumbents even though the incumbent's policies have no relevance for such conditions. Variation in  $u$  comes from positive economic shocks, such as economic boom and technological innovation, or negative economic shocks, such as financial and economic crisis. Voters' satisfaction level is positively correlated with the prevailing economic conditions. The economic component,  $u$ , takes the value 1, 0 or  $-1$  to represent three different states of the economy:  $u = 1$ , the economy is good with probability  $P_g$ ;  $u = -1$ , the economy is bad with probability  $P_b$ ;  $u = 0$ , the economy is stable with probability  $1 - P_g - P_b$ . Combining these two components, when  $\sigma$  is less than 0, the voter experiences endemic discontent. Thus, the voters' satisfaction with the incumbent government is negatively correlated with voters' endemic discontent.

In sum, voters voting behavior is determined by position issues, X, valence issues, Q, and voters' satisfaction level,  $\sigma$ . The expected utility of individual voter  $i$  who votes for incumbent A is as follows,

$$U_i(A) = U_i(X_A) + Q_i(A) + \sigma_i. \quad (3.5)$$

The expected utility of individual voter  $i$  to vote for challenger B is

$$U_i(B) = U_i(X_B) + Q_i(B). \quad (3.6)$$

Thus, individual  $i$  would vote for incumbent A if and only if

$$U_i(X_A) + Q_i(A) + \sigma_i \geq U_i(X_B) + Q_i(B), \quad (3.7)$$

and would vote for challenger B if only if

$$U_i(X_A) + Q_i(A) + \sigma_i < U_i(X_B) + Q_i(B). \quad (3.8)$$

Let us identify the swing voter  $j$ , a voter whose individual endemic discontent, given the candidates' platform and valence magnitude, makes him indifferent between incumbent A and challenger B:

$$U_j(X_A) + Q_j(A) + \sigma_j = U_j(X_B) + Q_j(B). \quad (3.9)$$

Given that  $\sigma_j = \delta_j + u$ ,

$$\begin{aligned} U_j(X_A) + Q_j(A) + \delta_j &= U_j(X_B) + Q_j(B) - u \\ \delta_j &= U_j(X_B) + Q_j(B) - U_j(X_A) - Q_j(A) - u \end{aligned} \quad (3.10)$$

All voters  $i$ , whose individual endemic discontent is smaller than the swing voter  $j$  with  $\delta_i > \delta_j$  (in other words, the voter  $i$ 's satisfaction level is higher than the swing voter  $j$  will vote for incumbent A. Note that there is large odd number of voters, as that number approaches infinity. Also given our distributional assumption that  $\delta_j$  is a random variable with standard normal distribution, the share of vote for incumbent A, denoted by  $V_A$ , is approximated by

$$V_A = 1 - \int_{-\infty}^{\delta_j} \frac{1}{\sqrt{2\pi}} e^{-\frac{\delta^2}{2}} d\delta = 1 - \Phi(\delta_j) \quad (3.11)$$

Obviously, B's share of vote is  $1 - V_A$ . From the perspective of the candidates, the electoral outcome is a random event. Incumbent A's probability of winning, denoted by  $P_A$ , is the probability that the share of vote for incumbent A is greater than some threshold  $\gamma^7$ , which is  $P_A = Prob(V_A \geq \gamma)$ . Challenger B's probability of winning is  $1 - P_A$ . Recall that both candidates are office-motivated, so the candidates' objective is to maximize the share of votes by choosing the policy positions and time allocation. The candidates' objective function is given by

$$\max V_r, r = A, B. \quad (3.12)$$

### 3.2 Sequence of Events

Before deriving the equilibrium results and doing comparative statics of different economic conditions, I will first explain the timeline of the model.

**Stage 1:** Candidate A is the incumbent. To maximize his share of votes, the incumbent chooses the policy position  $X_A$  and allocates his time efficiently between the electoral campaign and legislative work  $(t_A^e, t_A^l)$ .

**Stage 2:** Given information on the incumbent's choice  $X_A, t_A^e$ , challenger B decides if he should run for the election and compete with A. Challenger B will run for the election if his expected payoff from winning the office is greater than the opportunity cost of running for the election.<sup>8</sup> Otherwise, he will not participate in the election.<sup>9</sup> If B runs for the election, he will announce a policy position,  $X_B$ , and spend  $t_B^e$  amount of time on the electoral campaign to maximize the expected share of votes.

<sup>7</sup>Different voting systems have different value of  $\gamma$ . For example, in the baseline model, there are two candidates and there is only one winner. Under First-past-the-post voting system,  $\gamma$  is equal to  $\frac{1}{2}$ . Yet, under Approval voting system, each voter can vote for one or more than one candidates; the winner is the candidate who receives the most votes. In this case,  $\gamma$  is not equal to  $\frac{1}{2}$ .

<sup>8</sup>The condition of challenger B to choose not to run for the election will be shown in Proposition 4 in page 13.

<sup>9</sup>In this model, for simplicity I assume there are at most two candidates (the incumbent and one challenger) running for the election. Challenger B is the most competitive challenger among all potential ones. Thus, if in stage 2, challenger B decides not to participate in the election, other potential challengers whose expected share of votes are lower than challenger B will also not participate in the election. Thus, incumbent A becomes the only candidate running for the election.

**Stage 3:** The state of the economy ( $u$ ) is realized by both candidates and voters.

**Stage 4:** The election takes place.

## 4 Equilibrium and Comparative Statics

I apply the principles of backward induction. In stage 3, the state of the economy  $u$  is realized. By assumption, the economic condition,  $u$ , takes the value of 1 with probability  $P_g$ , the value of  $-1$  with probability  $P_b$ , and the value of 0 with probability  $1 - P_g - P_b$ . With uncertain economic future, the expected share of votes for incumbent A and challenger B is the sum of multiplication of each share of vote under different economic states and the probability of that economic condition occurs. By substituting equation (3.10) into equation (3.11), the expected share of votes for incumbent A and challenger B can be calculated. Given that  $j$  is a swing voter, the expected share of votes for incumbent A,  $V_A^*$ , and challenger B,  $V_B^*$ , is as follows,

$$\begin{aligned} V_A^* &= P_g * [1 - \Phi(U_j(X_B) + Q_j(B) - U_j(A) - Q_j(A) - 1)] \\ &\quad + P_b * [1 - \Phi(U_j(X_B) + Q_j(B) - U_j(A) - Q_j(A) + 1)] \\ &\quad + (1 - P_g - P_b) * [1 - \Phi(U_j(X_B) + Q_j(B) - U_j(A) - Q_j(A))] \end{aligned} \quad (4.1)$$

$$\begin{aligned} V_B^* &= P_g * \Phi(U_j(X_B) + Q_j(B) - U_j(A) - Q_j(A) - 1) \\ &\quad + P_b * \Phi(U_j(X_B) + Q_j(B) - U_j(A) - Q_j(A) + 1) \\ &\quad + (1 - P_g - P_b) * \Phi(U_j(X_B) + Q_j(B) - U_j(A) - Q_j(A)) \end{aligned} \quad (4.2)$$

The first terms in equations (4.1) and (4.2) represent the share of votes for incumbent A and challenger B respectively when the economic condition is good; the second term represents their share of votes when the economic condition is bad and the last term represents their share of votes when the economy is stable.

In stage 2, by knowing incumbent A's policy position and time allocation, challenger B chooses the policy position and proportion of time spending on the electoral campaign to maximize his share of votes. The optimal policy of challenger B is the same policy platform as the median voter  $m$ 's ideal point  $X_m$ . The equilibrium of challenger B's policy position and time allocation only exists if and only if his expected payoff of winning the election is greater than the opportunity cost of running for the election in stage 2. Otherwise, challenger B will not run for the election. Then in stage 1, by maximizing the expected share of votes, incumbent A will also choose the median voters' ideal policy position. The following proposition describes that both candidates' policy platform will cater to median voters' ideal point.

*Proposition 1:* If both incumbent A and challenger B are office-motivated only, and there exists median voters' ideal point  $X_m$ , then the equilibrium policy of challenger B is where  $X_B = X_m$ , the equilibrium policy of incumbent A is where  $X_A = X_m$ . The equilibrium results in both candidates converging to the median voters' ideal policy position, where  $X_A = X_B = X_m$ .

*Proof.* Suppose the median voters' ideal point is  $X$ , and that incumbent A chose the policy position  $Y$ , which is not equal to  $X$ . Challenger B is office-motivated, and cares only about winning. Even though

in our model we consider valence issues and endemic discontent, these two effects only change the height of the function of challenger B's probability of winning. The peak of the function will stay the same. Hence, the best optimal policy position of challenger B is consistent with the median voters' ideal point X. The closer to the median voters' ideal point, the more ballots he can secure to maximize the share of votes. Then since incumbent A is solely office-motivated too, he will have incentive to switch from Y and converge to X. Thus, in the model, both candidates will choose the same policy position as the median voters' ideal policy position.

The result of Proposition 1 is consistent with Downs'(1957) finding that when the ideological positions of voters are displayed as a single peak preference, the choice of candidates' policy positions will converge toward the median voters' ideal policy position. According to his study, the median voter is pivotal; the candidate chosen by the median voter wins the election. If the median voter votes for incumbent A, voters whose preferred platforms are to the left (right) of the median voters' policy position will cast their ballots for incumbent A (challenger B). Only if both candidates are office-motivated and solely interested in winning the election will the magnitude of valence issues and endemic discontent/content have no effect on candidates' policy positions. The effects of valence term and endemic discontent can also be considered as voter bias, Wittman's study (1983) shows that such a bias will not change candidates' policy positions.

Aside from his policy position, challenger B also allocates his time efficiently between his original job and electoral campaign. The optimum time allocated to the electoral campaign of challenger B is implicitly defined by the first-order conditions of his expected share of votes in respect to  $t_B^e$ . Yet since both candidates will converge to the same policy position, and the economic condition is realized in stage 3, the only choice variables are time allocation and the only effects that can alter and maximize by challenger B is the valence term. Thus, by maximizing challenger B's objective function in equation (3.12) is the same as maximizing the payoff function  $f(B)$ , which is  $Q_j(B) - Q_j(A) - t_B^e * \tau$ . Thus,

$$\begin{aligned} & \max f(B) \\ & = \max Q_j(B) - Q_j(A) - t_B^e * \tau \\ & = \max \frac{C_B * t_B^e - C_A * t_A^e}{C_A * t_A^e + C_B * t_B^e} - t_B^e * \tau. \end{aligned}$$

An interior equilibrium of  $t_B^{e*}$  is characterized by setting the first order condition of  $f(B)$  respect to  $t_B^e$  equal to zero.<sup>10</sup> Thus, challenger B's optimum time allocation is as follows,

$$\begin{aligned} t_B^e &= 1 \text{ if } \sqrt{\frac{C_A * t_A^e}{C_B}} * \left( \sqrt{\frac{2}{\tau}} - \sqrt{\frac{C_A * t_A^e}{C_B}} \right) > 1, \\ t_B^e &= \sqrt{\frac{C_A * t_A^e}{C_B}} * \left( \sqrt{\frac{2}{\tau}} - \sqrt{\frac{C_A * t_A^e}{C_B}} \right), \text{ if } 0 \leq \sqrt{\frac{C_A * t_A^e}{C_B}} * \left( \sqrt{\frac{2}{\tau}} - \sqrt{\frac{C_A * t_A^e}{C_B}} \right) \leq 1 \\ t_B^e &= 0 \text{ if } \sqrt{\frac{C_A * t_A^e}{C_B}} * \left( \sqrt{\frac{2}{\tau}} - \sqrt{\frac{C_A * t_A^e}{C_B}} \right) < 0. \end{aligned} \quad (4.3)$$

By knowing challenger B's response function in equation (4.3), incumbent A will allocate his time efficiently in stage 1 to maximize his expected share of votes. As mentioned previously, the only

<sup>10</sup>The second order condition of  $f(B)$  respect to  $t_B^e$  is negative. Challenger B's objective function is strict concavity, which guarantees the existence of local maximum.

choice variables in the model are time allocation. Thus, maximizing incumbent A's objective function in equation (3.12) is the same as maximizing the valence term, which is incumbent A's payoff function  $f(A)$ ,  $Q_j(A) - Q_j(B)$ , given equation (4.3).

$\max f(A)$  subject to equation (4.3)

$$\begin{aligned}
&= \max Q_j(A) - Q_j(B) \text{ subject to (4.3)} \\
&= \max \frac{(C_A * t_A^e - C_B * t_B^e)}{(C_A * t_A^e + C_B * t_B^e)} + Y(1 - t_A^e) \text{ subject to (4.3)} \\
&= \max \frac{(C_A * t_A^e - C_B * (\sqrt{\frac{C_A * t_A^e}{C_B}} * \sqrt{\frac{2}{\tau}} - \sqrt{\frac{C_A * t_A^e}{C_B}}))}{(C_A * t_A^e + C_B * (\sqrt{\frac{C_A * t_A^e}{C_B}} * \sqrt{\frac{2}{\tau}} - \sqrt{\frac{C_A * t_A^e}{C_B}}))} + Y(1 - t_A^e).
\end{aligned}$$

An equilibrium of  $t_A^e$  is characterized by deriving the first order condition of  $f(A)$  respect to  $t_A^e$ . Incumbent A's time allocation is

$$\begin{aligned}
t_A^e &= 1, \text{ if } \frac{C_A * \tau}{C_B} * \frac{1}{2 * Y^2} > 1, \\
t_A^e &= \frac{C_A * \tau}{C_B} * \frac{1}{2 * Y^2} \text{ if } 0 \leq \frac{C_A * \tau}{C_B} * \frac{1}{2 * Y^2} \leq 1, \\
t_A^e &= 0, \text{ if } \frac{C_A * \tau}{C_B} * \frac{1}{2 * Y^2} < 0.
\end{aligned}$$

Note that in equation (3.2) I normalize the total time allocated by incumbent A to be 1. Therefore, incumbent A faces the time constraint, forcing  $t_A^e$  to be bounded between 0 and 1. Besides, in our model, we focus on the effect of incumbency advantage on the electoral outcome, so the basic premise of the model is that the incumbent will run for reelection. Thus, we rule out the possibility that  $t_A^e = 0$ . Incumbent A has to spend positive amount of time on the electoral campaign to inform voters that he will run for reelection. The interior equilibrium of incumbent A's time allocation is,

$$t_A^e = \frac{C_A * \tau}{C_B} * \frac{1}{2 * Y^2}. \quad (4.4)$$

By substituting equations (4.4) into equation (4.3), challenger B's optimal time allocation is as follows,

$$\begin{aligned}
t_B^e &= 1, \text{ if } \frac{C_A}{C_B * Y} - \frac{C_A^2}{C_B^2} * \frac{\tau}{2 * Y^2} > 1, \\
t_B^e &= \frac{C_A}{C_B * Y} - \frac{C_A^2}{C_B^2} * \frac{\tau}{2 * Y^2} \text{ if } 0 \leq \frac{C_A}{C_B * Y} - \frac{C_A^2}{C_B^2} * \frac{\tau}{2 * Y^2} \leq 1, \\
t_B^e &= 0, \text{ if } \frac{C_A}{C_B * Y} - \frac{C_A^2}{C_B^2} * \frac{\tau}{2 * Y^2} > 0.
\end{aligned}$$

Challenger B also faces the time constraint, with  $t_B^e$  bounded between 0 and 1. The interior equilibrium of challenger B's time allocation is

$$t_B^e = \frac{C_A}{C_B * Y} - \frac{C_A^2}{C_B^2} * \frac{\tau}{2 * Y^2}. \quad (4.5)$$

Imposing time constraints on equations (4.4) and (4.5), the necessary and sufficient conditions of interior equilibria can be obtained.

*Proposition 2:*

- 1) If the constraint  $C_A \geq C_B$  holds,
    - a) and the constraints  $Y \geq \frac{C_B}{C_A}$  and  $0 \leq \tau < 2 * Y * \frac{C_B}{C_A}$  hold,
    - b) and the constraints  $1 \leq Y \leq \frac{C_B}{C_A}$  and  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2} < \tau < 2 * Y * \frac{C_B}{C_A}$  hold,
    - c) and the constraints  $\frac{C_A}{C_A+C_B} \leq Y \leq 1$  and  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2} < \tau < 2 * Y^2 * \frac{C_B}{C_A}$  hold,
  - 2) or if the constraint  $C_A < C_B$  holds,
    - a) and the constraints  $Y \geq 1$  and  $0 < \tau < 2 * Y * \frac{C_B}{C_A}$  hold,
    - b) and the constraints  $\frac{C_A}{C_B} \leq Y \leq 1$  and  $0 < \tau < 2 * Y^2 * \frac{C_B}{C_A}$  hold,
    - c) and the constraints  $\frac{C_A}{C_A+C_B} \leq Y \leq \frac{C_B}{C_A}$  and  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2} < \tau < 2 * Y^2 * \frac{C_B}{C_A}$  hold,
- then the interior solution of  $t_A^e$  and  $t_B^e$  exists, where  $t_A^e = \frac{C_A * \tau}{C_B} * \frac{1}{2 * Y^2}$  and  $t_B^e = \frac{C_A}{C_B * Y} - \frac{C_A^2}{C_B^2} * \frac{\tau}{2 * Y^2}$ .

*Proof.* By imposing the time constraints to the equations (4.4) and (4.5), the constraints of the opportunity cost  $\tau$  can be obtained. A  $\tau$  value bounded between 0 and  $2 * Y^2 * \frac{C_B}{C_A}$  is to ensure that  $0 < t_A^e < 1$ . As perviously mentioned,  $\tau$  should be greater than 0. In addition, a  $\tau$  value bounded between  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2}$  and  $2 * Y * \frac{C_B}{C_A}$  is to ensure that  $0 < t_B^e < 1$ . The upper bound of  $\tau$  is the lowest threshold for the challenger to run for the election. The challenger faces the tradeoff between spending time on his "day" job and on electoral campaigning. If the cost is greater than or equal to the upper bound, the challenger's opportunity cost of running for the election is higher than his expected payoff. Then the challenger will not participate in the election. When  $\tau$  meets the lower bound, the barrier of the challenger to run for the election is low, he will give up his "day" job and devote all his time to the election. (It will be shown in the later proposition.) Then by substituting the constraints of opportunity cost into equation (4.4) and (4.5) respectively, the corresponding scope of incumbency advantage of interior solution can be obtained.

From Proposition 2, we know that the longer-term incumbent (driving a higher  $Y$ ) prefers spending less time on the electoral campaign ( $t_A^e \downarrow$ ). Longer-term incumbency results in more resources and experience, which derive from incumbency advantage. The comparative advantage of longer-term incumbent is to expand his incumbency advantage perceived by voters. Thus, the longer-term incumbent will prefer to spend more time on legislative work. Note that this finding is based on our key assumption that the candidates are office-motivated, caring only election. If this assumption does not hold, incumbent A will care not only about the reelection but also the benefits of the electoral campaign to his party, for example, increase the fundraising for his party or other politicians, then the conclusion might not be held. In addition, the ratio of incumbent A's charisma versus incumbency advantage ( $\frac{C_A}{Y}$ ) affects challenger B's time allocation. The next proposition describes challenger B's response under different scenarios.

*Proposition 3:*

- 1) If the charisma of incumbent A is higher than (or equal to) that of challenger B,  $C_A \geq C_B$ ,
  - a) and the constraints  $Y \geq \frac{C_A}{C_B}$  and  $0 < \tau < Y * \frac{C_B}{C_A}$  hold,  
or the constraints  $\frac{1}{2} * \frac{C_A}{C_B} \leq Y < \frac{C_A}{C_B}$  and  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2} < \tau < Y * \frac{C_B}{C_A}$  hold, then an increase in the ratio of incumbent A's charisma versus incumbency advantage ( $\frac{C_A}{Y}$ ) increases challenger B's incentive to spend time on the electoral campaign.
  - b) and the constraints  $Y \geq 1$  and  $Y * \frac{C_A}{C_B} < \tau < 2 * Y * \frac{C_B}{C_A}$  hold,

or the constraints  $\frac{1}{2} * \frac{C_A}{C_B} \leq Y < 1$  and  $Y * \frac{C_B}{C_A} < \tau < 2 * Y^2 * \frac{C_B}{C_A}$  hold,

or the constraints  $\frac{C_A}{C_A+C_B} \leq Y < \frac{1}{2} * \frac{C_A}{C_B}$  and  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2} < \tau < 2 * Y^2 * \frac{C_B}{C_A}$  hold,

then an increase in the ratio of incumbent A's charisma versus incumbency advantage ( $\frac{C_A}{Y}$ ) decreases challenger B's incentive to spend time on the electoral campaign.

2) If the charisma of incumbent A is smaller than that of challenger B,  $C_A < C_B$ ,

a) and the constraints  $Y \geq \frac{C_A}{C_B}$  and  $0 < \tau < Y * \frac{C_B}{C_A}$  hold,

or the constraints  $\frac{1}{2} \leq Y < \frac{C_A}{C_B}$  and  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2} < \tau < Y * \frac{C_B}{C_A}$  hold,

or the constraints  $\frac{C_A}{C_A+C_B} \leq Y < \frac{1}{2}$  and  $2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2} < \tau < 2 * Y^2 * \frac{C_B}{C_A}$  hold,

then an increase in the ratio of incumbent A's charisma versus incumbency advantage ( $\frac{C_A}{Y}$ ) increases challenger B's incentive to spend time on the electoral campaign.

b) and the constraints  $Y \geq 1$  and  $Y * \frac{C_A}{C_B} < \tau < 2 * Y * \frac{C_B}{C_A}$  hold,

or the constraints  $\frac{1}{2} \leq Y < 1$  and  $Y * \frac{C_B}{C_A} < \tau < 2 * Y^2 * \frac{C_B}{C_A}$  hold, then an increase in the ratio of incumbent A's charisma versus incumbency advantage ( $\frac{C_A}{Y}$ ) decreases challenger B's incentive to spend time on the electoral campaign.

Proof. The function of the equation (4.5) is differentiable in  $\frac{C_A}{Y}$ . Take the first order condition of  $t_B^e$  respect to  $\frac{C_A}{Y}$ . When  $\frac{\partial t_B^e}{\partial (\frac{C_A}{Y})} > 0$ , the constraint  $\tau < Y * \frac{C_B}{C_A}$  holds, whereas when  $\frac{\partial t_B^e}{\partial (\frac{C_A}{Y})} < 0$ , the opposite constraint  $\tau > Y * \frac{C_B}{C_A}$  holds. Proposition 3 will be obtained by combining the above constraints and the constraints of opportunity cost  $\tau$  derived in Proposition 2, along with the constraints of incumbency advantage. All these constraints ensure that there is no violation of time constraints for both candidates.

Proposition 3 1a) and 2a) show that the higher the ratio of  $\frac{C_A}{Y}$ , challenger B will spend more time on the electoral campaign. Proposition 3 1b) and 2b) show that the higher the the ratio of  $\frac{C_A}{Y}$ , challenger B will spend less time on the electoral campaign. Here, we separate the type of incumbents into two categories: one is relatively longer-term incumbency but with weaker charisma, where is when the constraint  $\tau < Y * \frac{C_B}{C_A}$  holds; the other is relatively shorter-term incumbency but with stronger charisma, where is when the constraint  $\tau > Y * \frac{C_B}{C_A}$  holds. From Proposition 3, we can find that compared to shorter-term incumbent with relatively stronger charisma, the challenger would prefer to compete against longer-term incumbent but with relatively weaker charisma. This finding further verifies that the spending on the electoral campaign of incumbents will decline over time. From the challenger's point of view, longer-term incumbent but with relative weaker charisma is less competitive, so even though the ratio of  $\frac{C_A}{Y}$  increases, the challenger's incentive to spend time on the electoral campaign still increases. Moreover, the magnitude of incumbent A's charisma vis-à-vis the challenger's ( $\frac{C_A}{C_B}$ ) also affects the candidates' equilibrium time allocation. If incumbent A's charisma is higher than his opponent's, incumbent A will prefer to spend more time on the electoral campaign than legislative work. Yet for the challenger, his time allocation depends on which types of the incumbent he will compete against. The computation and the constraints are the same as stated in Proposition 3. Thus, I will not repeat the proof, but only demonstrate the results here. When the challenger faces the longer-term incumbent but with relatively weaker charisma, an increase in the magnitude of incumbent A's charisma vis-à-vis the challenger's increases challenger B's incentive to spend time on the electoral campaign. However, when the challenger faces the shorter-term incumbent but with relatively stronger

charisma, an increase in the magnitude of incumbent A's charisma vis-à-vis the challenger's decreases challenger B's incentive to spend time on the electoral campaign. So far, we only consider the cases where both the incumbent and the challenger are willing to run for the election. Yet if the opportunity cost of the challenger running for the election is higher than his probability of winning, he will choose not to participate in the election. The next proposition performs one of the corner solutions that the incumbent is the only candidate running for the election.

*Proposition 4:* If  $\tau \geq 2 * Y * \frac{C_B}{C_A}$  and the constraint  $Y \geq 1$  holds, then challenger B has no incentive to spend time on the electoral campaign:  $t_B^e = 0$ . Incumbent A will spend at least the amount of time equal to the return of his incumbency advantage, where  $1 \geq t_A^e \geq \frac{1}{Y}$ .

*Proof.* The result can be obtained by substituting  $\tau \geq 2 * Y * \frac{C_B}{C_A}$  into equation (4.4) and (4.5). And we have to impose the constraint of incumbency advantage where  $Y \geq 1$  to ensure that incumbent A meets the time constraint, where  $t_A^e \leq 1$ .

Proposition 4 shows one of the corner solutions that the challenger will not participate in the election. When the challenger's opportunity cost is greater than (or equal to) the upper bound,  $2 * Y * \frac{C_B}{C_A}$ , and faces the competition from longer-term incumbents ( $Y \geq 1$ ), the challenger will not run for the election. This corresponds to the scare-off effect, one of the explanations of incumbency advantage stated in section two, where the incumbent can deter competitors from entering an election. It is worth noting that when  $Y = 1$  the return of incumbency advantage from the electoral spending is greater than that from legislative work, so the incumbent will spend all his time on the electoral campaign,  $t_A^e = 1$ . Yet due to diminishing return of incumbency advantage, the increase of  $Y$  decreases the return of incumbency advantage as well as the time the incumbent spends on the electoral campaign. If, on the contrary, the challenger faces the competition from a shorter-term incumbent, it is not surprising that the scare-off effect will be less prominent. The equilibrium outcomes will be shown in the next proposition.

*Proposition 5:* If  $\tau \geq 2 * Y^2 * \frac{C_B}{C_A}$  and the constraint  $\frac{C_A}{C_A + C_B} \leq Y < 1$  holds, then incumbent A will spend all his time on the electoral campaign, where  $t_A^e = 1$  and challenger B will respond by at most spending  $\frac{C_A}{C_B} * (\frac{1}{Y} - 1)$  amount of time on the electoral campaign, where  $0 \leq t_B^e \leq \frac{C_A}{C_B} * (\frac{1}{Y} - 1)$ .

*Proof.* By substituting  $\tau \geq 2 * Y^2 * \frac{C_B}{C_A}$  into equation (16) and (17), we obtain  $t_A^e = 1$  and  $t_B^e \leq \frac{C_A}{C_B} * (\frac{1}{Y} - 1)$ . And we have to impose the constraint of incumbency advantage where  $\frac{C_A}{C_A + C_B} \leq Y < 1$  to ensure that  $t_B^e \leq 1$ .

Proposition 5 shows even if the challenger's opportunity cost is greater than (or equal to) the upper bound, the short-term incumbents ( $Y < 1$ ) cannot completely deter the challenger from entering the election. In this scenario, the short-term incumbent will spend all his time on the electoral campaign because of high return of incumbency advantage from campaign spending. From the challenger's perspective, the lower the  $Y$  is, the less competitive the incumbent is, so the more incentive he will have to spend time on the electoral campaign. Thus, when  $Y$  meets the lower bound in this case, where  $Y = \frac{C_A}{C_A + C_B}$ , or even  $Y < \frac{C_A}{C_A + C_B}$ , the increase in the challenger's probability of winning is offset by the increase in his opportunity cost entering the election, so the challenger will give up his "day"

job and spend all his time on the electoral campaign. Under this condition, the equilibrium condition is where  $t_A^e = 1$  and  $t_B^e = 1$ . Note that when  $Y < \frac{C_A}{C_A+C_B}$ , challenger B has the incentive to spend more time than he has on campaigning. Yet due to time constraint, the constraint is binding, so  $t_B^e = 1$ .<sup>11</sup> The previous two propositions display two corner solutions where the opportunity cost of the challenger meets the upper bound. The next proposition will show the equilibrium of the opposite scenario where the opportunity cost of the challenger meets the lower bound.

*Proposition 6:* If  $\tau \leq 2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2}$  and the constraint  $\frac{C_A}{C_A+C_B} \leq Y < \frac{C_A}{C_B}$  holds, then incumbent A will spend at most  $\frac{1}{Y} - \frac{C_B}{C_A}$  amount of time on the electoral campaign, where  $0 < t_A^e \leq \frac{1}{Y} - \frac{C_B}{C_A}$ . Challenger B will respond by giving up his "day" job and devoting all his time to the election, where  $t_B^e = 1$ .

*Proof.* By substituting  $\tau \leq 2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2}$  into equation (4.4) and (4.5), we obtain  $t_A^e \leq \frac{1}{Y} - \frac{C_B}{C_A}$  and  $t_B^e = 1$ . And we have to impose the constraint of incumbency advantage where  $\frac{C_A}{C_A+C_B} \leq Y < \frac{C_A}{C_B}$  to ensure that  $t_A^e \leq 1$ .

Proposition 6 shows that when the opportunity cost of the challenger meets the lower bound, the challenger has a lower barrier to enter the election, so he will spend all his time on campaigning. While expecting to face such competitive challengers, the incumbent will spend at most  $\frac{1}{Y} - \frac{C_B}{C_A}$  amount of time on the electoral campaign. The length of the incumbency is bounded between  $\frac{C_A}{C_A+C_B}$  and  $\frac{C_A}{C_B}$ . If the incumbent is a longer-term incumbent, where  $Y \geq \frac{C_A}{C_B}$ , the incumbent's best response is to spend all his time on legislative work, because the comparative advantage of longer-term incumbent is to expand the incumbency advantage perceived by voters. Yet the incumbent has to inform voters that he will run for reelection, so a value of  $t_A^e$  can be very close to 0, but has to be greater than 0. An equilibrium exists when  $t_A^e > 0$  and  $t_B^e = 1$ .<sup>12</sup> In contrast, if the incumbent is a short-term incumbent, where  $Y \leq \frac{C_A}{C_A+C_B}$ , the incumbent's comparative advantage is to spend as much time as he can on the electoral campaign to expand his charismatic effect perceived by voters. Because of time constraint,  $t_A^e$  is at most equal to 1. An equilibrium of the competition between a short-term incumbent and a competitive challenger exists where  $t_A^e = 0$  and  $t_B^e = 1$ .

From Proposition 5 and Proposition 6, we can find that the shorter-term incumbent,  $Y \leq \frac{C_A}{C_A+C_B}$ , will spend all his time on the electoral campaign because the return from electoral spending is higher than that from legislative work. When the challenger faces such a shorter-term incumbent, no matter

<sup>11</sup>The same rule applies to the scenario where  $\tau < 2 * Y^2 * \frac{C_B}{C_A}$ . An equilibrium exists,  $t_A^e = 1$  and  $t_B^e = 1$ , once  $Y < \frac{C_A}{C_A+C_B}$  and the opportunity cost of the challenger is lower than the upper bound,  $2 * Y^2 * \frac{C_B}{C_A}$ .

<sup>12</sup>Note that we rule out the possibility that incumbent A will not run for reelection by forcing  $\tau > 0$ , so  $t_A^e > 0$ . The reason behind this setting is associated to our key assumption that both candidates are office-motivated, caring only winning the election. The challenger makes his strategic entry by leveraging the expected payoff of winning the election and the opportunity cost of running the election. Yet the electoral outcome is a random event, so the opportunity cost of time foregone cannot be negative in our model. However, in the real world case, challenger B might not only consider winning the election, but also other benefits from electoral campaigning, for example, increasing the visibility of his private corporation and further expanding his business, or increasing fundraising and raising money to his party. If this is the case, the opportunity cost of the challenger can be 0 or negative. Under this circumstances, when expecting to face such strong challenger, who spends all his time on the electoral campaign, incumbent A might choose not to participate in the election, so  $t_A^e = 0$ . An equilibrium exists,  $t_A^e = 0$  and  $t_B^e = 1$ , once  $\tau < 0$ .

the magnitude of opportunity cost is greater than the upper bound,  $\tau \geq 2 * Y^2 * \frac{C_B}{C_A}$ , or smaller than the lower bound,  $\tau \leq 2 * Y * \frac{C_B}{C_A} - 2 * Y^2 * \frac{C_B^2}{C_A^2}$ , he will give up his "day" job and devote all his time to campaigning. Thus, it is not surprising that when the opportunity cost is bounded between the upper bound and the lower bound, both candidates' best responses will be consistent, to allocate all their time on the electoral campaign. We can conclude that while competing with the shorter-term incumbent, the challenger cares less about the magnitude of the opportunity cost. An equilibrium,  $t_A^e = 1$  and  $t_B^e = 1$ , exists under this scenario.

Finally, we consider the importance of voters' endemic discontent/content in determining the electoral outcome (stage 3). Suppose the economy is likely to improve during the election. (Ex.  $P_g \uparrow$ ,  $P_b \downarrow$ ,  $1 - P_b - P_g$  is unchanged) In this case, the share of votes for incumbent A increases and incumbent A's probability of winning also increases. By the same token, the share of votes for challenger B decreases and challenger B's probability of winning also decreases. Thus, we find that macroeconomic conditions will affect voters' voting behavior and the electoral outcome. Strong economies favor the incumbent and weak economics favor the challenger. In sum, when the economic condition is realized in stage 3, it resembles exogenous shocks to candidates; so the candidates cannot respond to the shocks. Hence, the candidates' policy positions and time allocation will not be affected.

## 5 Ex-ante Equilibrium and Comparative Statics

From the previous section, we know that if macroeconomic conditions were realized just right before the election (stage 3), it will not affect the candidates' strategic decision-making, but it will affect voters' voting behavior and the electoral outcome. However, what if macroeconomic conditions were realized ex-ante, in an earlier stage? How would it affect the candidates' decision making and electoral outcome?

Suppose macroeconomic conditions are realized before stage 1. An application of the principal of backward induction shows that the equilibrium conditions of incumbent A and challenger B's policy positions will stay the same, converging to the median voters' ideal point. However, the state of economic conditions will affect the challenger's opportunity cost of running for the election. If the prospects for the economy are negative in the near future, the challenger is more likely to earn less and/or become unemployed, resulting in a lower opportunity cost for challenger B. Concurrently, a weak economy also leads to higher endemic discontent among voters, leading to an increase in the anticipated share of votes for challenger B and a decrease in the anticipated share of votes for incumbent A. The probability of challenger B entering and winning election is higher as well. As a result, the marginal benefit of challenger B run for the electoral campaign increases and he will be more willing to spend time on the electoral campaign ( $t_B^e \uparrow$  due to  $\tau \downarrow$ ). Since macroeconomic conditions are realized ex-ante, incumbent A anticipates competition with stronger challengers. Thus, incumbent A will allocate his time efficiently by spending more time on the legislative work and less time on the electoral campaign ( $t_A^e \downarrow$  due to  $\tau \uparrow$ ). When the economy is strong, the challenger's opportunity cost of running for the election is higher, and the voters' endemic discontent is lower, leading to a decrease in the anticipated share of votes for challengers and an increase in the anticipated share of votes for incumbent A. Simultaneously, the probability of challenger B entering and winning the election is lower as well. As a result, challenger B will be less willing to spend time on the electoral campaign and incumbent A, facing a less competitive challenger, will spend more time on the electoral campaign.

Thus, we find that in this scenario, macroeconomic conditions will not only affect voters' voting

behavior and the electoral outcome, but also the candidates' strategic decision-making. A weak economy increases the intensity of endemic discontent and favors the challenger. The anticipated share of votes for the challenger will increase; however, unlike the previous analysis, now the incumbent can adjust his campaign strategy and respond to the macroeconomic conditions before the election takes place. The electorate's voting behavior and the electoral outcome rely on the dynamic effect of the two candidates' campaign strategies.

## 6 Conclusion

The outcome of elections depend on the balance between the effects of incumbency advantage and incumbency disadvantage. This paper incorporates both effects into a simple unified voting model, where two candidates compete in an election by choosing their policy positions and allocating their time across different tasks. Although this model highlights the importance of valence issues while introducing endemic discontent as a new determinant, the results show that both effects will not affect the candidates' ideal policy positions. My work is consistent with Downs's (1957) finding that if both candidates are office-motivated, both candidates will converge to the median voters ideal point.

A major departure of this paper twist the previous literature is that it uses time allocation between the electoral campaign and legislative work as the candidates' choice variables. From our general equilibrium analysis, we find that an increase in the challenger's opportunity cost of running for the election is negatively associated with the challenger's campaign spending, but positively associated with the incumbent's campaign spending. In addition, an increase in the length of incumbency will lead to a decrease in the incumbent's campaign spending as well as challenger's. As the effect from the incumbent's campaign spending on time declines over time, senior incumbents gain little by electoral campaign spending. Thus, longer term incumbents will spend less time on the electoral campaign compared with shorter term incumbents. We also show that when facing a stronger challenger, the incumbent will prefer spending less time on the electoral campaign and more time on legislative work. It is because that credible challengers can spend competitively on the electoral campaign. Since for senior incumbents the marginal benefit of spending time on the electoral campaign spending is smaller than that on legislative work, they will choose to maximize their utility by spending time on legislative work. Other papers have studied candidates' resources allocation, such as money, to understand candidates' behavior and its effect on the electoral outcome and we can apply these findings to our theoretical results. Erikson and Palfrey (1998) use data from the candidates' campaign spending in the U.S House elections throughout the 1970s and 1980s to study the effect of the incumbents' campaign spending on their share of votes. Their study shows that campaign spending increases the share of votes for the incumbent. However, while the effect from campaign spending is strong at a beginning, it declines over time. Our analysis not only provides theoretical support for their finding but also shows that diminishing returns from campaign spending occurs not only on money but also on time over the career of the incumbent.

This paper also links macroeconomic conditions to electoral outcomes. Realizing the state of the economy immediately prior to the election is analogous to an exogenous shocks to candidates; the candidates do not have enough time to adjust their electoral campaign strategy in order to respond to the shocks. Hence, the candidates' policy positions and time allocation will not be affected. Still, voter behavior will be affected by the intensified/weakened endemic discontent and will further affect the electoral outcome. In this scenario, the state of the economy is a vital factor in deciding the

electoral outcome. A strong economy favors the incumbent and a weak economy favors the challenger. Nevertheless, if the economic conditions are realized ex-ante, we can still consider it to be an exogenous shock. But now, the candidates have more time to respond and adjust their campaigning strategy, allocating their time to intensify/weaken the effect of endemic content/discontent caused by economic fluctuation. In this case, macroeconomic conditions will affect the candidates' strategic decision-making on time allocation and voter behavior, but will have less of an impact on the electoral outcome.

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