Contests with a Generalized Difference Form

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ABSTRACT: We explore the properties and implications of a general class of "difference-form" contests that has been derived for settings in which rent-seeking involves persuasion. Our study characterizes equilibria and analyzes the relationship between the extent of rent dissipation and the underlying contest characteristics. In so doing, we find results that are different from those of the traditional ratio model. For instance, in the pure-strategy equilibrium, it is possible that one or both contestants expend zero effort. Applications of such outcomes include lobbying, election campaigns, industrial disputes and lawsuits where one-sided submission and two-sided peace between the parties can occur as a Cournot equilibrium. Also in contrast to the traditional ratio model, we find that the extent of rent dissipation is non-monotonic in the number of contestants N while in the traditional ratio model it is strictly increasing in N.

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1. Introduction

The literature on rent- seeking games has expanded quite rapidly in the past two decades. According to Anderton (2001), one of the necessary building blocks of a unifying micro-theory of conflict economics is the contest success function (CSF), which specifies how the appropriative efforts of agents lead to an appropriative outcome. To date, two families of CSFs have been developed. The first family of the CSF comes from the Tullock (1980) rent-seeking game in which a contestant's winning probability depends on the ratio of fighting efforts. In the second family of success functions, called ''difference-form'' success functions, a contestant's probability of winning depends upon the difference of fighting efforts (Hirshleifer 1995).

The game-theoretic rent-seeking model studied by Tullock (1980) marked a starting point for numerous studies on the subject. Perez-Castrillo and Verdier (1992) stressed the importance of the shape of the players' reaction curve in order to understand the impact of the technology[1] of rent-seeking on the structure of the outcome of the game. Their findings indicate that in the case when the rent-seeking technology displays constant or decreasing returns to scale[2], the reaction curve of the agent is continuous in the bets of the other agents: increasing at first when the outside competition is weak and decreases continuously as the outside competition increases. In the case of increasing returns to scale a sharp discontinuity in the reaction curve obtains. This discontinuity is essentially due to the nonconvexity of the profit function of the agent. Based on the type of technology of rent-seeking, the authors were also able to characterize the type of pure strategy equilibria that may result in the game. With constant or decreasing returns to scale and for a fixed number of agents, there exists a unique Nash equilibrium which is symmetric. With increasing returns to scale and if the number of agents is not too high, there also exists a unique symmetric Nash equilibrium. However if the number of agents is too high, then there exists a multiplicity of equilibria which are asymmetric with N* of agents devoting the same amount of resources to rent-seeking and N-N* agents remain inactive.

Nitzan (1994) surveyed alternative ways of modeling rent seeking contests where the primary concern was given to the relationship between the extent of rent dissipation[3] and the underlying contest characteristics: e.g., the number of players, the asymmetry among the players, the source of the rent, the nature of the rent or the nature of the rent setter. He showed that the extent of rent dissipation is increasing in the number of rent seekers and in the marginal return to lobbying outlays.

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[1] Following Tullock (1980) contestant i's probability of winning a contested prize is $p_i = e_i^r / (e_i^r + \sum_{\substack{j=1 \\ j \neq i}}^{N} e_j^r)$, when contestants j = 1,……,N expend "effort" $e_j \geq 0$. According to Perez-Castrillo and Verdier (1992), $r \geq 0$ characterizes the returns of scale of the technology of rent-seeking.

[2] When $r \leq 1$ the technology of rent-seeking may be considered with decreasing returns of scale while when $r > 1$ the technology is with increasing returns.

[3] Rent dissipation is defined as the ratio between total rent-seeking outlays in equilibrium and the value of the contested rent.

Hirshleifer (1989), studied difference-form contests with two contestants, obtaining results that are different from those of the Tullock game. Hirshleifer pointed out that a crucial flaw of the traditional ratio model is that neither one-sided submission nor two-sided peace between the parties can ever occur as a Cournot equilibrium. In contrast, both of these outcomes are entirely consistent with a model in which success is a function of the difference between the parties' resource commitments. Che and Gale (1998) characterized equilibria for all parameter values for a particular class of difference-form contest success function, namely the piecewise linear function. In their work, they find similarities between general difference-form contests and all-pay auctions.

Skaperdas and Vaidya (2005) propose a general class of "difference-form" contests that has been derived for settings in which rent-seeking involves persuasion. Examples of such settings include litigation, advertising, lobbying, electoral campaigning or argumentation in policy debates where contending parties expend resources to persuade an audience of the correctness of their view. They examine how the probability of persuading the audience depends on the resources expended by the parties, so that persuasion can be modeled as a contest. In our work we will attempt to explore the properties and implications of the functional form proposed by Skaperdas and Vaidya by providing a complete characterization of the players' reaction functions and the pure strategy equilibria. We will also discuss the relationship between the extent of rent dissipation and the underlying contest characteristics: the nature of the contested rent, the number of players, the cost per unit of persuasion activity, and the force of the evidence.

Our results show that when the persuasion function is symmetric, the reaction curve of our agent is independent of the efforts of the other agent and the non-cooperative persuasion equilibrium is symmetric in the equilibrium efforts expended by both agents. Moving to more complicated cases with asymmetric cost functions and asymmetric contestable rents, the non-cooperative persuasion equilibrium is asymmetric in the equilibrium efforts expended by both agents. The agent with the relatively lower cost of persuasion per unit of her valuation of the contestable rent expends greater persuasion effort in equilibrium. The increase in the agent's equilibrium persuasion effort is due to the concavity of the evidence production function and to the decrease in her marginal cost relative to the marginal cost of the other agent. With asymmetric evidence production functions, the non-cooperative persuasion equilibrium is also asymmetric in the equilibrium efforts expended by both agents with the agent on the side of the truth expending greater persuasion effort in equilibrium. In this case, the increase in the agent's equilibrium persuasion effort is due to the increase in her marginal benefit relative to the marginal benefit of the other agent.

We also characterize the type of pure strategy equilibria that may result in the game. First we discuss the case where the total number of agents potentially interested in persuasion is fixed to some number $N(\geq 2)$. If the number of agents N is such that the positive profits condition is satisfied then there exists a unique Nash Equilibrium which is symmetric. However if the number of agents is too high and the positive profits condition is no longer satisfied, then there exists a multiplicity of equilibria which are asymmetric with N^* of agents devoting the same amount of resources to persuasion and $N - N^*$ agents remain inactive. In contrast to Nitzan (1994), we show that the extent of rent dissipation is non-monotonic in the number of rent seekers N. We also show that the extent of rent dissipation is non-monotonic in the contestable rent and in the cost per unit of persuasion activity while increasing in the force of the evidence presented to a third party audience. The greater the force of the evidence presented to the third party audience, the greater the return to the resource investment by both contestants and the higher is the amount of persuasion effort expended by all parties in equilibrium.

Finally we examine the case when the persuasion function is asymmetric. Our results show that the reaction curve of our agent is determined by the amount of effort expended by the other agent and by the degree of asymmetry between the likelihood ratios of judgment held by the third party audience. The reaction curve of each agent is continuous in the efforts of the other agent with the reaction curve of the favored[4] agent increasing continuously as the outside competition increases while the reaction curve of the other agent decreases continuously as the outside competition increases.

The paper is organized as follows. Section 2 describes an alternative to the Tullock functional form proposed by Skaperdas and Vaidya (2005). Section 3 outlines the model under symmetry and characterizes the non-cooperative pure strategy equilibrium with two agents. Sections 4 through 8 focus on the symmetric case and characterize: the non-cooperative pure-strategy equilibrium with asymmetric cost functions and asymmetric contestable rents, the non-cooperative persuasion equilibrium with asymmetric evidence production functions, the non-cooperative pure strategy equilibria with N agents, persuasion with a fixed number of agents N, and the relationship between the extent of rent dissipation and the underlying contest characteristics. Section 9 examines the non-cooperative persuasion equilibrium with asymmetry. Section 10 concludes.

2. Persuasion Function as an Alternative to the Tullock Functional Form

To lay out the building blocks of persuasion, Skaperdas and Vaidya (2005) examine an alternative evidence production process. Two players, player 1 and player 2, compete to gather and present evidence so as to influence the verdict of a third party audience in their favor. With discrete evidence production, Player 1 can either produce evidence in her favor denoted by E_1, or offer no evidence, denoted by ϕ. Similarly, Player 2 can either produce evidence in her favor, denoted by E_2, or offer no evidence, ϕ. The production of such evidence is not deterministic. The amount of resources enhances the probability of finding a favorable piece of evidence. The authors let $h(r_1)$ denote the probability that player 1 will find evidence in her favor. This probability is increasing in r_1, the resources expended on finding that evidence. Similarly $h(r_2)$

[4] One agent is favored in terms of the force of the evidence presented to the third party audience and in terms of the negative bias in judgment by the third party audience.

denotes the probability that player 2 will find evidence in her favor, with that probability also increasing in the resources r_2 expended by the player. Thus in terms of evidence there are four possible states of the world that can be faced by the third party audience: $(E_1, E_2), (E_1, \phi), (\phi, E_2), and\ (\phi, \phi)$ occurring with the following probabilities: $h(r_1)h(r_2),\ h(r_1)[1-h(r_2)],\ [1-h(r_1)]h(r_2),$ and $[1-h(r_1)][1-h(r_2)]$ respectively. Given the posterior probability of player 1 winning (and of player 2 losing) that will be induced by each realized combination of evidence and given the function $h(.)$, the ex ante probability of player 1 winning (and of player 2 losing) can be straightforwardly calculated:

$$
\begin{aligned}
p_1(r_1, r_2) = {} & h(r_1)h(r_2)\pi^*(E_1, E_2) + h(r_1)[1-h(r_2)]\pi^*(E_1, \phi) \\
& + [1-h(r_1)]h(r_2)\pi^*(\phi, E_2) + [1-h(r_1)][1-h(r_2)]\pi^*(\phi, \phi)
\end{aligned} \tag{2}
$$

Equation (2) above can be rearranged as:

$$
p_1(r_1, r_2) = \pi + \pi\big[(\Gamma - 1)h(r_1) - (1-\delta)h(r_2)\big] + [\pi^*(E_1, E_2) + (1-\delta-\Gamma)\pi]h(r_1)h(r_2) \tag{3}
$$

where π^* represents the third party audience's posterior probability of winning for player 1 and π represents the third party audience's prior. Γ and δ are defined according to the restrictions below:

$$
\begin{aligned}
& \pi^*(\phi, \phi) = \pi\,; \\
& \pi^*(\phi, E_2) = \delta\pi \ \ for\ some\, \delta \in (0,1)\,; \\
& \pi^*(E_2, \phi) = \{ \begin{matrix} \Gamma\pi \ if\ \Gamma \le 1/\pi \\ 1\ if\ \ \Gamma > 1/\pi \end{matrix} \quad where\, \Gamma > 1
\end{aligned} \tag{4}
$$

Skaperdas and Vaidya also examine equation (3) above in the case with symmetry when $\alpha \equiv 1-\delta = \Gamma - 1$, $\pi^*(\phi, \phi) = \pi^*(E_1, E_2) = \pi$, and $\pi = \dfrac{1}{2}$. The probability that player 1 wins then takes the following simple form:

$$
p_1(r_1, r_2) = \pi + \alpha\pi\big[h(r_1) - h(r_2)\big] \qquad 1 > \alpha > 0, \tag{5}
$$

where r_1, r_2, and $h(.)$ are as defined above. α represents the force of the evidence (the higher is the force of the evidence the higher is the contestant's probability of winning the contest).

3. The symmetric case

Using the symmetric form of the persuasion function that was derived in Skaperdas and Vaidya (2005) and explained in section 2 above, we consider the basic rent-seeking contest with 2 contending parties confronting the opportunity of winning a fixed prize, the contestable rent, X. The contending parties expend resources to persuade a third party audience of the correctness of their view. The probability of persuading the audience depends on the resources expended by the parties so that the probability that agent 1 wins takes the simple form:

$$p_1(r_1,r_2) = \frac{1}{2} + \alpha\frac{1}{2}\left[h(r_1) - h(r_2)\right] \qquad 1 > \alpha > 0, \tag{6}$$

where the parameter α and the probability function $h(r)$ are as defined in equation (5) above. We also suppose that $h: U \subseteq \Re \to \Re$ *is* C^2, *i.e.* h'' *is continuous with* $h''(r) \leq 0$ *for all* $r \in U$.

The expected profit of agent 1 may then be written as:

$$V^1(r_1) = \{\frac{1}{2} + \alpha\frac{1}{2}\left[h(r_1) - h(r_2)\right]\}X - cr_1 \qquad 1 > \alpha > 0, \tag{7}$$

where c represents the cost per unit of persuasion activity and X represents the contestable rent. The agent's decision problem then becomes to maximize her expected profit V^1 taken r_2 as given and under the constraint $r \geq 0$. The first order condition yields:

$$r_1^* = (h')^{-1}(\frac{2c}{\alpha X}) \qquad 1 > \alpha > 0, \tag{8}$$

We note that r_1^* represents the optimal effort chosen by our agent. Proposition 1 below characterizes her best response function:

Proposition 1

Two cases are possible:

For $r_2 < G(c,\alpha,X)$, then r_1^* is strictly positive for $r_2 \geq 0$ and determined by the first order condition (8).

For $r_2 \geq G(c,\alpha,X)$, then $r_1^* = 0$.

where $G(c,\alpha,X) = h^{-1}[h(r_1^*) + \frac{1}{\alpha} - \frac{2c}{\alpha X}r_1^*]$ and $h''(r) \leq 0$.

Proof: See Appendix.

From proposition 1 above, our agent will choose to invest in persuasion activities determined by the first order condition (8) only if the persuasion efforts r_2 exerted by the outside agent is less than a certain level given by $G(c,\alpha,X)$. For persuasion efforts r_2 greater than or equal to $G(c,\alpha,X)$, our agent will choose not to participate in persuasion activities and her best response function r_1^* is equal to zero. The point $G(c,\alpha,X)$ marks the point of complete dissipation of the rents.

Proposition 1 defines the shape of the reaction function r_1^* which is represented in figure 1 below. From figure 1 below, we notice that the best response of our agent r_1^* falls discontinuously from a positive value determined by the first order condition (8) to zero as the effort exerted by the other agent r_2 passes through the threshold value of $G(c,\alpha,X)$.

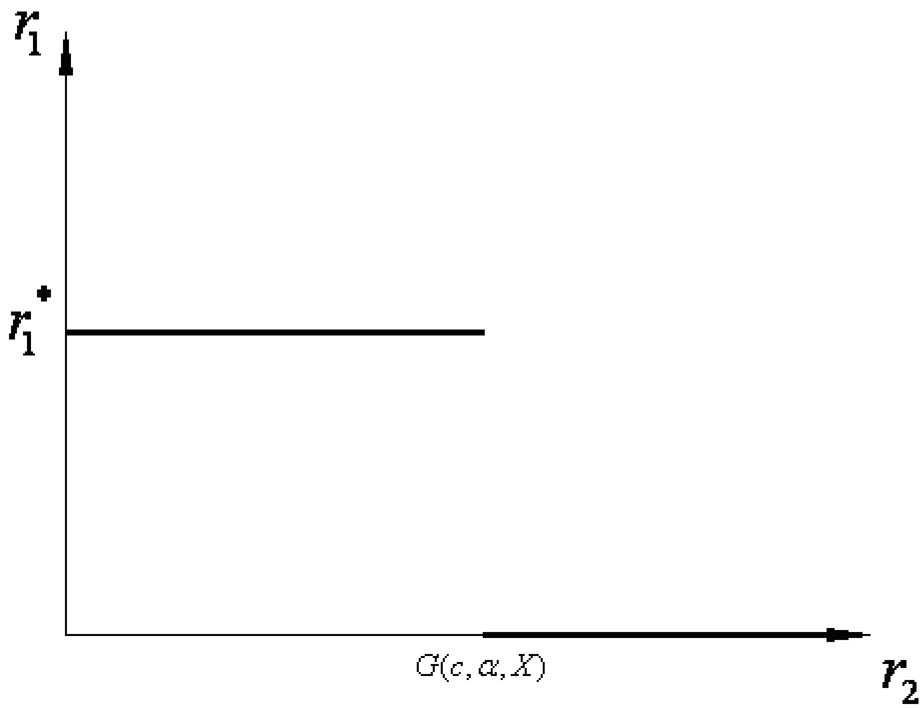

Figure 1. Agent 1's reaction curve

Proposition 2

A multiplicity of Nash equilibria are possible in this case. One possible Nash equilibrium is such that both agents invest the same amount of persuasion effort r^* in persuasion given by equation (8) above. A second possible Nash equilibrium is such that both agents invest zero persuasion effort in persuasion.

A necessary condition for the existence of the first Nash equilibrium (where both agents invest the same amount of persuasion effort r^*) is that $r_1^* < \frac{X}{2c}$ and $r_2^* < \frac{X}{2c}$. The equilibrium expected profit of agent 1 involved in persuasion is then:

$$V_1^* = [\frac{1}{2}X + \frac{\alpha}{2}X\,h(r_1^*) - \frac{\alpha}{2}X\,h(r_2^*)] - c\,r_1^* \qquad (9)$$

$$V_1^* = \frac{1}{2} X - c r^* \quad (10)$$

while agent 2's expected profit takes the following form:

$$V_2^* = [\frac{1}{2} X + \frac{\alpha}{2} X\, h(r_2^*) - \frac{\alpha}{2} X\, h(r_1^*)] - c r_2^* \quad (11)$$

$$V_2^* = \frac{1}{2} X - c r^* \quad (12)$$

Proof: See Appendix.

Figure 2 below maps a possible Nash equilibrium of the 2-player game described in proposition 2 above. We can see from the figure that the total cost of persuasion effort expended in equilibrium $2 c r^*$ cannot exceed the amount of the contestable rent X.

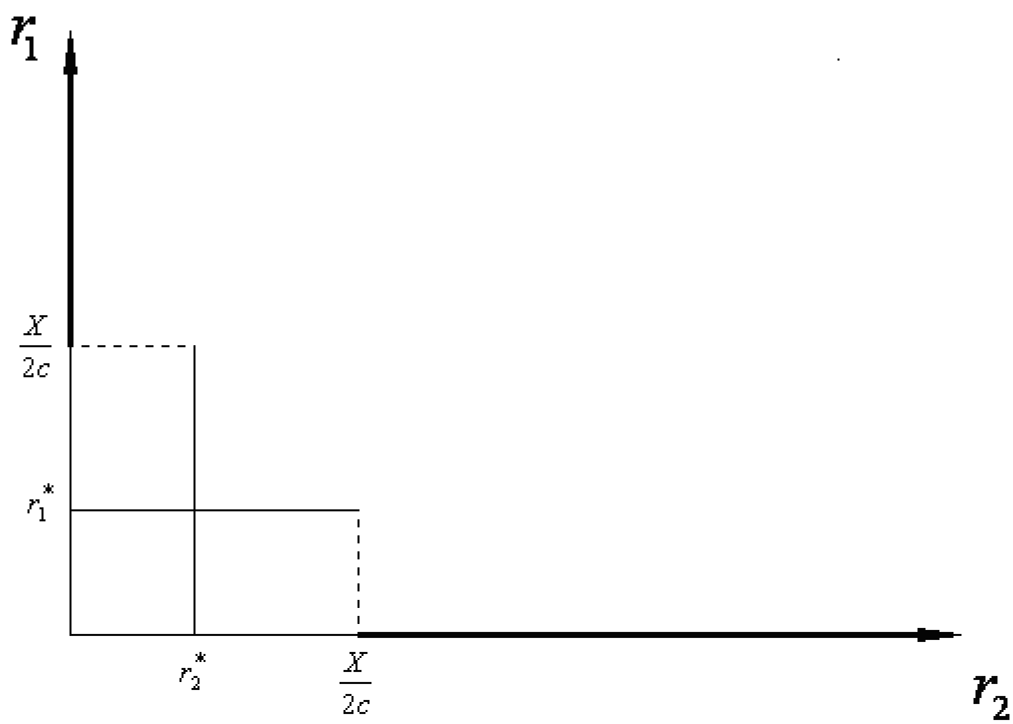

Figure 2 Non-cooperative pure strategy equilibrium

The analysis in sections 4 though 8 below will focus on the symmetric case discussed above. Section 9 will examine the non-cooperative persuasion equilibrium with asymmetry when $1-\delta \neq \Gamma - 1$ and $\pi^*(\phi,\phi) \neq \pi^*(E_1, E_2)$.

4. Non-cooperative persuasion equilibrium with asymmetric cost functions and asymmetric contestable rents

We consider the case with two agents under asymmetric cost structures. Agent 1's cost function is $C_1(r_1) = c_1 r_1$ while agent 2's cost function is $C_2(r_2) = c_2 r_2$ with $c_1 \neq c_2$. We will also consider that agent 1's interpretation of the contestable rent X differs from agent 2's and are given as X_1 and X_2 respectively. The expected profit of agent 1 may be then written as:

$$V^1(r_1) = \{\frac{1}{2} + \frac{\alpha}{2}\left[h(r_1) - h(r_2)\right]\} X_1 - c_1 r_1, \qquad 1 > \alpha > 0 \tag{13}$$

where agent's 1 decision problem is to maximize her expected profit V^1 taking r_2 as given and under the constraint $r \geq 0$. The first order condition yields:

$$r_1^* = (h^{'})^{-1}(\frac{2c_1}{\alpha X_1}), \qquad 1 > \alpha > 0 \tag{14}$$

We note that r_1^* represents the optimal effort chosen by agent 1. Similarly for agent 2:

$$r_2^* = (h^{'})^{-1}(\frac{2c_2}{\alpha X_2}), \qquad 1 > \alpha > 0 \tag{15}$$

Proposition 3

A multiplicity of Nash equilibria are possible in this case. One possible Nash equilibrium is such that the 2 agents invest different amounts of persuasion effort given by their optimal efforts in equations (14) and (15) above. A second possible Nash equilibrium is such that one agent exerts zero effort while the second agent exerts positive effort given by her first order condition in equations (14) or (15) above. The third possible Nash equilibrium is such that both agents invest zero persuasion effort.

$$\frac{r_1^*}{r_2^*} = (h^{'})^{-1}(\frac{c_1 X_2}{c_2 X_1}) , \tag{16}$$

such that $\frac{r_1^*}{r_2^*} > 1$ for $\frac{c_1}{X_1} < \frac{c_2}{X_2}$ and $\frac{r_1^*}{r_2^*} < 1$ for $\frac{c_1}{X_1} > \frac{c_2}{X_2}$.

Necessary conditions for the existence of such equilibria are

$r_1^* \leq [h(r_1^*) - h(r_2^*) + \frac{1}{\alpha}][\frac{\alpha X_1}{2c_1}]$ and $r_2^* \leq [h(r_2^*) - h(r_1^*) + \frac{1}{\alpha}][\frac{\alpha X_2}{2c_2}]$. The equilibrium expected profits to each agent involved in persuasion are:

$$V_1^* = [\frac{1}{2}X_1 + \frac{\alpha}{2}X_1 h(r_1^*) - \frac{\alpha}{2}X_1 h(r_2^*)] - c_1 r_1^* \quad (17)$$

$$V_2^* = [\frac{1}{2}X_2 + \frac{\alpha}{2}X_2 h(r_2^*) - \frac{\alpha}{2}X_2 h(r_1^*)] - c_2 r_2^* \quad (18)$$

Proof: See Appendix.

Figure 3 below maps one possible Nash equilibrium of the 2-player game described in proposition 3 above. The non-cooperative persuasion equilibrium is asymmetric in the equilibrium efforts expended by both agents. The agent with the relatively lower cost of persuasion per unit of her own interpretation of the contestable rent expends greater persuasion effort in equilibrium.

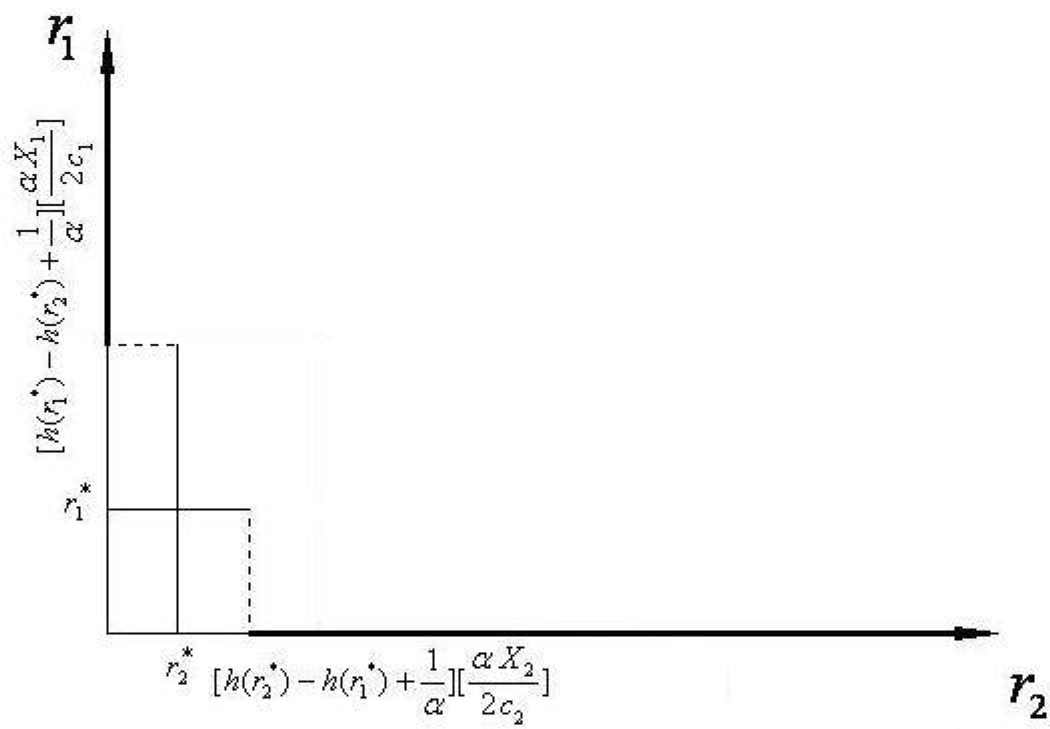

Fig. 3. Non-cooperative pure strategy equilibrium – $\frac{c_1}{X_1} < \frac{c_2}{X_2}$

5. Non-cooperative persuasion equilibrium – with asymmetric evidence production functions

We examine a setting where two players, player 1 and player 2, compete to gather and present evidence so as to influence the verdict of a third party audience in their favor. When player 1 is on the side of the truth, evidence production by player 1 and player 2 can be determined by the following functions:

$$e_1 = \theta h(r_1) \tag{19}$$

$$e_2 = (1-\theta) h(r_2) \tag{20}$$

$$\frac{1}{2} < \theta < 1 \tag{21}$$

Where e_1 and e_2 represent evidence production for player 1 and player 2 respectively. The function $h(.)$ is as defined in (6) above while the parameter θ captures the fact that the truth does matter in the production of evidence. Given, for example, that θ >1/2, if both parties devoted an equal amount of effort to gather and present evidence to the court, the outcome would favor player 1.

The expected profit to player 1 may be then written as:

$$V^1(r_1) = \{\frac{1}{2} + \frac{\alpha}{2}\left[\theta h(r_1) - (1-\theta) h(r_1)\right]\} X - cr_1, \qquad 1 > \alpha > 0 \tag{22}$$

where player 1's decision problem is to maximize her expected profit V^1 taking r_2 as given and under the constraint $r \geq 0$. The first order condition yields:

$$r_1^* = (h^{'})^{-1}(\frac{2c}{\alpha \theta X}), \qquad 1 > \alpha > 0 \tag{23}$$

We note that r_1^* represents the optimal effort chosen by player 1. Similarly for player 2:

$$r_2^* = (h^{'})^{-1}(\frac{2c}{\alpha (1-\theta) X}), \qquad 1 > \alpha > 0 \tag{24}$$

Proposition 4

A multiplicity of Nash equilibria are possible in this case. One possible Nash equilibrium is such that the 2 agents invest different amounts of persuasion efforts given by their optimal efforts in equations (23) and (24) above. A second possible Nash equilibrium is such that one agent exerts zero effort while the second agent exerts positive effort given by her first order condition in equations (23) or (24) above. The third possible Nash equilibrium is such that both agents invest zero persuasion effort in persuasion.

$$\frac{r_1^*}{r_2^*} = (h')^{-1}(\frac{1-\theta}{\theta}) \ , \qquad (25)$$

such that $\frac{1}{2} < \theta < 1$ and $\frac{r_1^*}{r_2^*} < 1$. The parameter θ captures the fact that the truth does matter in the production of evidence with player 1 being favored in this case.

Necessary conditions for the existence of such equilibrium are $r_1^* < [\theta\, h(r_1^*) - (1-\theta)\, h(r_2^*) + \frac{1}{\alpha}][\frac{\alpha X}{2c}]$ and $r_2^* < [(1-\theta)\, h(r_2^*) - \theta\, h(r_1^*) + \frac{1}{\alpha}][\frac{\alpha X}{2c}]$. The equilibrium expected profits to each agent involved in persuasion are:

$$V_1^* = [\frac{1}{2} X + \frac{\alpha}{2}\theta X\, h(r_1^*) - \frac{\alpha}{2}(1-\theta) X\, h(r_2^*)] - c r_1^* \qquad (26)$$

$$V_2^* = [\frac{1}{2} X + \frac{\alpha}{2}\theta X\, h(r_2^*) - \frac{\alpha}{2}(1-\theta) X\, h(r_1^*)] - c r_2^* \qquad (27)$$

Proof: See Appendix.

Figure 4 below maps a possible Nash equilibrium of the 2-player game described in proposition 4 above. The non-cooperative persuasion equilibrium is asymmetric in the equilibrium efforts expended by both players with the player on the side of the truth expending greater persuasion effort in equilibrium. The increase in the equilibrium persuasion effort exerted by the player is due to the increase in her marginal benefit relative to the marginal benefit of the other player.

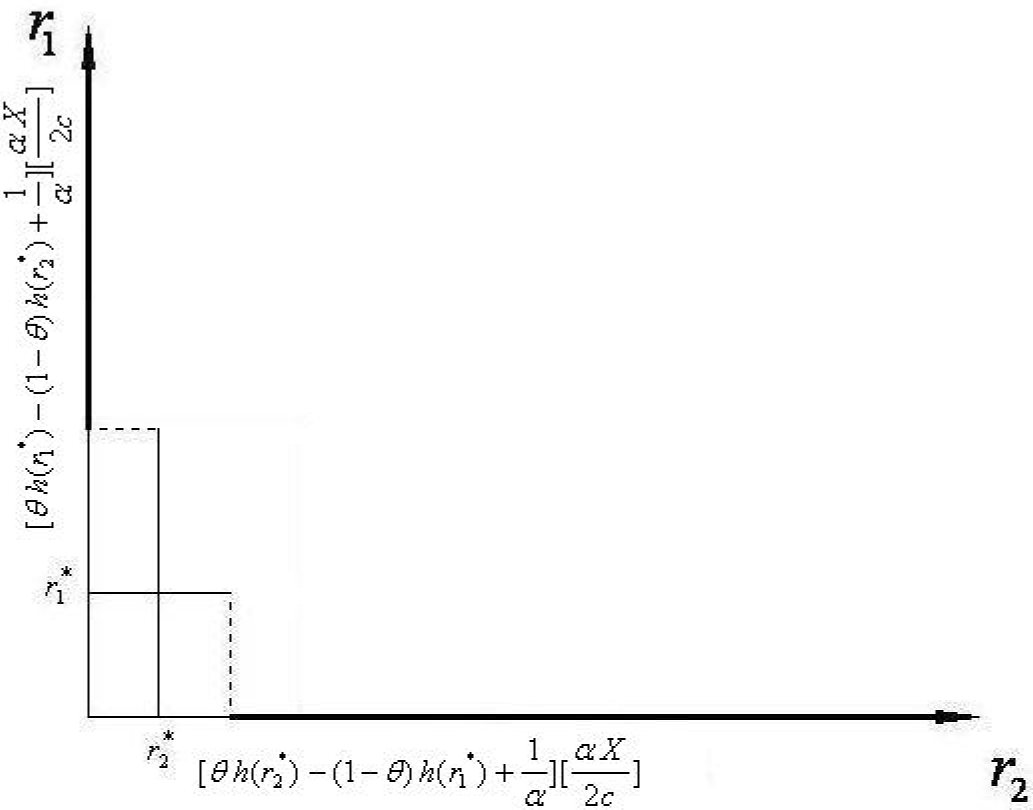

Fig. 4. Non-cooperative pure strategy equilibrium – $\theta > (1-\theta)$

6. Non-cooperative pure strategy equilibria with N agents

With N contending parties confronting the opportunity of winning a fixed prize X, we conjecture that the probability that agent i wins takes the following form:

$$p^i(r_i, r_{-i}) = \frac{1}{N} + \frac{\alpha}{N}\left[h(r_i) - \frac{\sum_{\substack{j=1 \\ i \neq j}}^{N} h(r_j)}{N-1} \right], \qquad 1 > \alpha > 0 \tag{28}$$

$h(.)$ is increasing and the other variables and functions are similarly defined to those in (6). The expected profit of agent i may be then written as:

$$V^i(r_i, r_{-i}) = \{\frac{1}{N} + \frac{\alpha}{N}\left[h(r_i) - \frac{\sum_{\substack{j=1 \\ i \neq j}}^{N} h(r_j)}{N-1} \right]\}X - cr_i, \qquad 1 > \alpha > 0 \qquad (29)$$

Where the agent's decision problem is to maximize her expected profit V^i taking the average effort exerted by other agents r_j as given and under the constraint $r \geq 0$. The first order condition yields:

$$r_i^* = (h')^{-1}(\frac{Nc}{\alpha X}), \qquad 1 > \alpha > 0 \qquad (30)$$

We note that r_i^* represents the optimal persuasion effort chosen by agent i. Proposition 5 below characterizes agent i's best response function:

Proposition 5

Two cases are possible:

For $r_j < G(N, c, \alpha, X)$, then r_i^* is strictly positive for r_j >0 and determined by the first order condition (30).

For $r_j > G(N, c, \alpha, X)$, then r_i^* = 0.

where $G(N, c, \alpha, X) = h^{-1}[h(r_i^*) + \frac{1}{\alpha} - \frac{Nc}{\alpha X} r_i^*]$ and $h''(r) \leq 0$

Proof: See Appendix.

Proposition 6

Nash equilibria with N agents are such that all agents invest the same amount of persuasion effort r^* in persuasion. One possible Nash equilibrium is such that all agents invest the same amount of effort given by equation (30) above. The other possible Nash equilibrium is such that all agents invest zero effort.

A necessary condition for the existence of such an equilibrium is $r^* \leq \frac{X}{Nc}$. The equilibrium expected profit to each agent involved in persuasion is:

$$V^* = [\frac{X}{N} + \frac{\alpha X}{N} h(r^*) - \frac{\alpha X}{N} h(r^*)] - c r^* \qquad (31)$$

$$V^* = \frac{X}{N} - c\,r^* \tag{32}$$

Proof: See Appendix.

Similar to section 3 above, the total cost of persuasion effort expended in equilibrium $N\,c\,r^*$ cannot exceed the amount of contestable rent X .

7. Persuasion with a fixed number of agents N

In this section we discuss how the total number of agents $N(\geq 2)$ potentially interested in persuasion can affect the symmetry of the Nash equilibrium in pure strategies.

Proposition 7

a) If $\frac{X}{c} > N(h^{'})^{-1}(\frac{N\,c}{\alpha\,X}) > 0$, then there is a unique Nash equilibrium in which all the N agents participate in persuasion activity with each of them investing $r^* = (h^{'})^{-1}(\frac{N\,c}{\alpha\,X})$. The equilibrium expected profit for each agent is equal to $V^* = \frac{X}{N} - (c)(h^{'})^{-1}(\frac{N\,c}{\alpha\,X}) > 0$.

b) If $\frac{X}{c} \leq N(h^{'})^{-1}(\frac{N\,c}{\alpha\,X})$, let N^* be the highest number of agents such that $\frac{X}{c} > N(h^{'})^{-1}(\frac{N\,c}{\alpha\,X}) > 0$. Then if $N^* > 1$, the Nash equilibria in pure strategies are asymmetric. There exists an equilibrium with N^* agents participating in persuasion activity and $N - N^*$ non-participating agents in which each of the participating agents devotes $r^* = (h^{'})^{-1}(\frac{N\,c}{\alpha\,X})$ of resources to persuasion and receives a profit $V^* = \frac{X}{N} - (c)(h^{'})^{-1}(\frac{N\,c}{\alpha\,X}) > 0$. Each of the non-participating agents invests nothing and has zero profit.

8. Non-cooperative persuasion equilibrium and the extent of rent dissipation

The existing rent-seeking literature is concerned with the existence and characterization of Nash-equilibria and, in particular, with the relationship between total rent-seeking outlays in equilibrium and the value of the contested rent. The ratio D between these two values is called the extent of rent dissipation. This ratio is important for empirical applications since it can serve as a direct measure for inferring the value of the resources spent on the contested rent from its value [Nitzan (1994)]. Our results show that the extent of rent dissipation is non-monotonic in the number of agents N while in the Tullock function it is strictly increasing in N. We also show that the extent of rent dissipation is decreasing in the degree of concavity of the evidence production function similar to the Tullock function where it is decreasing in the degree of concavity of the effort production function.

Proposition 8

When the probabilistic contest success functions are symmetric and of the form shown in (30) above and if an interior Nash equilibrium in pure strategies exists, then the extent of rent dissipation is at most 1 as $D = (\frac{N c}{X})(h^{'})^{-1}(\frac{N c}{\alpha X})$. The extent of rent dissipation can be divided into the two multiplicative parts $\frac{N c}{X}$ and $(h^{'})^{-1}(\frac{N c}{\alpha X})$ where $\frac{N c}{X}$ represents the total cost of persuasion activity per unit of the contestable rent while the term $(h^{'})^{-1}(\frac{N c}{\alpha X})$ represents the symmetric equilibrium effort. The extent of rent dissipation is non-monotonic in the contestable rent X, in the number of rent seekers N and in the cost per unit of persuasion activity c while increasing in the parameter α. The extent of rent dissipation is increasing in the number of agents N and in the cost per unit of persuasion activity c when the symmetric equilibrium effort is decreasing at a slower rate than the rate of increase in the total cost of persuasion activity per unit of the contestable rent $\frac{1}{N}$ and $\frac{1}{c}$ respectively. Conversely the extent of rent dissipation is decreasing in the contestable rent X when the symmetric equilibrium effort is decreasing at a faster rate than the rate of decrease in the total cost of persuasion activity per unit of the contestable rent $-\frac{1}{X}$. The parameter α represents the force of the evidence (the higher is α the higher is the contestant's probability of winning the contest). So the higher the α, the greater the return to the resource investment by both contestants and the higher is the amount of persuasion effort expended by all parties in equilibrium.

Proof: See Appendix.

In the appendix, we show that when the rate of decrease in the symmetric equilibrium effort $A = -\frac{\frac{\partial (h')^{-1}}{\partial N}(\frac{N c}{\alpha X})}{(h')^{-1}(\frac{N c}{\alpha X})}$ is less than the rate of increase in the total cost of persuasion activity per unit of the contestable rent $\frac{1}{N}$ then the extent of rent dissipation D is increasing in the number of agents N and decreasing otherwise. From a more technical viewpoint, A is a measure of the degree of concavity of the probability function $h(.)$. It measures the speed at which the marginal probability of finding evidence in the agent's favor is decreasing. So when the speed at which the marginal probability of finding evidence in the agent's favor is decreasing at a slower rate than the rate of increase in the total cost of persuasion activity per unit of the contestable rent $\frac{1}{N}$ then the extent of rent dissipation D is increasing in the number of agents N and decreasing otherwise. A Similar relationship can be established between the cost per unit of persuasion activity c and the extent of rent dissipation D while a converse relationship can be established between the contestable rent X and the extent of rent dissipation D.

9. Non-cooperative persuasion equilibrium - asymmetric case

With asymmetry, the probability that agent 1 wins takes the following functional form:

$$\begin{aligned} p_1(r_1, r_2) &= \pi + \pi\big[(\Gamma - 1)h(r_1) - (1-\delta)h(r_2)\big] \\ &\quad + [\pi^*(E1, E2) + (1-\delta-\Gamma)\pi]h(r_1)h(r_2), \end{aligned} \tag{33}$$

while agent 2's probability takes the following form:

$$\begin{aligned} p_2(r_1, r_2) &= 1 - \pi + \pi\big[(1-\delta)h(r_2) - (\Gamma - 1)h(r_1)\big] \\ &\quad + [(\delta + \Gamma - 1)\pi - \pi^*(E1, E2)]h(r_1)h(r_2), \end{aligned} \tag{34}$$

where Γ, δ, π and $h(.)$ are as defined in (3) and (4) above. The expected profit of agents 1 and 2 may be then written as:

$$\begin{aligned} V^1(r_1) &= \{\pi + \pi\big[(\Gamma - 1)h(r_1) - (1-\delta)h(r_2)\big] \\ &\quad + [\pi^*(E1, E2) + (1-\delta-\Gamma)\pi]h(r_1)h(r_2)\}X - cr_1, \end{aligned} \tag{35}$$

$$\begin{aligned} V^2(r_2) &= \{1 - \pi + \pi\big[(1-\delta)h(r_2) - (\Gamma - 1)h(r_1)\big] \\ &\quad - [\pi^*(E1, E2) + (1-\delta-\Gamma)\pi]h(r_1)h(r_2)\}X - cr_2, \end{aligned} \tag{36}$$

Where the agent's decision problem is to maximize her expected profit V taken the resources expended by the other agent as given and under the constraint $r \geq 0$.

The first order condition yields:

$$r_1^* = (h')^{-1}\left(\frac{c}{[\pi(\Gamma-1)X + [\pi^*(E1, E2) + (1-\delta-\Gamma)\pi]Xh(r_2)]}\right), \tag{37}$$

while,

$$r_2^* = (h^{'})^{-1}\left(\frac{c}{[\pi(1-\delta)X - [\pi^*(E1,E2)+(1-\delta-\Gamma)\pi]Xh(r_1)]}\right), \tag{38}$$

We note that r_1^* represents the best response of agent 1 to a given effort r_2 exerted by agent 2. We also note that the term $[\pi^*(E1,E2)+(1-\delta-\Gamma)\pi]$ is positive providing agent 1 with the advantage in the persuasion contest. We can rewrite the term $[\pi^*(E1,E2)+(1-\delta-\Gamma)\pi]$ as:

$$[\pi^*(\phi,\phi)-\pi^*(\phi,E2)]-[\pi^*(E1,\phi)-\pi^*(E1,E2)] \tag{39}$$

which is positive if the following condition holds:

$$\left|\ \pi^*(E1,E2)-\pi^*(E1,\phi)\ \right| \le \left|\ \pi^*(\phi,E2)-\pi^*(\phi,\phi)\ \right| \tag{40}$$

The above condition is always satisfied since in the early stage of the contest (when no evidence is presented), the marginal probability increase (that is the extra contribution) of the evidence $E2$ presented by agent 2 when no evidence ϕ has been presented by agent 1, $\{\pi^*(\phi,E2)-\pi^*(\phi,\phi)\}$, is greater than the marginal probability increase of the same evidence $E2$ presented by agent 2 when evidence $E1$ has already been presented by agent 1, $\{\pi^*(E1,E2)-\pi^*(E1,\phi)\}$. Proposition 9 below characterizes the agents best response functions:

Proposition 9

The following cases are possible:

r_1^* is strictly positive and increasing in r_2 for r_2 >0 and determined by the first order condition (37).

For $r_1 < G(c,X,\pi,\delta,\Gamma)$, then r_2^* is strictly positive and decreasing in r_1 for r_1>0 and determined by the first order condition (38).

For $r_1 \ge G(c,X,\pi,\delta,\Gamma)$, then r_2^* = 0.

where $G(c,X,\pi,\delta,\Gamma) = h^{-1}\left[\frac{(1-\delta)\pi}{\pi^*(E1,E2)+(1-\delta-\Gamma)\pi}\right]$ and $h^{''}(r) \le 0$

Nash equilibria are defined at the intersection of r_1^* and r_2^*

Proof: See Appendix.

From proposition 9 above, we can see that agent 2 will choose to invest in persuasion activities determined by the first order condition (38) only if the persuasion effort r_1 exerted by agent 1 is less than a certain level given by $G(c,X,\pi,\delta,\Gamma)$. For persuasion effort r_1 greater than $G(c,X,\pi,\delta,\Gamma)$, agent 2's marginal return to persuasion becomes negative and she will choose not to participate in persuasion activities with her best response function r_2^* being equal to zero.

Proposition 9 defines the Nash equilibria of the 2-player game and it also defines the shape of the reaction functions r_1^* and r_2^*. In figure 5 below, the Nash equilibrium of the game occurs at the point of intersection of the two reaction curves r_1^* and r_2^* which is indicated by the point of intersection of the two lightly shaded curves in the middle of the graph. In figure 6 below the Nash equilibrium of the game also occurs at the point of intersection of the two reaction curves r_1^* and r_2^* which is indicated by the point of intersection $(h^{'})^{-1}(\frac{c}{[\pi(\Gamma-1)X})$ at the bottom of the graph.

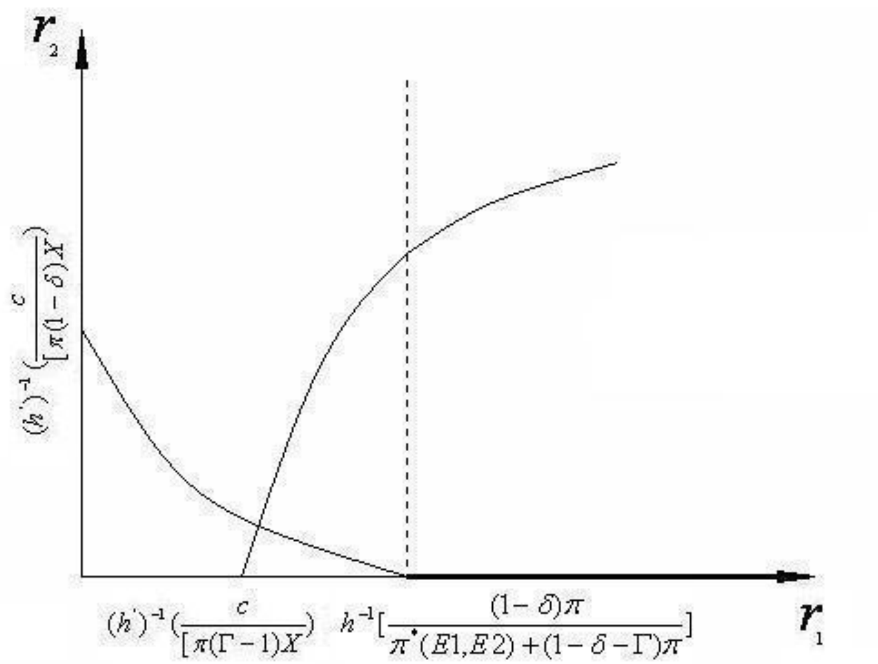

Fig. 5. Non-cooperative persuasion equilibrium - asymmetric example 1

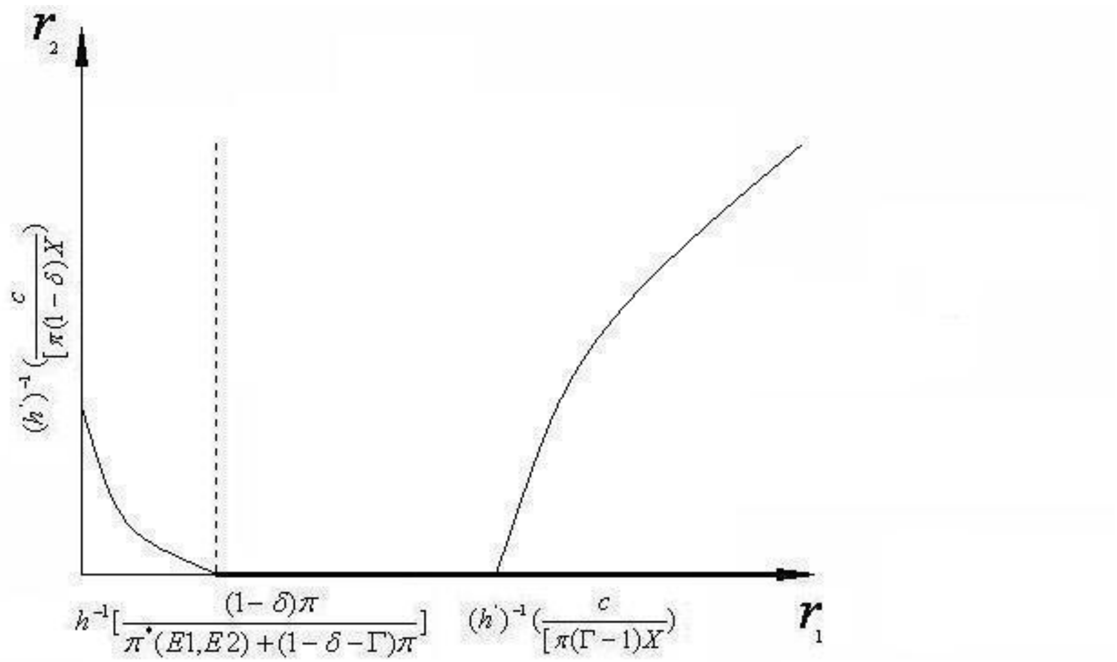

Fig. 6. Non-cooperative persuasion equilibrium - asymmetric example 2

Proposition 10

Nash equilibria with 2 agents under asymmetry are such that both agents invest different amounts of effort r^* in persuasion. One possible set of Nash equilibria is such that both agents invest different amount of effort given by equation (41) below. The other possible set of Nash equilibria is such that one agent exerts zero effort while the second agent exerts positive effort given by the first order condition (37) above.

$$\frac{r_1^*}{r_2^*} = (h')^{-1}(\frac{\pi(1-\delta)-[\pi^*(E1,E2)+(1-\delta-\Gamma)\pi]h(r_1^*)}{\pi(\Gamma-1)+[\pi^*(E1,E2)+(1-\delta-\Gamma)\pi]h(r_2^*)}), \quad (41)$$

Necessary conditions for the existence of such equilibrium are

$r_2 < h^{-1}[\frac{1}{1-\delta}+\frac{c}{\pi(1-\delta)X}\frac{h(r_1^*)}{h'(r_1^*)}-\frac{c}{\pi(1-\delta)X}r_1^*]$,

$r_1 < h^{-1}[\frac{1-\pi}{\pi(\Gamma-1)}+\frac{c}{\pi(\Gamma-1)X}\frac{h(r_2^*)}{h'(r_2^*)}-\frac{c}{\pi(\Gamma-1)X}r_2^*]$. The equilibrium expected profit to each agent involved in persuasion is:

$$V_1^* = \pi X + \frac{c\,h(r_1^*)}{h'(r_1^*)} - \pi(1-\delta)X\,h(r_2^*) - cr_1^* \quad (42)$$

$$V_2^* = (1-\pi)\,X + \frac{c\,h(r_2^*)}{h'(r_2^*)} - \pi(\Gamma-1)\,X\,h(r_1^*) - cr_2^* \tag{43}$$

Proof: See Appendix.

Our results show that the reaction curve of agent 1 is determined by the amount of effort expended by agent 2 and by the degree of asymmetry between the likelihood ratios of judgment held by the third party audience. The reaction curve of each agent is continuous in the efforts of the other agent with the reaction curve of agent 1 (the agent with the advantage as shown in equations (39) and (40) above) increasing continuously in the persuasion efforts of agent 2 while the reaction curve of agent 2 decreases continuously in the persuasion efforts of agent 2.

10. Conclusion

In this paper, we have offered a review of the general properties and implications of the new game-theoretic rent-seeking model proposed by Skaperdas and Vaidya (2005). We also offered a comparison with the traditional ratio model studied by Tullock (1980). As argued by Hirshleifer (1989), a crucial flaw of the traditional ratio model is that neither one-sided submission nor two-sided peace between the parties can ever occur as a Cournot equilibrium. In contrast, both of these outcomes are entirely consistent with the new game-theoretic rent-seeking model proposed by Skaperdas and Vaidya (2005) in which success is a function of the difference between the parties' resource commitments. We argue that the new proposed functional form is suitable for a wider range of applications than the traditional ratio model proposed by Tullock. Examples of such applications include lobbying, military combats, election campaigns, industrial disputes and lawsuits where one-sided submission and two-sided peace between the parties can occur as a Cournot equilibrium.

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Appendix

Proof of proposition 1:

The first order condition equation states:

$$r_1^* = (h')^{-1}\left(\frac{2c}{\alpha X}\right), \qquad 1 > \alpha > 0 \tag{A.1}$$

Agents will expend positive effort only in the case when they can achieve positive expected profits:

$$V^1(r_1^*) > 0 \quad \Leftrightarrow \quad \left\{\frac{1}{2}X + \frac{\alpha}{2}X\,h(r_1^*) - \frac{\alpha}{2}X\,h(r_2) - c\,r_1^*\right\} > 0 \tag{A.2}$$

$$\Leftrightarrow\ r_2 < h^{-1}\left[h(r_1^*) + \frac{1}{\alpha} - \frac{2c}{\alpha X}r_1^*\right] \tag{A.3}$$

$$\Leftrightarrow\ G(c,\alpha,X) = h^{-1}\left[h(r_1^*) + \frac{1}{\alpha} - \frac{2c}{\alpha X}r_1^*\right] \tag{A.4}$$

Proof of proposition 2:

The equilibrium expected profit of agent 1 involved in persuasion is:

$$V_1^* = [\frac{1}{2}X + \frac{\alpha}{2}X\,h(r^*) - \frac{\alpha}{2}X\,h(r^*)] - c\,r^* \tag{A.5}$$

$$V_1^* = \frac{1}{2}X - c\,r^* \tag{A.6}$$

similarly agent 2's equilibrium expected profit takes the following form:

$$V_2^* = [\frac{1}{2}X + \frac{\alpha}{2}X\,h(r^*) - \frac{\alpha}{2}X\,h(r^*)] - c\,r^* \tag{A.7}$$

$$V_2^* = \frac{1}{2}X - c\,r^* \tag{A.8}$$

Equilibria can exist only if the equilibrium profits of both agents are positive or if

$$V_1^* > 0 \tag{A.9}$$

$$V_2^* > 0 \tag{A.10}$$

This is equivalent to the conditions that

$$r_1^* < \frac{X}{2c} \tag{A.11}$$

$$r_2^* < \frac{X}{2c} \tag{A.12}$$

<u>Proof of proposition 3:</u>

A Nash equilibrium with 2 agents under asymmetric cost functions and asymmetric contestable rents is such that both agents invest different amounts of effort r^* in persuasion with

$$r_1^* = (h')^{-1}(\frac{2c_1}{\alpha\,X_1}), \qquad 1 > \alpha > 0 \tag{A.13}$$

$$r_2^* = (h')^{-1}(\frac{2c_2}{\alpha\,X_2}), \qquad 1 > \alpha > 0 \tag{A.14}$$

and

$$r_1^* = (h')^{-1}(\frac{c_1\,X_2}{c_2\,X_1})\,r_2^* \tag{A.15}$$

The equilibrium equation above characterizes the shape of all possible Nash equilibria between players 1 and 2 and is determined by the magnitude of

$\frac{r_1^*}{r_2^*} = \frac{(h')^{-1}(\frac{2c_1}{\alpha X_1})}{(h')^{-1}(\frac{2c_2}{\alpha X_2})} = (h')^{-1}(\frac{c_1 X_2}{c_2 X_1})$. From Hao and Zheng (1998), we know that the inverse of an increasing (decreasing) monotone function is also increasing (decreasing) which implies that $(h')^{-1}(.)$ is a decreasing function since $h''(.) \le 0$. This implies that $\frac{r_1^*}{r_2^*} > 1$ for $\frac{c_1}{X_1} < \frac{c_2}{X_2}$ and $\frac{r_1^*}{r_2^*} < 1$ for $\frac{c_1}{X_1} > \frac{c_2}{X_2}$.

Proof of proposition 4:

See proof of proposition 3 above.

Proof of proposition 5:

The first order condition equation states:

$$r_i^* = (h')^{-1}(\frac{N c}{\alpha X}), \qquad \alpha > 0 \tag{A.16}$$

Agents will expend positive effort only in the case when they can achieve positive expected profits:

$$V(r_i^*) > 0 \iff \{\frac{X}{N} + \frac{\alpha X}{N} h(r_i^*) - \frac{\alpha X}{N}\{\frac{\sum_{\substack{j=1 \\ i \neq j}}^{N} h(r_j)}{N-1}\} - c\, r_i^*\} > 0 \tag{A.17}$$

$$\iff r_j < h^{-1}[h(r_i^*) + \frac{1}{\alpha} - \frac{N c}{\alpha X} r_i^*] \tag{A.18}$$

$$\iff G(N, c, \alpha, X) = h^{-1}[h(r_i^*) + \frac{1}{\alpha} - \frac{N c}{\alpha X} r_i^*] \tag{A.19}$$

Proof of proposition 6:

With N agents there can only be symmetric equilibria. This equilibrium can exist only if the equilibrium profit is positive or if

$$V \quad = [\frac{X}{N} + (\frac{\alpha X}{N}) h(r^*) - (\frac{\alpha X}{N})(\frac{\sum_{\substack{j=1 \\ i \neq j}}^{N} h(r^*)}{N-1})] - cr^* > 0 \tag{A.20}$$

This is equivalent to the condition that

$$r^* < \frac{X}{N c} \tag{A.21}$$

<u>Proof of proposition 8:</u>

If an interior Nash equilibrium in pure strategies exists, then the extent of rent dissipation is $D = (\frac{N c}{X})(h^{'})^{-1}(\frac{N c}{\alpha X})$. A necessary condition for the existence of an interior Nash equilibrium in pure strategies is $(h^{'})^{-1}(\frac{N c}{\alpha X}) \leq \frac{X}{N c}$ which implies that the extent of rent dissipation is:

$$D \leq (\frac{N c}{X})(\frac{X}{N c}) \tag{A.22}$$

$$\Leftrightarrow \; D \leq 1 \tag{A.23}$$

The change in D with respect to N can be shown by the representation below:

$$\frac{\partial D}{\partial N} = (\frac{c}{X})(h^{'})^{-1}(\frac{N c}{\alpha X}) + (\frac{N c}{X})\{\frac{\partial}{\partial N}((h^{'})^{-1})(\frac{N c}{\alpha X})\} \tag{A.24}$$

From equation (A.16) and from proposition 4 above, it can be easily shown that the extent of rent dissipation D is increasing in the number of agents N when $\frac{X}{N c} > (h^{'})^{-1}(\frac{N c}{\alpha X}) > -N \frac{\partial}{\partial N}((h^{'})^{-1})(\frac{N c}{\alpha X})$, and decreasing when $0 < (h^{'})^{-1}(\frac{N c}{\alpha X}) < -N \frac{\partial}{\partial N}((h^{'})^{-1})(\frac{N c}{\alpha X})$. Similarly we can show that the extent of rent dissipation D is increasing in the cost per unit of persuasion activity c when $\frac{X}{N c} > (h^{'})^{-1}(\frac{N c}{\alpha X}) > -N \frac{\partial}{\partial c}((h^{'})^{-1})(\frac{N c}{\alpha X})$, and decreasing when $0 < (h^{'})^{-1}(\frac{N c}{\alpha X}) < -N \frac{\partial}{\partial c}((h^{'})^{-1})(\frac{N c}{\alpha X})$. It can be also shown that the extent of rent dissipation D is decreasing in the contestable rent X when

$$\frac{X}{N\,c} > (h^{'})^{-1}(\frac{N\,c}{\alpha\,X}) > X\,\frac{\partial}{\partial\,X}((h^{'})^{-1})(\frac{N\,c}{\alpha\,X})$$, and increasing when

$$0 < (h^{'})^{-1}(\frac{N\,c}{\alpha\,X}) < X\,\frac{\partial}{\partial\,X}((h^{'})^{-1})(\frac{N\,c}{\alpha\,X}).$$

<u>Proof of proposition 9:</u>

The first order condition equation states:

$$r_1^* = (h^{'})^{-1}(\frac{c}{[\pi(\Gamma-1)X + [\pi^*(E1,E2) + (1-\delta-\Gamma)\pi]Xh(r_2)]}), \qquad \text{(A.25)}$$

Agent 1 will expend positive effort only in the case when:

$$r_1^* > 0 \quad \Leftrightarrow \quad \pi(\Gamma-1)X + [\pi^*(E1,E2) + (1-\delta-\Gamma)\pi]Xh(r_2) > 0 \qquad \text{(A.26)}$$

$$\Leftrightarrow \; r_2 < h^{-1}[\frac{\pi(\Gamma-1)}{\pi^*(E1,E2) + (1-\delta-\Gamma)\pi}] \qquad \text{(A.27)}$$

<u>Proof of proposition 10:</u>

With two agents there can only be asymmetric equilibria. This equilibrium can exist only if the equilibrium profits are positive or if

$$V_1^* = \pi\,X + \frac{c\,h(r_1^*)}{h'(r_1^*)} - \pi(1-\delta)\,X\,h(r_2^*) - cr_1^* \qquad \text{(A.28)}$$

and

$$V_2^* = (1-\pi)\,X + \frac{c\,h(r_2^*)}{h'(r_2^*)} - \pi(\Gamma-1)\,X\,h(r_1^*) - cr_2^* \qquad \text{(A.29)}$$

This is equivalent to the conditions that

$$r_1 < h^{-1}[\frac{1-\pi}{\pi(\Gamma-1)} + \frac{c}{\pi(\Gamma-1)\,X}\frac{h(r_2^*)}{h^{'}(r_2^*)} - \frac{c}{\pi(\Gamma-1)\,X}r_2^*] \qquad \text{(A.30)}$$

and

$$r_2 < h^{-1}[\frac{1}{1-\delta} + \frac{c}{\pi(1-\delta)\,X}\frac{h(r_1^*)}{h^{'}(r_1^*)} - \frac{c}{\pi(1-\delta)\,X}r_1^*] \qquad \text{(A.31)}$$