

# **Judicial Review and the Power of the Executive and Legislative Branches**

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## ABSTRACT

The legal system can affect what policies a government can implement. In particular, when there is separation of powers, the strength of the judiciary to review and overturn actions of the executive and legislative branches can affect such things as how much redistribution these policy-making branches can do. Surprisingly, having judicial review helps the policy-making branches—the stronger is the judiciary, the more redistribution they are able to do. This occurs because the policy-making branches must make promises on and off the equilibrium path to individuals in order to make redistribution possible. However, in many circumstances, the government wants to renege on these promises, either to do more redistribution than promised or to not carry out severe threats against any individuals who lied. Judicial review can prevent renegeing on these promises, thus making them credible.

We develop this in the context of an optimal income tax model with a finite number of individuals where the government knows the exact distribution of types but not which individual is of which type. In this finite model, the government can detect misrevelation by even a single individual so that an individual's taxes can depend not just on one's own actions but also on others' actions. Piketty [*JET*, 1993] showed that the government could implement any full-information Pareto optimal allocation if the government could commit to its announcements, even to infeasible allocations in circumstances after some individuals misreveal. We derive the sequential equilibrium allocations when individuals reveal their types by simple announcements when feasibility on and off the equilibrium path is imposed. Increasing the degree of judicial review expands the set of achievable allocations on the full-information utility possibility frontier. We also relate the different possible legal rules to different solution concepts in economics.

## 1. Introduction

Economists study a wide variety of government policies as mechanism design problems in which the government as a principal seeks to induce certain behavior from citizens as agents. The government lacks specific information about individuals and is constrained to implement policies that are incentive compatible. One important example is the optimal income taxation problem where the government seeks to redistribute from one type of individual to another without knowing the type of any specific individual. The standard analysis of such problems treats the mechanism designer as outside the game. The designer sets rules to achieve goals but cannot adjust rules or renege on promises in the course of the game. One interpretation is that the designer is able to commit fully to whatever mechanism she selects. Hurwicz [2008] raises the issue that those who monitor a game must themselves be monitored. He suggests that this requires creating, if not an infinite regress, at least a circle of guardians in which everyone is monitored by someone else. Myerson [2009] follows up on this analysis by considering the situation of a leader (think of a monarch) and a governor subservient to him. The leader may design mechanisms to induce appropriate behavior on the part of the governor. The question then arises of why won't the leader fire the governor to avoid paying the amount promised. Myerson's model requires the leader to randomize explicitly in monitoring behavior. However, the leader is not indifferent between his actions, so he must be closely monitored by others who have the power to punish the leader. "Who has such power over a leader of a sovereign political institution?" [Myerson, 2009, p. 73].

Separation of powers is one way in which democratic societies confront this problem. The executive and legislative branches act as the mechanism designer (and principal), and the judicial branch can monitor their behavior and, in some cases, overrule their actions. In effect, a

mechanism involves promises by the policy-making branches to citizens to carry out certain actions conditional on citizens' behavior. The judicial branch may overrule the other branches if they renege on these promises (although this still leaves open the issue of who monitors the judiciary). What is crucial is that the government makes promises not only about what will happen on the equilibrium path if individuals act in a way the government wants but also about what will happen off that path if individuals deviate from the desired behavior. To induce desired behavior, the government could make harsh threats of punishments if someone deviated. If a deviation occurs, it might not be in the government's interest to carry out the punishment. That is, it would desire to renege on its promise in such circumstances. If judicial review prevented this renegeing, it would make the threat credible and strengthen the government's ability to induce the desired behavior.

This view contrasts with the conventional interpretation that judicial review is an essential part of separation of powers by acting as a check on the power of the other branches of the government as encapsulated in the phrase "checks and balances". If the judiciary has the authority to overrule actions of the executive or the legislature, it would seem straightforward that this limits the ability of the policy-making branches to do what they want. However, while this interpretation may have validity from an *ex post* view, it is misleading from an *ex ante* perspective. As an example, consider a government trying to redistribute from more able individuals to less able ones where the government knows how many there are of each type but does not know the type of any particular individual. The government must rely on individuals to self-report their types. These reports may not always be truthful. If the redistribution is extensive, it may benefit high-ability individuals to declare themselves to be of lower ability to avoid the tax and to receive the transfer. To induce revelation, the government must promise to

limit how much redistribution it will do if individuals reveal truthfully and must threaten large enough punishments when it knows that some individuals have lied about their types. However, individuals must believe that the government would do what it promised if they are to reveal their types accurately. Once individuals reveal their information, the government could gain by deviating from its promises or threats. After the fact, if individuals were truthful the government could gain by doing more redistribution than it promised. If individuals lied, it might not be in the government's interests to actually carry out the threats, especially if they were draconian. Judicial review would prevent this from happening. Without judicial review, individuals would anticipate that the government would not keep its promise of doing limited redistribution if they were truthful and would not carry out the punishments if they lied and thus individuals would be reluctant to reveal their information preventing the government from doing any redistribution. Thus, *ex ante*, judicial review, by blocking actions of the other branches in particular cases, strengthens their power to carry out policy by allowing them to make credible threats and promises which are needed to make certain policies effective

The standard view may be correct with respect to tort law. If the government harms an individual, without judicial review, there is no effective recourse except perhaps the good will of the executive. Even if the executive had waived sovereign immunity, separation of powers is needed to guarantee recourse in the courts. This view does not necessarily hold in situations in which implementing a policy is more like contract law. In enacting a policy, the government must specify what it will do in a variety of circumstances depending upon how individuals respond to the policy. This is like a complicated contract with many contingencies depending on how many individuals reveal themselves to be of each type. Failure to carry out a promise in any contingency is like a breach of a contract. Judicial review by punishing breaches strengthens the

ability of the government to enter contracts. The government needs a strong judiciary *ex ante* to keep it from breaching its contracts, even though *ex post* it gains from such breaches.<sup>1</sup>

It is important to note that judicial review is not an all-or-nothing procedure. There are different levels depending upon whether the judiciary is active—it can seek on its own to overrule other branches—or passive—it can only act in response to complaint. If the judiciary is passive, its strength in carrying out judicial review depends upon standing, that is, who is allowed to bring a complaint. A standard principle of the doctrine of standing is that only someone who has been harmed is allowed to bring a complaint.<sup>2</sup> Hence, under this doctrine, if the government violates promises in a way that helps someone but harms no one, then no complaint can be filed and judicial review cannot be invoked. A possible further limit is the doctrine of “clean hands”. In order to file a complaint, an individual must not have acted in a wrongful manner.<sup>3</sup> Thus individuals who have misrevealed their information would not have access to the courts. Individuals can only complain about policy changes which harm them based on preferences they reported and not on their true preferences if the two differ.

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<sup>1</sup> There are many other contributions in the literature on the problem of ensuring government commitment. Kydland and Prescott [1977] consider explicit rules as a replacement for continuing optimization by a government to avoid suboptimal outcomes from reacting to citizens’ behavior. Shepsle [1991] discusses the role of legislative committees as a commitment device to make it more difficult for future legislatures to overturn decisions. Myerson [2008] considers the problem of an autocrat needing to establish a “personal constitution” for his active supporters. In contrast to us, these authors primarily focus on commitment on the equilibrium path, as opposed to making credible actions taken after individuals deviate from equilibrium behavior.

<sup>2</sup> See Black [1979, p. 1261]. The Administrative Procedures Act authorizes actions by “any person ... adversely affected ... by agency action.”

<sup>3</sup> See Black [1979, p. 227]. A party cannot seek relief “if such party in his prior conduct has violated conscience or good faith...” In the American legal system, “clean hands” only applies in limited circumstances and may not be applicable in the optimal tax problem we consider. However, a legal system could extend the doctrine to such cases, and we study it because it is analogous to a particular solution concept in economics.

Depending upon whether judicial review exists and what restrictions if any are placed upon it, a hierarchy of judicial strengths exists:

(1) A unitary government (such as an absolute monarch) where all power resides in a single authority, and any branches exist solely for convenience or efficiency in carrying out the government's wishes.

(2) Separation of powers with an independent but passive judiciary with restrictions on access to the judiciary due to a "clean hands" rule. In this case, the judiciary can overrule the other branches but only if a case is brought to it by a proper party.

(3) Separation of powers with an independent but passive judiciary without "clean hands" restrictions. As in case 2, the judiciary can overrule other branches but only if a case is brought to it. In this case, any one harmed can sue even if they themselves engaged in wrongful conduct.

(4) Separation of powers with an active judiciary. In this case, the judiciary does not have to wait for a case to be brought to it. Thus, the judiciary can, indeed must, enforce contracts even if no one objects to modifications (because individuals would have taken different prior actions if they knew the executive branch would change the contract terms).

The standard view would seem to be that the power of the executive and legislative branches is strongest in case 1 and is sequentially weaker in case 2, then case 3, and finally case 4. In case 1, there are no checks on the government's power, while these checks get stronger in the other cases. As shown below, not only can the existence of judicial review strengthen the other branches, but the stronger is the judiciary's ability to review, the more powerful are the other branches in their ability to implement policies. Again for tort law, the standard view may be valid. The less limited is the judiciary, the more recourse individuals have and the less power

the executive has. Note that the effect of an active instead of a passive judiciary might come in to play in circumstances in which many individuals suffer small harm, so that no one individual gains enough from bringing a case and joint action is costly.

In policy-making situations such as redistribution that are analogous to contract settings, the more the government can commit, the more it is able to do. Not only can the existence of judicial review strengthen the other branches, but the stronger is the judiciary's ability to review, the more powerful are the other branches in their ability to implement policies. With separation, depending on the nature of the judiciary, the government can commit to its promises to a stronger or weaker extent. With an active judiciary, it can fully commit and is able to do as much redistribution as it could if it knew every individual's type and did not have to rely on self-reports. The intermediate cases allow the government to commit but not fully. We show that the amount of redistribution that the government can do increases (at least weakly) as judicial power increases. A stronger judiciary, rather than weakening the government, may actually strengthen it.

In applying this to the case of optimal taxation, it is of interest that the different cases that arise from different legal rules correspond naturally to different equilibrium notions in the mechanism design and game theory literature. Case 1 of a unitary government is a no-commitment equilibrium encapsulating the ideas underlying sub-game perfection. Case 4 with an active judiciary is one of full commitment. The intermediate cases allow the government to commit but not fully. Case 2 where clean hands is imposed is one of partial commitment, while case 3 without clean hands corresponds to the notion of renegotiation proofness. The conceptual difference between case 2 and 3 relates to the role of individuals who misrevealed in blocking renegotiations. In renegotiation-proof equilibria whether in games (see, for example, Farrell and

Maskin [1989]) or in principal-agent problems (see, for example, Fudenberg and Tirole [1990]), renegotiation is allowed only if it leads to a Pareto improvement. Even the “liars” must benefit according to their true preferences and not those they claimed to hold. In partial commitment, someone who misrevealed can be harmed according to their true preferences. In effect, they are committed to the preference they revealed and cannot admit to lying after revelation. Baron and Besanko [1987] analyze a principal-agent model in which the principal can guarantee only that any truth-telling agent will earn nonnegative profit in the second period of a regulatory regime. Hamilton and Slutsky’s [2004] use of partial commitment in nonlinear pricing is similar, except that the principal can choose to guarantee any feasible utility level to truth-telling agents with the levels of these guarantees conditioned on the aggregate reports.

In particular, to develop these results, we formally model a variation of the classic optimal income tax problem by assuming a finite population instead of a continuum as in Mirrlees [1971] (with a continuous ability distribution) and Stiglitz [1982] (with a finite number of types). With a finite population, since misrevelation by even one individual can be detected by the government, it is necessary to be specific about what the government does off the equilibrium path when some individuals misreveal even though these outcomes arise with zero probability in the play of the game.<sup>4</sup> Such off-equilibrium behavior can be crucial in determining the equilibrium. With an infinite number of individuals, the government cannot detect misrevelation by a single individual and does not need to be explicit about what would happen if a mass of individuals misrevealed.<sup>5</sup> In the finite case, Piketty [1993] showed that any full-

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<sup>4</sup> Of course, when there is “noise” in the structure of the problem (for example, if the planner does not know the precise distribution of types), then any outcome may be on the equilibrium path with positive probability, and the distinction between on- and off-the-equilibrium path restrictions disappears.

<sup>5</sup> Although Stiglitz [1982] has a finite number of types, at least implicitly he assumes a continuum of individuals of each type, so he also models an infinite number of individuals.

information optimum can be sustained as a unique equilibrium for some choice of taxes off the equilibrium path. Government budget balance in such off-equilibrium situations is not assumed and, in some cases, a deficit is needed off-equilibrium to achieve truthful revelation in equilibrium. Hamilton and Slutsky [2007] consider the effect in this model of imposing budget balance in all off-equilibrium situations. When individuals reveal their types by choosing an action (such as earning a specified income level), imposing budget balance out of equilibrium upsets Piketty's result. However, when individuals reveal type by simple announcements, Hamilton and Slutsky show that Piketty's result continues to hold even with the requirement of budget balance off the equilibrium path.

Budget balance off the equilibrium path is important for the credibility of the tax mechanism. If a deficit were required, the outcome off the equilibrium path would not be feasible. In effect, to sustain a particular outcome under truthful revelation, the government must make threats of what will happen in the event that some misreveal. Individuals only respond to these threats if they believe that the threats will be carried out. Clearly, feasibility is necessary for credibility. Furthermore, individuals will not respond to feasible threats if the government cannot commit to its action and it is not in the government's interest to carry out that action. What the government can commit to depends upon the legal institutions. We determine the set of sustainable equilibrium outcomes that exist under the four cases specified above.

Specifically, in case 1, the government cannot commit to any policy it announces before individuals reveal but will always reoptimize after individuals announce their type. In case 4, the government can fully commit so the results are those given in Hamilton and Slutsky [2007]. In case 2, the government makes a partial commitment by announcing its policies. In any state of the world, the planner can replace its announced allocation with one which is Pareto superior

under the assumption that individuals' announcements were truthful. In other words, the government can make adjustments but only if they cause no harm to any individual who has revealed a type, based upon the preferences of the type that individual claims to be. The government can revise the allocation given to a type of consumer as long as individuals truly of that type are not made worse off, although individuals who had falsely reported that they were that type may be made worse off by such revisions. While the government cannot use information it acquires about an individual's type in order to redistribute more resources from a truthful type to other groups, it can reoptimize. In case 3, the government can similarly reoptimize from its initial promises but only if the change is benefits everyone including those who may have misrevealed. Incorporating reoptimization by the planner after initial announcements subject to some commitment restrictions means the mechanism design problem no longer has the planner standing outside the game setting the rules under which individuals will play. The planner is now another player in the game, able to make certain moves but subject to the restrictions of optimizing at every information set.<sup>6</sup>

We show that the amount of redistribution that can be achieved in cases 2 and 3 is the same. If there is a passive judiciary with standard standing restrictions, whether or not clean hands is also imposed has no substantive effect on the outcome. We show that the scope of redistribution that a planner can achieve is greater in case 4 than in cases 2 or 3 which in turn is greater than in case 1. That is, the constrained utility possibility frontier that is achievable by the government in cases 2 or 3 is a subset of the full-information frontier achievable in case 4. In addition, the set of feasible allocations achievable in cases 2 or 3 grows if the economy is

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<sup>6</sup> Baliga, Corchón, and Sjöström [1997] consider a principal who is unable to commit and thus becomes an active player in the game in which agents have complete information. They focus on refinements of perfect Bayesian equilibria of the “cheap-talk” game, in contrast to our focus on off-equilibrium behavior in games with unique equilibrium outcomes.

replicated by increasing the number of individuals of each type. In contrast to the Mirrlees-Stiglitz model, all achievable allocations in cases 2, 3, and 4 are undistorted. In case 1, some achievable allocations lie inside the full-information frontier.

Section 2 specifies the formal model. Section 3 analyzes the outcomes of this mechanism. Section 4 compares outcomes to the conventional optimal income tax model. Finally, section 5 offers some conclusions.

## 2. The Model

### A. Structure

The economy has a finite number  $n$  of individuals, who are of two different types. Let  $n^i$  ( $i = 1, 2$ ) denote the number of each type. Each individual consumes a net trade bundle  $X^i$  with two components, the first being labor income and the second consumption. A linear production technology determines feasible pairs of net trades,  $p \cdot (n^1 X^1 + n^2 X^2) \leq 0$ . Without loss of generality, we can specify the units of measurement so that  $p = (-1, 1)$ . If  $p \cdot X^i = 0$ , then that net trade bundle involves no redistribution across types.

Each individual has a utility function  $U^i(X^i)$  over the net trade vectors. The utility functions can differ across types because of taste, productivity, or endowment differences. We make the following assumptions about preferences:

- (i)  $U^i(X^i)$  is quasiconcave and continuously differentiable. Since labor  $X_1^i$  is a bad,  $\partial U^i / \partial X_1^i < 0$ , while  $\partial U^i / \partial X_2^i > 0$ .
- (ii)  $\lim_{x_2^i \rightarrow 0} \frac{\partial U^i}{\partial X_2^i} \rightarrow \infty$ . Thus, for any bundle  $X$  with  $p \cdot X < 0$ , there is an  $\hat{X}_2 > 0$  such that  $U^i(X) = U^i(0, \hat{X}_2)$ .

- (iii) Individuals have a maximum possible labor supply  $\bar{L}$  and thus have maximum possible pre-tax incomes  $\bar{X}_1^1$  and  $\bar{X}_1^2$ . Furthermore,  $\lim_{x_1^i \rightarrow \bar{x}_1^i} \frac{\partial U^i}{\partial X_1^i} \rightarrow -\infty$ .

From these assumptions, we can characterize the indifference curves of each type. Indifference curves slope up ( $MRS^i(X) = -\frac{\partial U^i / \partial X_1^i}{\partial U^i / \partial X_2^i} > 0$ ). The indifference curves through any bundle

$X^*$  with  $p \cdot X^* < 0$  must cross the line  $p \cdot X = 0$  twice—at a point where  $X_1 < X_1^*$  and at a point where  $X_1^* < X_1 < \bar{X}_1^i$ .

Next, we assume that neither consumption nor leisure is inferior. Consider lines  $p \cdot X = K$  and let  $X^i(K)$  be the bundle which solves  $\text{Max } U^i(X)$ , s.t.  $p \cdot X = K$ . Then,

- (iv) If  $K' > K''$ , then  $X_1^i(K') \leq X_1^i(K'')$  and  $X_2^i(K') \geq X_2^i(K'')$ .

In addition, we make a standard single crossing assumption across types:

- (v) At any bundle  $X$ ,  $MRS^1(X) > MRS^2(X)$  and hence  $\bar{X}_1^1 < \bar{X}_1^2$ .

Assumption (v) holds in the standard case in which individuals differ only in ability with type 2s more able than type 1s. Figure 1 displays indifference maps of the two types satisfying these assumptions.

## B. Timing of Actions

Nature makes the first move in the game by determining each individual's type. The government learns the preferences and the exact number of each type, but it does not learn the

type of any particular individual. All individuals know their own types and the total numbers of each type.<sup>7</sup>

In the second stage, the government announces a menu of provisional tax policies conditional on how many individuals reveal themselves as each type. Let  $N^1$  denote the number of individuals who later reveal themselves to be type 1s, and let  $Y^1(N^1)$  and  $Y^2(N^1)$  be the net trade bundles provisionally assigned to the two types. The government may modify these bundles later in the game, subject to the constraints of the legal system. To insure that the final bundles are feasible, these provisional bundles must themselves be feasible for each possible revelation pattern:

$$p \cdot [N^1 Y^1(N^1) + (n - N^1) Y^2(N^1)] \leq 0, \quad 0 \leq N^1 \leq n \quad (1)$$

In addition to feasibility, the  $Y^i(N^1)$  bundles must be selected to ensure a unique outcome in the next stage in which individuals reveal their types. These latter constraints are in effect incentive compatibility constraints. We present the formal statement of these restrictions below after specifying the rest of the game.

The provisional tax policies determine utility levels for each type. Let  $\bar{U}^i(N^1) \equiv U^i(Y^i(N^1))$  be the utility level achieved by a true type  $i$  under the provisional bundle assigned when  $N^1$  people reveal themselves as type  $i$ 's. A utility proposal  $\bar{U}$  is the set of  $\bar{U}^i(N^1)$  for  $i = 1$  and  $2$  and each value of  $N^1$  between  $0$  and  $n$ . A utility proposal is feasible if the  $Y^i(N^1)$  defining it satisfy the no-deficit constraints in (1). A feasible utility proposal can be viewed as guarantee levels made by the government to truthful individuals conditioned on the numbers who reveal themselves of each type. Let  $\mathcal{U}$  be the set of all such feasible utility proposals. In stage

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<sup>7</sup> Except for an individual's knowledge of her own type, individuals and the government have the same information about the aggregate distribution. This differs from models such as Abreu and Sen [1991] where individuals have complete information about the type of every individual.

2, the government can be viewed as specifying feasible utility guarantees to individuals instead of explicit provisional bundles.

In the third stage after observing the government's provisional tax policies, individuals simultaneously reveal their types to the government and are committed to that revelation in the remainder of the game. Individuals simply declare a type without taking any action, and the governments' provisional bundles at different values of  $N^1$  need have no relation to each other. The government has the flexibility to give a type completely different bundles depending on what others have revealed.<sup>8</sup>

In the fourth stage, having learned  $N^1$ , the government can revise the taxes imposed in that revelation circumstance. These are specified as bundles  $X^i(N^1)$  that for the revealed  $N^1$  presumably raise the government's welfare and satisfy the budget constraint:

$$p \cdot [N^1 X^1(N^1) + (n - N^1) X^2(N^1)] \leq 0 \quad (2)$$

Then, in the fifth stage, the judicial authority considers any objections to the revisions. In case 1, in the absence of an independent judiciary, no objections are possible, so the government can make any revisions it wants. In case 4, an active judiciary objects and blocks any changes to the stage 2 announcements, so the government is committed to those policies. In cases 2 and 3, individuals who are harmed by the revisions can bring objections. These are sustained in case 2 if the objection is based upon the individual's announced preferences and in case 3 if the objection is based upon the individual's true preferences (even if the individual had misrevealed). When making revisions in the fourth stage, the government can look ahead to what the judiciary will allow in the next stage and only makes revisions that will pass judicial

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<sup>8</sup> Under Piketty's [1993] action revelation, the bundles for different values of  $N^1$  must all have the same value of the labor component since it is by choosing this component that individuals reveal their type.

review. In effect, judicial review imposes on the government's revisions the restrictions that no individual who is entitled to object can be worse off than the stage 2 utility level:

$$U^i(X^i(N^1)) \geq \bar{U}^i(N^1), \quad i = 1, 2 \quad (3)$$

We specify the reoptimization constraints in terms of utility guarantees because they are what would be enforceable in a legal proceeding. If the allocation were revised to make an individual were no worse off, she would have no claim for redress.

Figure 2 displays the extensive form game tree through the first four stages for the case where  $n = 2$ .<sup>9</sup> The two individuals are named A and B. The tree shows only the case in which nature chooses one individual to be of each type. Nature is equally likely to make A the type 1 or the type 2. The other cases (both type 1 or both type 2) are not shown—they imply complete knowledge by the government of each individual's type. In general, given the assumptions that the government and individuals know the exact numbers of each type, even if other distributions were considered, they would form separate subgames and could be treated independently.

### C. Social Preferences

This game has  $n + 1$  strategic players:  $n$  individuals and the government. Individuals' utility functions define their payoffs. The government's preferences derive from a weighted utilitarian welfare function. It depends not only on the bundles assigned but on the number who reveal themselves of each type. Let  $N^{ij}$  be the number of individuals truly of type  $i$  who have revealed themselves as type  $j$ 's, where  $N^{11} + N^{21} = N^1$  and  $N^{12} + N^{22} = n - N^1$ . Let

$\alpha(N^1) = (\alpha_{11}(N^1), \alpha_{12}(N^1), \alpha_{21}(N^1), \alpha_{22}(N^1))$  be the vector of weights given to the types where

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<sup>9</sup> The different forms of judicial review lead to different subgames at the end. With backward induction, the action choices in the fourth stage are never overturned in the fifth stage.

$\alpha_{ij}(N^1)$  is the weight given to a type  $i$  who claims to be a type  $j$ . Thus, we can write the welfare function as:

$$W(X^1(N^1), X^2(N^1), N^1) \equiv \alpha_{11}(N^1)N^{11}U^1(X^1) + \alpha_{21}(N^1)N^{21}U^2(X^1) + \alpha_{12}(N^1)N^{12}U^1(X^2) + \alpha_{22}(N^1)N^{22}U^2(X^2) \quad (4)$$

We normalize  $\alpha_{ij}$  so that  $\alpha_{11}(N^1) + \alpha_{12}(N^1) + \alpha_{21}(N^1) + \alpha_{22}(N^1) = 1$ . The argument  $N^1$  appears in the function for two reasons.<sup>10</sup> First, if  $N^1 \neq n^1$ , some individuals have misrevealed and receive the bundle intended for the other type; their utility does not equal the utility of those for whom the bundle is intended. Second, when  $N^1 \neq n^1$ , the government knows some have misrevealed and may give them less weight in the welfare function. Our framework is flexible in that respect. If  $\alpha_{11} = \alpha_{12}$  and  $\alpha_{21} = \alpha_{22}$ , then the government gives the same weight to an individual whether or not he reveals truthfully. If  $\alpha_{12} = \alpha_{21} = 0$ , then the government gives zero weight to any individual who lies and positive weight only to truth-tellers.<sup>11</sup>

The government might gain by acting as if its preferences differ from its true ones. For example, it might gain by giving zero weight to liars even though it really does value their utility. We assume that the government cannot commit to a false welfare function and thus must act in stage 4 according to its true preferences. As we show below, setting particular values of the utility guarantee in stage 2 can be equivalent to placing no weight on liars in stage 4. We assume that individuals know the government's true preferences and thus can accurately predict what policies it will implement in stage 4 conditional on the  $N^1$  value.

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<sup>10</sup> These properties would also need to hold for a Bergson-Samuelson welfare function other than a weighted utilitarian function.

<sup>11</sup> The  $\alpha_{ij}$  might depend on  $N^1$  because the weight given to someone who lied would vary with the number of liars. If only a few lie, then the liars might be given little weight in social preferences. If many lie, this may not be possible in a democracy.

#### D. Solution Concept

Given the sequential play where individuals observe the provisional government policies in stage 3 before revealing their types and the government observes individual revelations in stage 4 before making revisions subject to judicial review, the appropriate solution concept is a refinement of Nash equilibrium which incorporates backward induction reasoning. Since information sets contain multiple nodes and no proper subgames exist in the game tree of Figure 2, subgame perfection is not a strong enough equilibrium concept. We use the stronger notion of sequential equilibrium (Kreps and Wilson [1982]), where an equilibrium is defined by strategies and beliefs. The player at each information set maximizes given her beliefs, and these beliefs must be consistent with the equilibrium strategies, where possible.

Consider the government at its fourth stage information sets. We assume it believes that an information set arises from the minimum number of deviations from truthful revelation by individuals. For example, in Figure 2, there are the information sets where one person reveals themselves to be of type 1 and the other of type 2. This could arise either because both revealed honestly or both lied. In this case, the government believes both were honest. Similarly, when  $n > 2$  and  $n^1 < N^1 < n$ , the government believes it is at a node at which all those who revealed themselves as type 2 were honest and only  $N^1 - n^1$  of type 2's lied. The government believes that the different nodes in the information set which satisfy this are equally likely and does not believe that some individuals within a type are more likely than others to lie.<sup>12</sup>

For such beliefs to be part of a sequential equilibrium, they must be consistent with the equilibrium play of individuals where possible. That means that at stage 3, the mechanism should induce individuals to reveal truthfully in equilibrium, if possible. In case 1, this is not

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<sup>12</sup> Assuming that any information set off the equilibrium path arises from the minimal number of deviations satisfies the limit restriction of Kreps and Wilson [1982, p. 875].

always possible. A unitary government cannot be held to any promises it makes. Therefore, provisional policies announced in stage 2 have no effect on individual decisions. In stage 4, after individual revelation, the government chooses policies without any subsequent judicial review that maximize its preferences independent of any effect that foreseeing these policies might have had on individuals' earlier decisions. Thus, the government has no ability to induce revelation in this case—individuals may reveal truthfully or lie depending upon what the government's preferences lead it to do in stage 4. In case 4, an active judiciary compels the other branches to carry out all promises. Thus, any policies announced in stage 2 will be carried out by the government since any modifications, even those which are Pareto improving, will be overturned in judicial review. Since the stage 2 policies will generally redistribute from one type to the other, individuals will potentially have incentives to misreveal. Restrictions must be imposed to rule this out. With a finite number of individuals, the standard Mirrlees-Stiglitz self-selection constraints are not appropriate since the government can detect misrevelation in the aggregate by even one person.<sup>13</sup> Following Piketty [1993], we assume that truthful revelation is induced by dominance solvability rather than the stronger notion that truth is a dominant strategy. Hamilton and Slutsky [2007] developed these conditions when budget balance is imposed in all revelation situations, not just under truthful revelation on the equilibrium path.

Similar conditions are needed in cases 2 and 3. However, since the government moves both prior to and subsequent to individual revelation, the required incentive constraints are more complicated. Individuals observe the government's provisional policies or utility guarantees but also react to their beliefs of what adjustments the government will make after revelation that survive judicial review. Since the final stage adjustments occur after revelation, they cannot be

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<sup>13</sup> Misrevelation also upsets budget balance and thus the government cannot maintain the same consumption bundles for all types that it proposed if all revealed truthfully.

directly constrained to yield dominance solvability. To ensure this requires more constraints on the utility guarantees than belonging to the set  $\mathcal{U}$  of stage 2 utility guarantees which satisfy budget balance under all patterns of revelation.

To specify this formally, consider case 2 with clean hands (similar conditions apply in case 3 when “clean hands” is not imposed). Knowing the governments’ beliefs and preferences, individuals can predict what the government will do in the final stage. At any information set defined by  $N^1$ , the government will choose  $X^1(N^1)$  and  $X^2(N^1)$  to maximize  $W(X^1(N^1), X^2(N^1), N^1)$  subject to (2) and (3), yielding the solution values  $\hat{X}^1(N^1, \bar{U}^1(N^1), \bar{U}^2(N^1), \alpha(N^1))$ . When redistribution is from type 2s to type 1s, the dominance solvability conditions are:

$$U^1(\hat{X}^1(N^1 + 1, \bar{U}^1(N^1 + 1), \bar{U}^2(N^1 + 1), \alpha(N^1 + 1))) \geq U^1(\hat{X}^2(N^1, \bar{U}^1(N^1), \bar{U}^2(N^1), \alpha(N^1))), \quad (5)$$

$$0 \leq N^1 \leq n - 1$$

$$U^2(\hat{X}^2(N^1, \bar{U}^1(N^1), \bar{U}^2(N^1), \alpha(N^1))) \geq U^2(\hat{X}^1(N^1 + 1, \bar{U}^1(N^1 + 1), \bar{U}^2(N^1 + 1), \alpha(N^1 + 1))), \quad (6)$$

$$n^1 \leq N^1 \leq n - 1$$

Condition (5) imposes that truth telling is a dominant strategy for type 1s. Condition (6) then makes it a dominant strategy for type 2s to reveal truthfully given that type 1s are truthful.<sup>14</sup> The government must choose utility levels in stage 2 to be feasible (that is to be in  $\mathcal{U}$ ) and to satisfy (5) and (6). Given these restrictions and the beliefs in stage 4, the unique equilibrium has

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<sup>14</sup> When redistribution is toward type 1s, then type 2s are more likely to gain by misrevealing since they would avoid a tax and gain a subsidy. Imposing that truth is a dominant strategy for type 1s and then that type 2s are truthful if type 1s are is therefore a weaker requirement than having truth be a dominant strategy for type 2s and then requiring that type 1s be truthful if no type 2 misreveals. For redistribution toward type 2s, it is weaker to require that type 2s always reveal correctly and then, given that, that type 1s are truthful.

truthful revelation by individuals in stage 3. Given these actions, the beliefs in stage 4 are justified.

### E. Full Information Benchmark

It will be useful to compare our equilibrium outcomes to the standard benchmark of what the government would do if it had full information about individuals' types. In this case, it would solve the following optimization:

$$(I) \quad \begin{aligned} & \text{Max}_{X^1, X^2} \alpha n^1 U^1(X^1) + (1 - \alpha) n^2 U^2(X^2) \\ & \text{s.t.} \quad p \cdot (n^1 X^1 + n^2 X^2) \leq 0. \end{aligned}$$

Choice of the parameter  $\alpha$  allows this problem to describe any undistorted allocation. Let  $X^i(n^1, \alpha)$  denote the solution to (I) for any  $n^1$  and  $\alpha$ . There exists an  $\alpha^0$  such that the solution to (I) entails no redistribution ( $p \cdot X^1(n^1, \alpha^0) = p \cdot X^2(n^1, \alpha^0) = 0$ ). For  $\alpha > \alpha^0$ , there will be transfers from the type 2's to type 1's ( $n^1 p \cdot X^1(n^1, \alpha) = -n^2 p \cdot X^2(n^1, \alpha) > 0$ ) and the reverse for  $\alpha < \alpha^0$ .

## 3. Results

### A. Case 4 – An Active Judiciary

Consider the following optimization problem:

$$(II) \quad \begin{aligned} & \text{Max}_{X^1(N^1), X^2(N^2)} \alpha n^1 U^1(X^1(n^1)) + (1 - \alpha) n^2 U^2(X^2(n^1)) \\ & \text{s.t.} \quad U^1(X^1(N^1 + 1)) \geq U^1(X^2(N^1)), \quad 0 \leq N^1 \leq n - 1 \end{aligned} \quad (7)$$

$$U^2(X^2(N^1)) \geq U^2(X^1(N^1 + 1)), \quad n^1 \leq N^1 \leq n - 1 \quad (8)$$

$$p \cdot [N^1 X^1(N^1) + (n - N^1) X^2(N^1)] = 0, \quad 0 \leq N^1 \leq n \quad (9)$$

As argued above, with a finite number of individuals, even if only one individual misreveals, the planner at some stage will detect that the revealed pattern does not accord with what it knows for

certain to be the actual pattern. If the types' net trade bundles use different resources ( $p \cdot X^1 \neq p \cdot X^2$ ), misrevelation causes either a noticeable surplus or a deficit. The government is then able to condition the bundles offered to each type on what others reveal (this conditioning must be anonymous in the sense that it only depends on the aggregate announcements of others). In fact, if a deficit results, the planner not only can but must alter taxes to maintain feasibility. Thus the government chooses the vectors  $X^1(N^1)$  and  $X^2(N^1)$  for every  $N^1$  even though only the truthful revelation bundles  $X^1(n^1)$  and  $X^2(n^1)$  enter the objective function.

Conditions (7) and (8), building upon Piketty [1993], are the dominance solvability conditions that guarantee that the individual revelation game has a unique equilibrium of truthful revelation. These apply when the government seeks to redistribute from type 1s to type 2s where (7) imposes that 1s are always truthful and (8) imposes that 2s are truthful given that all 1s have revealed correctly. Condition (9) is the requirement that the government's budget balances on and off the equilibrium path. The constraints (9) mean that the government can commit only to feasible policies.<sup>15</sup>

Optimization problem **(II)** describes what happens in the five-stage game when any revisions the government might try to make in stage 4 to the policies it announced in stage 2 are blocked by judicial review in stage 5. This means that the government is committed to the policies it announces in stage 2. Hamilton and Slutsky [2007] show that the solutions to **(I)** and **(II)** are the same. That is, with an active judiciary that can force the government to keep any feasible promises it makes, the government can achieve any full-information outcome it desires. The government's ability to redistribute is at its maximal possible level.

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<sup>15</sup> Actually, the government could commit to policies which ran a surplus with the  $=$  in (9) replaced by  $\leq$ . It would then be necessary to specify what is done with any surplus that might arise. We assume here that the government is forbidden to run a surplus.

## B. Case 2 – A Passive Judiciary with Standing and Clean Hands

It is easier to study the case of standing and clean hands and then consider dropping the clean hands requirement. We start with the simple case with one individual of each type. In stage 2, the government makes the utility guarantees  $\bar{U}^2(0), (\bar{U}^1(1), \bar{U}^2(1)),$  and  $\bar{U}^1(2)$  for the three information sets which could arise after individual revelation.<sup>16</sup> Then, in stage 4, looking ahead to what will be allowed under judicial review in stage 5, the government solves the three maximization problems described below to determine its final policies.

Assume that  $\alpha_{12}(0) \leq \alpha_{11}(1) \leq \alpha_{11}(2)$ . When  $N^1 = 0$ , the type 1 has lied, so he might be given less relative weight than when truthful. When  $N^1 = 2$ , the type 2 has lied and might be given less relative weight. The first of these stage 4 problems is when both individuals claim to be type 2:

$$\begin{aligned} \text{Max}_{X(0)} \quad & \alpha_{12}(0)U^1(X(0)) + (1 - \alpha_{12}(0))U^2(X(0)) & (10) \\ \text{s.t.} \quad & p \cdot X(0) = 0 \quad \text{and} \quad U^2(X(0)) \geq \bar{U}^2(0) \end{aligned}$$

with the solution  $\hat{X}(0, \bar{U}^2(0), \alpha_{12}(0))$ . The constraint  $U^2(X(0)) \geq \bar{U}^2(0)$  comes from potential judicial review. If this did not hold, the true type 2 individual would have standing to object to the new proposal and would succeed in blocking the attempted change.

The second one is when both claim to be type 1:

$$\begin{aligned} \text{Max}_{X(2)} \quad & \alpha_{11}(2)U^1(X(2)) + (1 - \alpha_{11}(2))U^2(X(2)) & (11) \\ \text{s.t.} \quad & p \cdot X(2) = 0 \quad \text{and} \quad U^1(X(2)) \geq \bar{U}^1(2) \end{aligned}$$

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<sup>16</sup> The government makes the same guarantees in the two information sets where each type is revealed by one individual. They differ only in the names of the individuals who reveal each type, and the names are irrelevant to the government's actions.

with the solution  $\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2))$ . As in (10), the constraint  $U^1(X(2)) \geq \bar{U}^1(2)$  comes from potential judicial review.

Let  $X^{i*}$  be person  $i$ 's most preferred point on the no-redistribution line. Hence,  $X^{i*}$  solves:

$$\text{Max}_X U^i(X) \text{ s.t. } p \cdot X = 0.$$

Then, for problems (10) and (11) to be feasible,  $\bar{U}^2(0) \leq U^2(X^{2*})$  and  $\bar{U}^1(2) \leq \bar{U}^1(X^{1*})$  must hold. Let  $\tilde{X}(0, \alpha_{12}(0))$  and  $\tilde{X}(2, \alpha_{11}(2))$  be the solutions to (10) and (11) without the judicial review constraints. That is, these are the government's best points on the no-redistribution line given  $\alpha_{12}(0)$  and  $\alpha_{11}(2)$ . Then in (10),  $\hat{X}(0, \bar{U}^2(0), \alpha_{12}(0)) = \tilde{X}(0, \alpha_{12}(0))$  when  $\bar{U}^2(0) \leq U^2(\tilde{X}(0, \alpha_{12}(0)))$  with  $\hat{X}(0, \bar{U}^2(0), \alpha_{12}(0))$  moving to  $X^{2*}$  as  $\bar{U}^2(0)$  increases from  $U^2(\tilde{X}(0, \alpha_{12}(0)))$  to  $U^2(X^{2*})$ . Similarly, in (11),  $\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2)) = \tilde{X}(2, \alpha_{11}(2))$  for  $\bar{U}^1(2) \leq U^1(\tilde{X}(2, \alpha_{11}(2)))$  with  $\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2))$  moving to  $X^{1*}$  as  $\bar{U}^1(2)$  increases from  $U^1(\tilde{X}(2, \alpha_{11}(2)))$  to  $U^1(X^{1*})$ .

The third problem (when all reveal truthfully) is:

$$\begin{aligned} \text{Max}_{X^1(1), X^2(1)} & \alpha_{11}(1)U^1(X(1)) + (1 - \alpha_{11}(1))U^2(X^2(1)) & (12) \\ \text{s.t.} & \quad p \cdot [X^1(1) + X^2(1)] = 0 \\ & \quad U^1(X^1(1)) \geq \bar{U}^1(1) \quad \text{and} \quad U^2(X^2(1)) \geq \bar{U}^2(1) \end{aligned}$$

with the solution  $(\hat{X}^1(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1)), \hat{X}^2(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1)))$ . In this problem, there are two judicial review constraints since both individuals have clean hands and can successfully appeal changes which might lower their utility.

To analyze (12), let  $R$  be the amount of resources redistributed from person 2 to person 1.

Then the resource constraint in (12) can be written as the pair of equations  $p \cdot X^1(1) = R$

and  $p \cdot X^2(1) = -R$ , with  $R$  treated as a choice variable in the maximization along with

$X^1(1)$  and  $X^2(1)$ . A necessary condition for the  $\hat{X}^i(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))$  to be solution values in (12) is that there exists some value of  $R$  such that  $\hat{X}^1(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))$  solves:

$$\begin{aligned} \text{Max} \quad & U^1(X^1(1)), \text{ s.t. } p \cdot X^1(1) = R \\ & X^1(1) \end{aligned}$$

and  $\hat{X}^2(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))$  solves:

$$\begin{aligned} \text{Max} \quad & U^2(X^2(1)), \text{ s.t. } p \cdot X^2(1) = -R. \\ & X^2(1) \end{aligned}$$

In other words, for any feasible  $\bar{U}^1(1)$  and  $\bar{U}^2(1)$ , the  $\hat{X}^i(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))$  are undistorted bundles that solve problem **(I)** for some value of  $\alpha$ , which is determined by the values of  $\alpha_{11}(1)$ ,  $\bar{U}^1(1)$ , and  $\bar{U}^2(1)$ .

Consider the case in which the government desires to redistribute from type 2 to type 1 so that  $\alpha_{11}(1)$  is greater than  $\alpha^0$ , the no-redistribution value of  $\alpha$  in the undistorted benchmark **(I)**.

Then the dominance solvability conditions (5) and (6) apply and reduce to

$$U^1(\hat{X}^1(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))) \geq U^1(\hat{X}(0, \bar{U}^2(0), \alpha_{12}(0))) \quad (13)$$

$$U^1(\hat{X}(2, \bar{U}^1(2), \alpha_{12}(2))) \geq U^1(\hat{X}^2(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))) \quad (14)$$

$$U^2(\hat{X}^2(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))) \geq U^2(\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2))) \quad (15)$$

Constraint (13) is weaker at smaller values of  $U^1(\hat{X}(0, \bar{U}^2(0), \alpha_{12}(0)))$ . From the discussion of

(10) above, this occurs when  $\hat{X}(0, \bar{U}^2(0), \alpha_{12}(0))$  is set at  $X^{2*}$ , which is the feasible value farthest

away from  $X^{1*}$ . Then  $\bar{U}^2(0)$  equals  $U^2(X^{2*})$  and  $\hat{X}(0, U^2(X^{2*}), \alpha_{12}(0)) = X^{2*}$ . At this value, (13) must be satisfied with strict inequality.  $\hat{X}^1(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))$  is type 1's most preferred bundle on a budget line  $p \cdot X^1 = R > 0$  whereas  $X^{2*}$  is inferior to type 1's most preferred bundle on the inferior budget line  $p \cdot X^1 = 0$ .

Constraints (14) and (15) are weakest when  $\bar{U}^1(2)$  is chosen to lead to a value of  $\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2))$  which makes  $U^1(\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2)))$  as large as possible and  $U^2(\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2)))$  as small as possible. This will occur if  $\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2))$  is as close to  $X^{1*}$  but as far from  $X^{2*}$  as is possible. From the discussion of (11) above, this occurs when  $\bar{U}^1(2) = U^1(X^{1*})$  and  $\hat{X}(2, \bar{U}^1(2), \alpha_{11}(2))$  equals  $X^{1*}$ . For this value of  $\bar{U}^1(2)$ , condition (14) must be satisfied with strict inequality. The bundle  $X^{1*}$  is type 1's most preferred bundle on the budget line  $p \cdot X^1 = 0$  while  $\hat{X}^2(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))$  is, from 1's perspective, not the best bundle on an inferior budget line  $p \cdot X^1 = -R$ .

Therefore, in stage 2, the government can always choose  $\bar{U}^1(2)$  and  $\bar{U}^2(0)$  so that (13) and (14) must hold with strict inequality and can be deleted. The remaining constraint (15) becomes the following restriction on  $\bar{U}^1(1)$  and  $\bar{U}^2(1)$ :<sup>17</sup>

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<sup>17</sup> Actually, in the two-person case, conditions (16) and (17) will never bind. The government would not choose provisional policies in stage 2 which would lead either to be violated. Assume  $\alpha_{11}(1) > \alpha^0$  and that (16) is violated. The type 2 individual would misreveal and  $X^{*1}$  would be the allocation selected in stage 4. This bundle is Pareto inferior to an undistorted bundle that could have been implemented under stage 2 announcements consistent with (16). To see this, consider the line  $p \cdot X = -K^*$  tangent to the type 2 indifference curve through  $X^{*1}$ . For any  $K', 0 < K' < K^*$ , let  $\hat{X}^1(K')$  be the type 1 best bundle on the line  $p \cdot X = K'$  and let  $\hat{X}^2(K')$  be the best bundle on  $p \cdot X = K'$ . Then  $U^1(\hat{X}^1(K')) > U^1(X^{*1})$  and  $U^2(\hat{X}^2(K')) > U^2(X^{*1})$ . In addition,

$$U^2(\hat{X}^2(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))) \geq U^2(X^{*1}) \quad (16)$$

If  $\alpha_{11}(1)$  were less than  $\alpha^0$ , with desired redistribution from 1s to 2s, a similar analysis would reduce the appropriate dominance solvability constraints to:

$$U^1(\hat{X}^1(1, \bar{U}^1(1), \bar{U}^2(1), \alpha_{11}(1))) \geq U^1(X^{*2}) \quad (17)$$

In effect, the government does best by putting zero weight on the utilities of those who misreveal with  $\alpha_{12}(0) = 0$  and  $\alpha_{11}(2) = 1$ . If these are not its true preferences, it cannot directly commit to behaving this way once misrevelation has occurred. However, by setting the appropriate utility guarantees in stage 2, it acts as if it gives no weight to misrevealers in stage 4. It would like to implement bundles based on their having positive weight, but the judicial review constraints prevent it from doing so.

We can now specify the equilibrium outcome in this case.

**Theorem 1:** Assume that  $n^1 = n^2 = 1$ . In the sequential equilibrium when the government faces judicial review with clean hands and standing, there exist values  $\beta'$  and  $\beta''$  ( $0 < \beta' < \alpha^0 < \beta'' < 1$ ) such that the equilibrium allocations solve:

(a) the full-information optimization **(I)** with  $\alpha$  set equal to  $\alpha_{11}(1)$ , for any

$$\beta' \leq \alpha_{11}(1) \leq \beta''.$$

(b) the full-information optimization **(I)** for  $\alpha = \beta''$  when  $\alpha_{11}(1) > \beta''$ .

(c) the full-information optimization **(I)** for  $\alpha = \beta'$  when  $\alpha_{11}(1) < \beta'$ .

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these bundles can be sustained by the partial commitment requirements if they are provisionally announced in stage 2 and they yield higher social welfare than giving  $X^{*1}$  to both individuals. Even if (16) were not imposed, optimal decision making would lead to choices which satisfy it. Imposing (16) is, however, useful in eliminating such suboptimal policies from consideration. Note that if the government in stage 2 did not guarantee  $U^1(X^{*1})$ , a similar argument would still follow for any  $X$  between  $X^{*1}$  and  $X^{*2}$ .

Given the beliefs the government was assumed to hold as part of a sequential equilibrium, these are unique equilibrium actions in stages 3 and 4 of the game. Regardless of the government's preferences on or off the equilibrium path, the equilibrium sustains an undistorted bundle which therefore lies on the full-information utility possibility frontier. However, some points on the full-information frontier cannot be sustained. When  $\alpha_{11}(1)$  exceeds  $\beta''$ , the government cannot redistribute more toward the type 1 than if  $\alpha_{11}(1)$  equaled  $\beta''$ . Although efficient, less redistribution than desired is possible. Similarly, if  $\alpha_{11}(1)$  is less than  $\beta'$ , the government can redistribute only as much as it would if  $\alpha_{11}(1)$  equaled  $\beta'$ .

With only one individual of each type, the government faces only a small number of information sets. In the two information sets in which one individual misrevealed, both individuals claimed to be of the same type, so they had to be treated identically. This reduced the flexibility available to the government in reoptimization in these information sets. It could only choose a single bundle on the  $p \cdot X = 0$  line.

With more than one of each type, after misrevelation, the government retains considerable flexibility. As long as some agents claim to be different types, the government can redistribute between those making different reports. In choosing the original utility commitments, the government chooses how much to give each group and not simply how to balance their preferences in choosing a zero-redistribution bundle. Having more consumers of both types increases the subset of the full-information utility possibility frontier which can be sustained in equilibrium. We show this for  $n^1 = n^2$  in the next theorem.

**Theorem 2:** Assume that  $n^1 = n^2 = k$ . Under the judicial review requirements of case 2, the sequential equilibrium outcomes are the same as those in Theorem 1 with  $\alpha_{11}(1)$  replaced by  $\alpha_{11}(k)$  and  $\beta'$  and  $\beta''$  replaced by  $\beta'(k)$  and  $\beta''(k)$ . Furthermore,  $\beta''(k)$  increases and  $\beta'(k)$  decreases as  $k$  increases.

We have formally shown in the proof that it is possible to do more redistribution with  $k > 4$  than the maximum redistribution with  $k = 4$ . The same logic can be used to show that the maximum redistribution for a given number of workers can be increased when there are more workers of both types by reverting to the no-redistribution bundles when large numbers of type 2s misreport and using bundles analogous to those in Figure 6 (in the proof) when only a small number misreport. Thus, more of the full-information frontier is sustainable as the number of workers of both types grows. It is an open question whether this process continues without bound so that the entire full-information Pareto frontier can be implemented in the partial commitment equilibrium with announcement revelation when the number of consumers is large.<sup>18</sup>

One difficulty with this mechanism deserves mention. In the Stiglitz framework, agents do not care about other's announcements because their consumption bundle does not depend on them. But with mechanisms in which agents' bundles depend directly on the aggregate of all reports, the planner must be able to prevent trade between agents. Suppose that a coalition of type 2 consumers agree that one agent will claim to be type 1. The agents claiming to be type 2

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<sup>18</sup> While the construction in the proof uses equal numbers of both types, adding more individuals of only one type would similarly give the government more flexibility in the allocations it could sustain. However, the balanced budget line would shift away from the 45° line to reflect the new  $n_1/n_2$  ratio. One could replicate this unbalanced economy and extend the attainable portion of the full-information utility possibility frontier as in Theorem 2.

then obtain a better bundle than when all type 2 agents reveal truthfully. It may be that the coalition of type 2 consumers can trade among themselves to make them all better off. This is even more of a problem if agents can identify each other's types *ex ante*, even though the planner cannot. It is thus possible that the mechanism described above is not coalition-proof. Adding restrictions on the  $X^2$  bundles to prevent these coalitions from forming might limit the range of implementable first-best allocations.

### **C. Case 3 – A Passive Judiciary with Standing but not Clean Hands**

In case 3, as compared to case 2, potentially more objections can be raised to revisions by the government in stage 4. An individual who misrevealed is allowed to object to a revision even if the harm occurs with the true preferences, but not under the preferences asserted. Since such objections never arise given the government's chosen policies, the outcomes are the same with or without clean hands. The intuition is straightforward. On the equilibrium path, all individuals reveal truthfully and thus can object to changes whether or not clean hands is imposed. Off the equilibrium path, some lie, so clean hands could matter. However, for any redistribution that the government seeks to achieve on the equilibrium path, the tax structure off the equilibrium path is designed solely to support truthful revelation in equilibrium. Thus, the government will try to punish those who misreveal as harshly as it can, given its information and constraints. The punishments it promises in stage 2 may be so extreme that it is not in the government's interests to carry them out in stage 4. This means that any *ex post* adjustments will raise, not lower, the utility of the misrevealers who therefore will not object. If the government were to gain by making a revision that lowers the welfare of a misrevealer, it would have been better to have proposed the revised policy at the earlier stage. Overall, at any off-equilibrium information set, the set of proposals that will not be overturned under clean hands is a subset of

those not overturned without clean hands, but this subset includes the most effective threats that can be used to achieve truthful revelation. Thus, the set of feasible outcomes under the doctrine of standing is independent of whether or not the clean hands doctrine also applies.<sup>19</sup>

For a specific example, consider the result in Theorem 1 when there is only one individual of each type. The bundles which are subject to revision are the off-the-equilibrium path bundles after misrevelation. These bundles have both individuals claiming to be of the same type. Hence, under budget balance, the only feasible off-equilibrium outcomes are on the no-redistribution budget line. Since the tax authority wants to induce truthful revelation, it gains by making the maximal credible threats to the misrevealer and giving the best feasible utility guarantee to the truth-telling type. Consider the ideal points of the two individuals on that budget line. Under either partial commitment or renegotiation proofness (the solution concepts that correspond to imposing or not imposing clean hands), as the extreme points on the no-redistribution budget line of the points that are credible *ex post*, these are the strongest credible threats. Bundles on the no-redistribution line that are not between these two extreme points would not be credible since the utility of the truth-telling type is higher at the extreme point. With single crossing, the utility of the misrevealer also increases with a move toward the nearer of the extreme points. It is only on the line segment between these two extreme points that one type's utility rises and the other type's utility falls by a small move along the no-redistribution line—and thus only in this region does the set of allowable revised bundles differ depending on whether or not clean hands applies. The strongest allowable punishment to the misrevealing type in either case is then to choose the ideal point of the other type. Since both cases have the same maximal threat to ensure truth telling, the redistributions that can be accomplished are the same.

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<sup>19</sup> If the threats are set exogenously (as in Baron and Besanko [1987]), then the outcome can change when both standing and clean hands are required.

#### D. Case 1 – A Unitary Government

Now consider the case of no commitment with  $n_1 = n_2 = 1$  where  $\tilde{X}(0, \alpha_{12}(0))$ ,  $(\tilde{X}^1(1, \alpha_{11}(1)), \tilde{X}^2(1, \alpha_{11}(1)))$ , and  $\tilde{X}(2, \alpha_{11}(2))$  solve (10-12) with no judicial review constraints. Although the government has no instruments to induce truth telling (since stage 2 announcements have no effect on individuals' decisions), truthful revelation occurs for some government preferences. Assuming  $\alpha_{11}(1) > \alpha^0$ , the dominance solvability constraints for this are:

$$U^1(\tilde{X}^1(1, \alpha_{11}(1))) \geq U^1(\tilde{X}(0, \alpha_{12}(0))) \quad (18)$$

$$U^1(\tilde{X}(2, \alpha_{11}(2))) \geq U^1(\tilde{X}^2(1, \alpha_{11}(1))) \quad (19)$$

$$U^2(\tilde{X}^2(1, \alpha_{11}(1))) \geq U^2(\tilde{X}(2, \alpha_{11}(2))) \quad (20)$$

As in the case 2, the constraints related to type 1s are always satisfied. In (18),  $\tilde{X}^1(1, \alpha_{11}(1))$  is the type 1's best bundle on a budget line with more resources than is  $\tilde{X}(0, \alpha_{12}(0))$  so must be better. In (19), the worst value for type 1 of  $\tilde{X}(2, \alpha_{11}(2))$  for any value of  $\alpha_{11}(2)$  is  $X^{2*}$  which arises when  $\alpha_{11}(2)$  is 0. From noninferiority (assumption (iv)),  $X_1^{2*} \leq X_1^2(1, \alpha_{11}(1))$  and  $X_2^{2*} \geq \tilde{X}_2^2(1, \alpha_{11}(1))$  must hold. Hence, from monotonicity, type 1 always prefers  $\tilde{X}(2, \alpha_{11}(2))$  to  $\tilde{X}^2(1, \alpha_{11}(1))$ . Only (20) might be violated for some  $\alpha_{11}(1)$  and  $\alpha_{11}(2)$  with truth telling for the type 2 occurring if and only if it is satisfied.

Similarly, when  $\alpha_{11}(1) < \alpha^0$ , the dominance solvability condition that is necessary and sufficient for the type 1 to reveal truthfully is:

$$U^1(\tilde{X}^1(1, \alpha_{11}(1))) \geq U^1(\tilde{X}(0, \alpha_{12}(0))). \quad (21)$$

**Theorem 3:** Assume that  $n^1 = n^2 = 1$ . In the sequential equilibrium for a unitary government, there exist  $\theta'$  and  $\theta''$ , which depend on the off-equilibrium weights  $\alpha_{11}(2)$  and  $\alpha_{22}(0)$ , satisfying  $0 < \beta' \leq \theta' < \alpha^0 < \theta'' \leq \beta'' < 1$  (with  $\beta'$  and  $\beta''$  as given in Theorem 1) such that:

- (A) there exist values of  $\alpha_{11}(2)$  and  $\alpha_{22}(0)$  such that the equilibrium allocations solve the full-information optimization **(I)** with  $\alpha = \alpha_{11}(1)$ , for  $\beta' \leq \alpha_{11}(1) \leq \beta''$ ;
- (B) both consumers receive  $\tilde{X}(2, \alpha_{11}(2))$  for any  $\alpha_{11}(1) > \theta''$ ;
- (C) both consumers receive  $\tilde{X}(0, \alpha_{12}(0))$  for any  $\alpha_{11}(1) < \theta'$ .

From the results in Theorem 3 and the arguments in the proof, there are three regions for  $\alpha_{11}(1)$ . First, for values close to  $\alpha^0$ , the full-information allocation attained in Theorem 1 is the equilibrium for any off-equilibrium preferences  $\alpha_{11}(2)$  and  $\alpha_{22}(0)$ . Second, when  $\alpha_{11}(1)$  is farther away from  $\alpha^0$ , but between  $\theta'$  and  $\theta''$  and (20) and (21) are satisfied, then there exist off-equilibrium preferences such that the same set of full-information equilibria can be attained as under complete commitment in Theorem 1. However for other off-equilibrium preferences, the full-information allocation is not achievable. To achieve the full-information allocation when  $\alpha_{11}(1)$  is near  $\theta'$  or  $\theta''$ , the off-equilibrium preferences must be extreme—the government can give no weight to anyone who misreveals. In other words, when everyone, even liars, gets positive weight, then the attainable set is strictly smaller than under commitment. That is,  $\beta' < \theta'$  and  $\theta'' < \beta''$ ; the inequalities are strict. Third, when  $\alpha_{11}(1) > \theta''$  or  $\alpha_{11}(1) < \theta'$ , the full-information allocation is unattainable for any off-equilibrium preferences. When an individual misreveals, either in region 2 or 3, no redistribution is possible, and the allocation is inside the full-information frontier. The precise allocation selected depends on the government's preferences

after one type misreveals. These off-equilibrium outcomes in turn affect what the government can do in equilibrium so the equilibrium outcomes depend not only upon the government's preferences under truthful revelation but also on the preferences after misrevelation.

Paradoxically, only a government desiring to do a small amount of redistribution is capable of doing any.<sup>20</sup>

Our next result is that the set of full-information allocations with no commitment shrinks as the number of consumers of each type grows.

**Corollary 1:** Assume that  $n^1 = n^2 > k > 1$ . In the sequential equilibrium when the government cannot commit to policies, there exist values of  $\alpha_{11}(2)$  and  $\alpha_{22}(0)$  such that the equilibrium allocations solve the full-information optimization **(I)** with  $\alpha = \alpha_{11}(k)$ , for  $\theta'_k \leq \alpha_{11}(1) \leq \theta''_k$  where  $\theta' < \theta'_k < \theta''_k < \theta''$ . For other values of  $\alpha$ , the equilibrium allocations lie below the full-information utility possibility frontier.

The attainable section of the full-information utility possibility frontier shrinks as the economy grows since a single deviator obtains more utility when the number of truthful individuals of his type is larger. In Theorem 2, the attainable section of the full-information utility possibility frontier with a passive judiciary grows as the economy is replicated, but it shrinks with replication under a unitary government that cannot commit.

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<sup>20</sup> This result is analogous to that of Roberts [1984] in an infinite horizon dynamic model. In that case, any individual who reported that she was the more able type would be forced to pay a lump-sum tax every period after the report. The government could not transfer enough resources to the individual in the initial period to make her willing to report being of the more able type. Because of the infinite horizon, any redistribution from an individual has a similar impact to that of values of  $\alpha$  far from  $\alpha^0$  in our model when commitment is not possible.

With no commitment, for  $n^1 = n^2 = 2$ , some redistribution may still occur even when some consumers misreveal. When one type 2 misreveals, that consumer is better off than the truthful type 2, and both type 2s are better off than when they both misreveal. Hence, the government may have preferences which lead to a single type 2 individual misrevealing with redistribution occurring between that individual and the remaining ones who claim to be type 1s. See Figure 3 for an illustration of how the frontier shifts for the economy with  $n^1 = n^2 = 1$  and  $n^1 = n^2 = 2$ .

#### 4. Comparisons to the Stiglitz Model

The nature of the optimal tax system as derived here differs significantly from the standard model. In Stiglitz [1982], the government can fully commit to its policies, and taxes depend only on each individual's actions and not others' actions (with a continuum of consumers, the government cannot detect misrevelation by a single individual, and it does not affect the budget constraint). To induce truthful reporting, the government offers each individual a choice from the same pair of net trade bundles, where each bundle is intended for a particular consumer type and the bundles satisfy self-selection or incentive compatibility constraints. The planner's optimization problem is:

$$\begin{aligned} & \text{Max}_{X^1, X^2} \alpha n^1 U^1(X^1) + (1 - \alpha) n^2 U^2(X^2) \\ & \text{s.t.} \quad U^1(X^1) \geq U^1(X^2) \\ & \quad \quad U^2(X^2) \geq U^2(X^1) \\ & \quad \quad p \cdot (n^1 X^1 + n^2 X^2) \leq 0. \end{aligned}$$

Brito *et al.* [1990] show that the nature of the optimal solution varies with the value of  $\alpha$ . There exist  $\delta'$ ,  $\delta''$ ,  $\gamma'$ , and  $\gamma''$  with  $0 \leq \gamma' < \delta' < \alpha^0 < \delta'' < \gamma'' \leq 1$  such that, for  $\alpha$  between  $\delta'$  and

$\delta''$ , the solution is on the full-information frontier. For  $\alpha$  between  $\gamma'$  and  $\delta'$  and between  $\delta''$  and  $\gamma''$ , the optimum gives a distorted bundle to the type whose utility is greater than at  $\alpha^0$ . In these regions, as  $\alpha$  increases, type 1's utility increases and type 2's decreases. For  $\alpha$  above  $\gamma''$  or below  $\gamma'$ , redistribution is constant with the allocation at  $\alpha > \gamma''$  the same as at  $\gamma''$  and at  $\alpha < \gamma'$ , the same as at  $\gamma'$ . A larger region of the full-information frontier is achievable with partial commitment than in the standard model,  $\beta'' > \delta''$  and  $\beta' < \delta'$ . This follows since  $U^2(X^2(\beta'')) = U^2(X^{*1})$  defines  $\beta''$ , while  $U^2(X^2(\delta'')) = U^2(X^1(\delta''))$  defines  $\delta''$ . Since consumption is not inferior for type 1s, then  $U^2(X^1(\delta'')) > U^2(X^{*1})$  must hold making the constraint defining  $\delta''$  tighter. Interestingly, if consumption were inferior, then  $\delta''$  could be less than  $\beta''$ .

However, the comparisons between  $\beta''$  and  $\gamma''$  or  $\beta'$  and  $\gamma'$  are ambiguous. In Figure 4A,  $\beta''$  exceeds  $\gamma''$  while in Figure 4B,  $\gamma''$  exceeds  $\beta''$ . It is therefore possible that the maximum utility that can be achieved by the type favored in redistribution is less in our framework than in the standard self-selection model.

## 5. Conclusions

Legal rules can significantly and substantively affect the policies implemented by the government, sometimes in counterintuitive ways. As we have shown, the weaker is the ability of the judiciary to review and overturn executive or legislative acts, the weaker are those branches in their ability to redistribute through the income tax system. More redistribution is possible with an active judiciary than with a passive judiciary, and a passive judiciary allows more redistribution than one which cannot overturn other branches' actions. When the planner has precise information about the aggregate distribution of types with a finite number of individuals,

the planner can determine if some individuals have misrevealed. A mechanism designer (such as a tax authority) can then choose different allocations when all individuals reveal types truthfully and when some misreveal (and it must do so to maintain budget balance in all outcomes).

Different legal principles can constrain the tax authority in choosing allocations off the equilibrium path. An active judiciary allows the tax authority to commit to its tax schedules (conditional on the aggregate announcements by individuals). In this case, the tax authority can implement any allocation on the full-information utility possibility frontier. If the judiciary can only act when individuals with standing object to the tax authority revising consumption bundles, the tax authority can only implement a subset of the allocations on the full-information frontier. The doctrine of “clean hands” would limit the judiciary’s jurisdiction, but it has no impact on the range of achievable outcomes. If the judiciary has no power over the other branches, the set of possible allocations on the full-information frontier shrinks further, and the tax authority may choose an allocation below the frontier. Thus, separation of powers may allow the government to achieve better outcomes than with a unitary government.

One important implication when the government cannot commit to its initially announced policies but can reoptimize after individual decisions is that it raises questions about Stiglitz’s [1986] “new new welfare economics” approach. That approach essentially seeks to separate allocation from distribution, with allocation issues incorporated in finding the information-constrained Pareto frontier and distributional issues involved in choosing a point on that frontier. In our model, individuals cannot make their individual choices without predicting how government will readjust its announced policies. Making such predictions requires knowing the government’s distributional preferences. Distributional preferences do not simply imply choosing a point from a feasible set, but they in part determine what is feasible. Theorem 3, with

no commitment by the government, demonstrates this most strongly. The strength of the government's distribution preferences affects whether any redistribution is possible.

Some strong assumptions were made in the specific analysis of tax policy done above for reasons of tractability. The conclusions about the role of judicial review would hold more generally. Two crucial factors in the analysis were the budget balance requirements off the equilibrium path and the tax authority's precise knowledge of the distribution of types. To see the role of budget balance, compare our optimal tax results to the related problem of a discriminating monopolist. Levine and Pesendorfer [1995] analyze a monopolist selling to a finite set of customers who can fully commit to contract offers. In results analogous to Piketty's, they show that the monopolist can fully extract the surplus from consumers. Hamilton and Slutsky [2004] impose restrictions similar to the ones in this paper, of dominance solvability and reoptimization with partial commitment both on and off the equilibrium path. The full-surplus extraction result of Levine and Pesendorfer [1995] continues to hold unlike our results for taxation. The difference is the absence of budget balance restrictions.

That the tax authority has exact information about the aggregate distribution of types can be relaxed somewhat. For example, assume that there are two types, high and low. Assume the tax authority is uncertain about the exact ability of these types but that the lowest possible high is known to be more able than the highest possible low. The tax authority can in effect pool all possible highs at the lowest possible values and similarly for the lows. The tax authority can then do the analysis here with the pooled types. There will be some loss in social welfare from the pooling, but the tax authority can still do better than in the standard model. In the nonlinear pricing problem [Hamilton and Slutsky 2004], we show that the monopolist can gain by using a

mechanism in this vein even when there is some uncertainty about the precise distribution of types.

Finally, although we examined judicial review's impact on optimal income taxation, there are important lessons from this analysis for any mechanism design problem with a finite number of agents. If only to avoid imposing infeasible actions off the equilibrium path, policies affecting one agent cannot be independent of what other agents do. The principal may be able to alter policies from those initially announced. If so, the partial commitment restrictions we considered here may be an appropriate and realistic possibility lying between complete commitment and no commitment.

## Appendix

### Note on the Proofs:

Conditions (5) and (6) in the text are weak inequalities. Typically, dominance solvability conditions involve iterated elimination of strictly dominated strategies. When a game is dominance solvable in that sense, it has a unique Nash equilibrium. If one eliminates weakly dominated strategies instead, the process may eliminate some Nash equilibria when multiple equilibria exist. (See Gretlein [1983] for an explanation of the difficulties that arise from eliminating weakly dominated strategies.) Here we want the mechanism to have a unique equilibrium and intend to eliminate only strictly dominated strategies. However, since these conditions serve as constraints in an optimization problem, if we specify strict inequalities, the feasible set would not be closed, potentially leading to difficulties. As developed in the proofs, most of these conditions hold with strict inequality. (See, for example, the discussion in the text of conditions (13), (14), and (15).) At least one constraint holds with equality. This is similar in effect to having standard self-selection constraints hold with weak inequalities. Individuals might be indifferent between revealing truthfully or falsely, but we assume they reveal truthfully. A bundle within an arbitrarily small  $\varepsilon$  of the solution exists which would make the condition hold with strict inequality.

**Proof of Theorem 1:** Define  $\beta'$  and  $\beta''$  by the following equations:  $U^1(X^1(1, \beta')) = U^1(X^{2*})$  and  $U^2(X^2(1, \beta'')) = U^2(X^{1*})$ , where  $X^{i*}$  is type  $i$ 's best bundle on the  $p \cdot X = 0$  line and  $X^i(1, \alpha)$  is the solution to the full-information optimization (I) for some  $\alpha$ . Since  $\alpha$  is the welfare weight on type 1's utility,  $U^1(X^1(1, \alpha))$  is increasing in  $\alpha$  and  $U^2(X^2(1, \alpha))$  is decreasing in  $\alpha$ . For any  $\alpha$  such that  $\beta' \leq \alpha \leq \beta''$ , consider the following stage 2 provisional policies for the government:

$Y(0) = X^{2*}$ ,  $Y(2) = X^{1*}$ ,  $Y^1(1) = X^1(1, \alpha)$ , and  $Y^2(1) = X^2(1, \alpha)$ . Given the reoptimization possible under partial commitment, these provisional policies would be implemented in stage 4 since any feasible deviation from them would violate a truth-teller's entitlement. Given the monotonicity in  $\alpha$ , (16) and (17) are satisfied for  $\alpha$  between  $\beta'$  and  $\beta''$ . If  $\alpha > \beta''$ , then  $X^1(1, \alpha)$  and  $X^2(1, \alpha)$  violate (16) and for  $\alpha < \beta'$ , they violate (17). No other allocations can be implemented consistent with (16) or (17) and reoptimization in stage 4 since from the optimization in (12), the truth-telling allocations must be undistorted and the  $Y(0)$  and  $Y(2)$  announcements yield the least restrictive dominance solvability constraints.

Now (a) follows because, given  $\alpha_{11}(1)$  between  $\beta'$  and  $\beta''$ , the government selects  $X^1(1, \alpha_{11}(1))$  and  $X^2(1, \alpha_{11}(1))$ , since they are feasible and maximize social welfare. No other allocations do better. For  $\alpha_{11}(1) > \beta''$ ,  $X^1(1, \alpha_{11}(1))$  and  $X^2(1, \alpha_{11}(1))$  cannot be implemented since (16) is violated. The best feasible bundle is  $X^1(1, \beta'')$  and  $X^2(1, \beta'')$ , showing (b). For  $\alpha_{11}(1) < \beta'$ , similarly,  $X^1(1, \beta')$  and  $X^2(1, \beta')$  are the best feasible policies for the government, showing (c). **QED**

**Proof of Theorem 2:** First, note that if we restrict attention to situations in which individuals of the same type are treated equally then the full-information utility possibility frontier is the same for all  $k$ . Second, any allocation which can be implemented for  $k = 1$  can be implemented for any  $k > 1$ . From Theorem 1, the allocations which can be implemented when  $k = 1$  are  $X^1(1, \alpha)$  and  $X^2(1, \alpha)$ ,  $\beta' \leq \alpha \leq \beta''$ . For  $k > 1$ , consider the following stage 2 announcements by the government.  $Y^1(n^1) = X^1(1, \alpha)$ ,  $Y^2(n^1) = X^2(1, \alpha)$ ,  $Y^1(N^1) = X^{1*}$ ,  $N^1 \neq n^1$  and  $Y^2(N^1) = X^{2*}$ ,  $N^1 \neq n^1$ . Since for every  $N^1$  the provisional announcements are an undistorted

allocation, they cannot be altered in the reoptimizations of stage 4 because some truthful individuals would have their utility reduced below their guarantee level. These announcements then satisfy the dominance solvability conditions. For example, for  $\alpha > \alpha^0$ , (5) and (6) must hold. For  $N^1 \neq n^1$  or  $n^1 - 1$ , (5) reduces to  $U^1(X^{1*}) \geq U^1(X^{2*})$  which must hold with strict inequality from single crossing. For  $N^1 = n^1$ , (5) is  $U^1(X^{1*}) \geq U^1(X^2(1, \alpha))$ , which holds with strict inequality since  $X^{1*}$  is type 1's best bundle on a higher resource line than  $X^2(1, \alpha)$ . For  $N^1 = n^1 - 1$ , (5) is  $U^1(X^1(1, \alpha)) \geq U^1(X^{2*})$ . This also holds with strict inequality because  $X^1(1, \alpha)$  is type 1's best bundle on a higher resource line than  $X^{2*}$ . Hence, type 1 has a dominant strategy to be truthful. For type 2, for  $N^1 > n^1$ , (6) becomes  $U^2(X^{2*}) \geq U^2(X^{1*})$  which holds from single crossing. For  $N^1 = n^1$ , (6) is  $U^2(X^2(1, \alpha)) \geq U^2(X^{1*})$  which is the same as (16), so it must hold with strict inequality. All necessary conditions are satisfied. A similar argument holds for  $\alpha < \alpha^0$ . Therefore,  $\beta''(k) \geq \beta''(1)$  and  $\beta'(k) \leq \beta'(1)$  hold for all  $k$ .

Third, we will show that  $\beta''(k)$  strictly increases in  $k$  with a similar argument that  $\beta'(k)$  strictly decreases. Consider  $k = 2$  and any  $\alpha_{11}(2)$  with  $\alpha_{11}(2) > \alpha^0$ . There are five possible information sets,  $N^1 = 0, 1, 2, 3$ , or  $4$ . Denote the equilibrium allocation to be sustained as  $X^1(2, \alpha_{11}(2))$  and  $X^2(2, \alpha_{11}(2))$ . The government must then choose utility guarantees such that the off-equilibrium-path allocations satisfy the following dominance solvability constraints

$$U^1(X(4)) \geq U^1(X^2(3)) \tag{A1}$$

$$U^1(X^1(3)) \geq U^1(X^2(2, \alpha)) \tag{A2}$$

$$U^1(X^1(2, \alpha)) \geq U^1(X^2(1)) \tag{A3}$$

$$U^1(X^1(1)) \geq U^1(X(0)) \tag{A4}$$

$$U^2(X^2(2, \alpha)) \geq U^2(X^1(3)) \quad (\text{A5})$$

$$U^2(X^2(3)) \geq U^2(X(4)) \quad (\text{A6})$$

Note that, unlike when  $n^1 = n^2 = 1$ , the planner's choice of  $X^1(3)$  is not constrained to lie on the no-redistribution budget line. Committing to a utility guarantee  $U^2(X^2(3)) > U^2(2, \alpha_0)$  makes it credible for the planner to choose  $X^1(3)$  below the no-redistribution line, which allows the planner to reduce  $U^2(X^2(2, \alpha))$ . As with the  $X(2)$  bundle with a single consumer of each type, offering efficient bundles (on the  $MRS^i = 1$  locus) allows the planner to discourage misreporting as much as possible among all bundles on a given budget line (given the credibility restrictions from the reoptimization).

The bundles  $X^1(3)$ ,  $X(4)$ , and  $X^2(3)$  each appear on the LHS and RHS of different inequalities. Since  $X(4)$  must lie on the no-redistribution line, the incentive constraints (A1), (A2), and (A6) and the budget constraint  $p \cdot [3X^1(3) + X^2(3)] = 0$  may restrict the set of efficient allocations the planner can achieve. Observe that the set of efficient allocations the planner can sustain is greater than when there is only one consumer of each type. In Figure 5, by choosing the maximum  $X^2(3)$  given (A1), the planner can commit to a low level for  $U^2(X^1(3))$ , allowing more redistribution from type 2 to type 1 under truthful reporting.

Figure 6 displays the set of utility guarantees and off-equilibrium-path bundles to sustain the maximum transfer from type 2 to type 1 when  $n^1 = n^2 = 4$ . Observe that the utility guarantees to truthful type 2's involve redistribution both toward and away from that group. The bundle  $X^2(5)$  is constructed to be as costly as possible to make it credible that  $U^2(X^1(5))$  is so low. Any full-information allocation with  $U^2(4, \alpha^0) > U^2 > U^2(X^2(4, \alpha))$  can be implemented

as a dominance-solvable equilibrium. Given this, the maximal transfer when  $n^1 = n^2 = 2$  is less than the maximal transfer when  $n^1 = n^2 = 4$ . Note that  $X^2(7)$  in Figure 6 is the same as  $X^2(3)$  in Figure 5. Let  $D$  be the value of the transfer to truthful type 2's in the  $X^2(7)$  bundle, and let  $E$  be the value of the tax on those reporting to be type 1's in the  $X^1(7)$  bundle. The amount  $D$  is fixed by the point where type 1's indifference curve which is tangent to the no-redistribution line intersects the locus of allocations where  $MRS^2 = 1$  (the efficient bundles to give type 2); it is independent of  $n^1, n^2$  and the number of misreporting agents. From budget balance, for  $n^1 = n^2 = 2$ ,  $3E = D$ . With  $n^1 = n^2 = 4$ ,  $E'$  is the maximum net tax that can be extracted from type 1 reporters when only one type 2 misreports. From budget balance,  $5E' = 3D'$ . Since  $D' > D$ , then  $E' > E$  and, thus, in equilibrium under truthful revelation, type 2's can be made worse off when  $n^1 = n^2 = 4$  than when  $n^1 = n^2 = 2$ . (This holds as well for the case of  $n^1 = n^2 = 3$ .)

When  $n^1 = n^2 > 4$ , let the no-redistribution efficient bundles be  $X^1(N^1)$  and  $X^2(N^1)$  for all  $N^1 \geq n^1 + 4$ . Then let  $X^2(n^1 + 3)$  be the same bundle as  $X^2(7)$  when  $n^1 = n^2 = 4$ . Let  $\bar{E}$  be the value of the net tax paid by type 1's when  $N^1 = n^1 + 3$ , let  $\bar{F}$  be the net tax paid by type 2's when  $N^1 = n^2 + 2$ , let  $\bar{F}'$  be the net transfer to type 1's when  $N^1 = n + 2$ ,  $\bar{D}'$  be the net transfer to type 2's when  $N^1 = n^1 + 1$  and let  $E'$  be the net tax paid by type 1's when  $N^1 = n^1 + 1$ .  $D$  is the same when  $n^1 = 4$  and  $n^1 > 4$ . Since  $3E = D$  and  $(n^1 + 3)\bar{E} = (n^1 - 3)D$ ,  $\bar{E} > E$ , thus  $\bar{F} > F$  since 2's indifference curve through  $X^1(n^1 + 3)$  is lower. Since  $\bar{F} > F$  and  $(n^1 + 2)\bar{F}' = (n^1 - 2)\bar{F}$ ,  $\bar{F}' > F'$  for  $n^1 > 4$ . Since  $\bar{F}' > F'$ ,  $\bar{D}' > D'$  since  $X^2(n^1 + 1)$  is on a higher indifference curve for 2. Since  $(n^1 + 1)\bar{E}' = (n^1 - 1)\bar{D}'$ ,  $\bar{E}' > E'$  follows.

Thus, the planner can extract more resources from type 2 as  $n^1$  and  $n^2$  grow in proportion. **QED**

**Proof of Theorem 3:**  $\tilde{X}(2, \alpha_{11}(2))$ , as the solution to (11) without the partial commitment constraint, lies between  $X^{1*}$  and  $X^{2*}$  depending upon  $\alpha_{21}(2) = 1 - \alpha_{11}(2)$ . If  $\alpha_{21}(2) = 0$ , then  $\tilde{X}(2, \alpha_{11}(2)) = X^{1*}$ . Since  $\tilde{X}^2(1, \alpha_{11}(1))$  is the same as the solution to (I) with  $\alpha = \alpha_{11}(1)$ , then when  $\alpha_{21}(2) = 0$ , (20) is the same as (16). Hence,  $\theta''$ , the largest value of  $\alpha_{11}(1)$  at which (20) is not violated, equals  $\beta''$ .

If  $\alpha_{21}(2) > 0$ , then  $\tilde{X}(2, \alpha_{11}(2))$  diverges from  $X^{1*}$  in a way that type 2 prefers. Now (20) is more restrictive than (16). At  $\alpha = \beta''$ , (20) would be violated. Hence,  $\theta'' < \beta''$  must hold.

Using  $\alpha_{12}(0) = 0$ , similar arguments show that  $\theta'$ , the smallest value of  $\alpha_{11}(1)$  at which (21) is satisfied, equals  $\beta'$  and exceeds it when  $\alpha_{12}(0) > 0$ . Conditions (20) and (21) hold for any  $\alpha_{11}(1)$  between  $\beta'$  and  $\beta''$  when  $\alpha_{12}(0)$  and  $\alpha_{21}(2)$  equal their extreme values, showing (A).

For  $\alpha > \theta''$ , type 2s will misreveal and the government implements the solution to (11) without the partial commitment constraint, showing (B), while for  $\alpha < \theta'$ , the solution to (10) without the partial commitment constraint is implemented, showing (C). **QED**

**Proof of Corollary 1:** Let  $k = 2$  and assume that the planner wishes to tax type 2s. When one type 2 individual claims to be type 1, the planner chooses bundles  $X^1$  and  $X^2$  to maximize:

$$2\alpha_{11}(3)U^1(X^1) + \alpha_{21}(3)U^2(X^1) + \alpha_{22}(3)U^2(X^2) \text{ subject to a budget constraint } p \cdot (3X_1 + X_2) \leq 0.$$

Let  $\tilde{X}^1(3, \alpha_{11}(3))$  and  $\tilde{X}^2(3, \alpha_{11}(3))$  denote the bundles chosen.  $\tilde{X}^1(3, \alpha_{11}(3))$  lies on a higher budget line than the bundle received when  $k = 1$  and type 2 misreveals since the planner can still

redistribute from one type 2 to two type 1s and the misrevealing type 2. This bundle also lies (weakly) between the undistorted bundles for the two types on this budget line. Thus, the type 2 who misreveals prefers the bundle  $\tilde{X}^1(3, \alpha_{11}(3))$  to the bundle received when type 2 misreveals when  $k = 1$ . Hence, the type 2 bundle when all reveal truthfully must satisfy

$U^2(\tilde{X}^1(2, \alpha_{11}(2))) \geq U^2(\tilde{X}^1(3, \alpha_{11}(3)))$ . Since the RHS utility value exceeds the minimum in Condition (A) of Theorem 3, less redistribution is possible when agents reveal truthfully for  $n^1 = n^2 = k$  than when  $n^1 = n^2 = 1$ .

For any larger value of  $k$ , the bundle given to type 1s when one type 2 misreveals lies on a higher budget line (there are more type 2s to tax relative to the number of type 1s to benefit), so the worst bundle for type 2s consistent with truthful revelation must lie on a higher budget line.

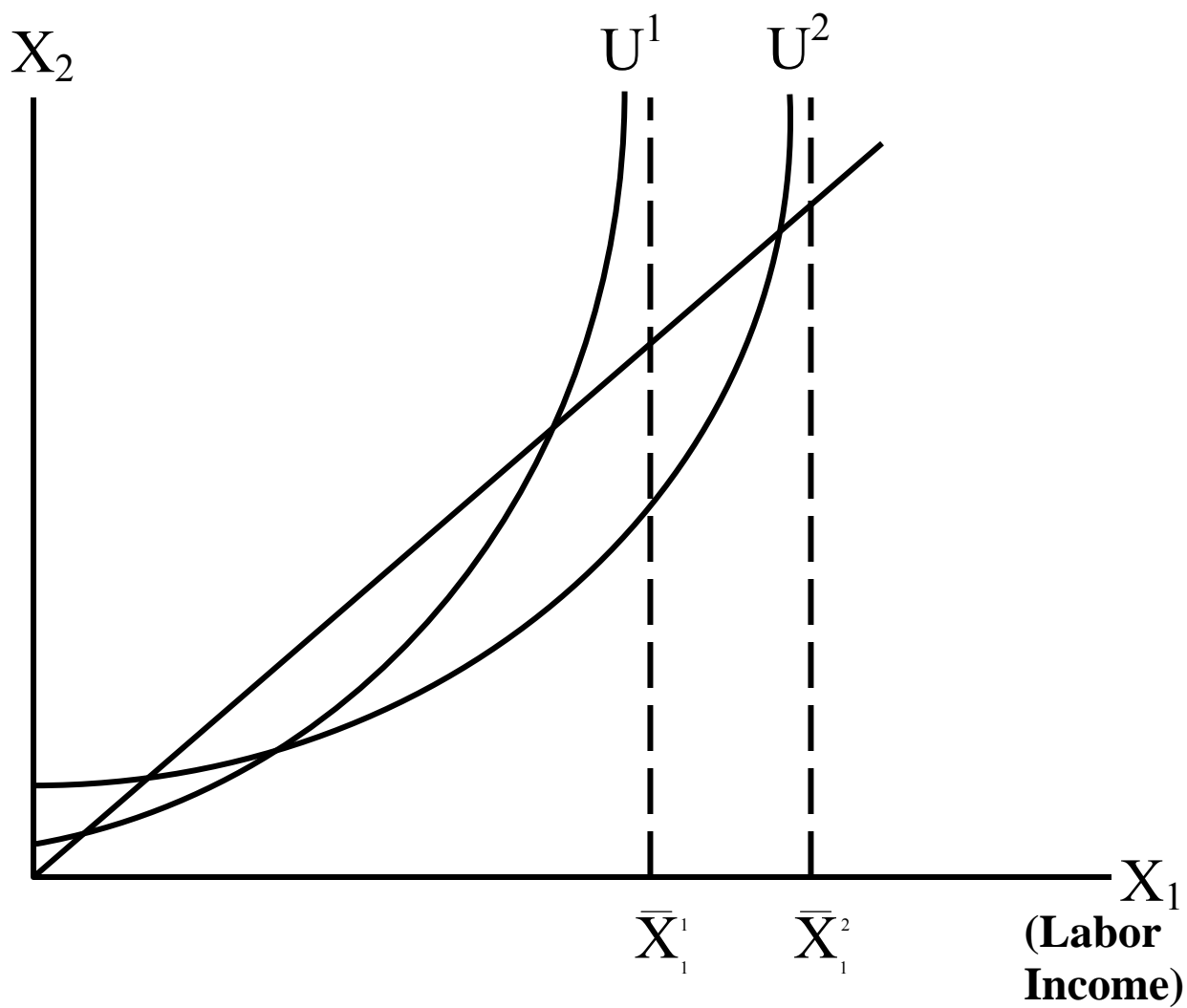
**QED**

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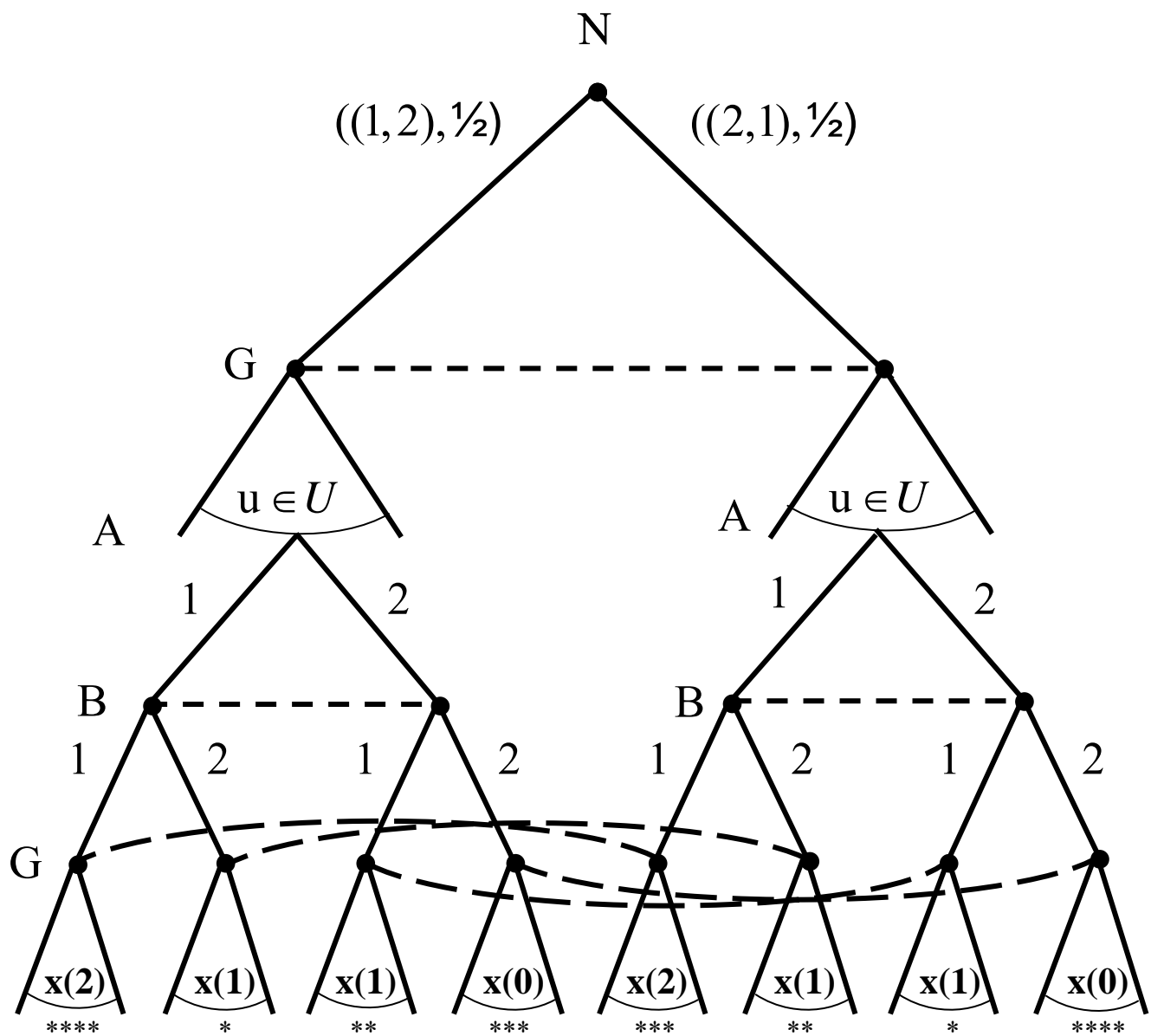
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**(Consumption)**

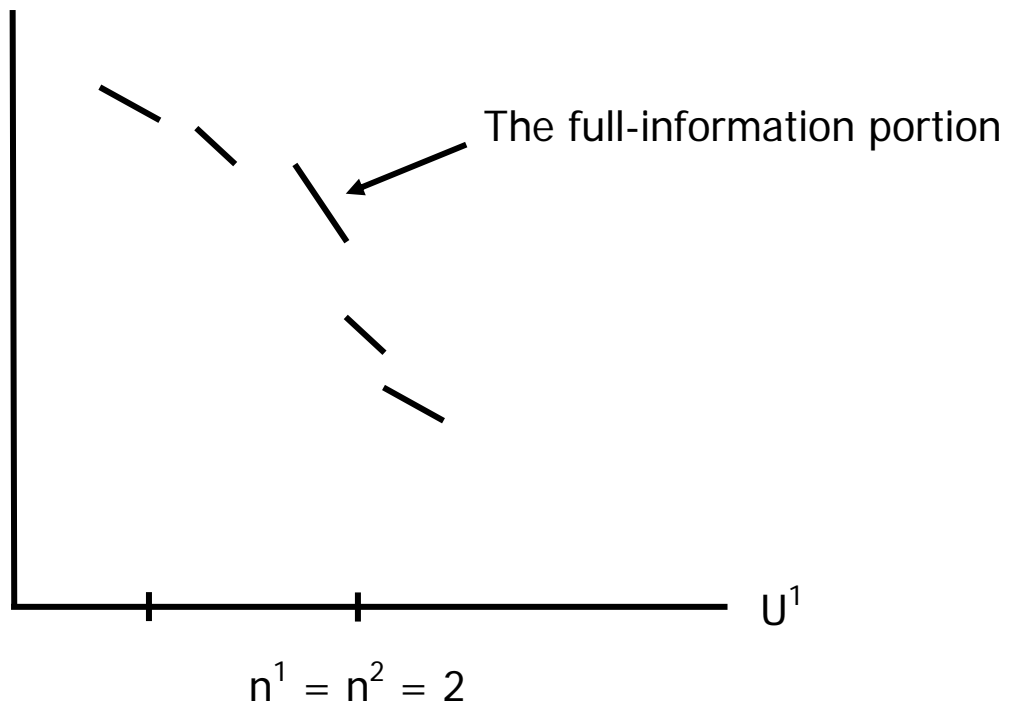
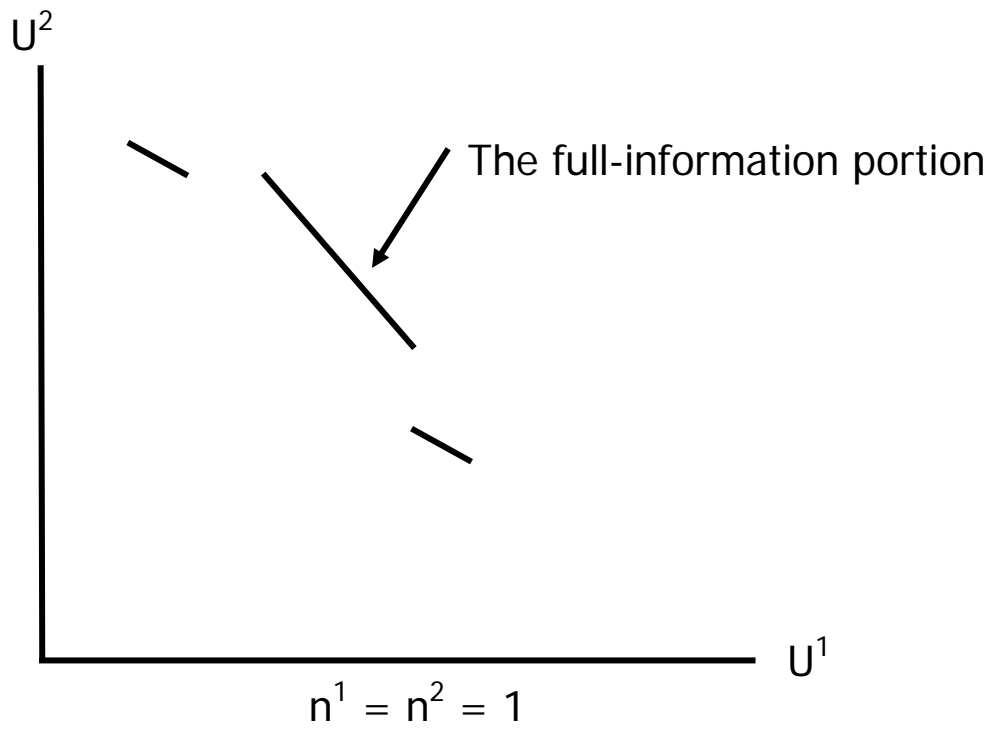


**Figure 1**



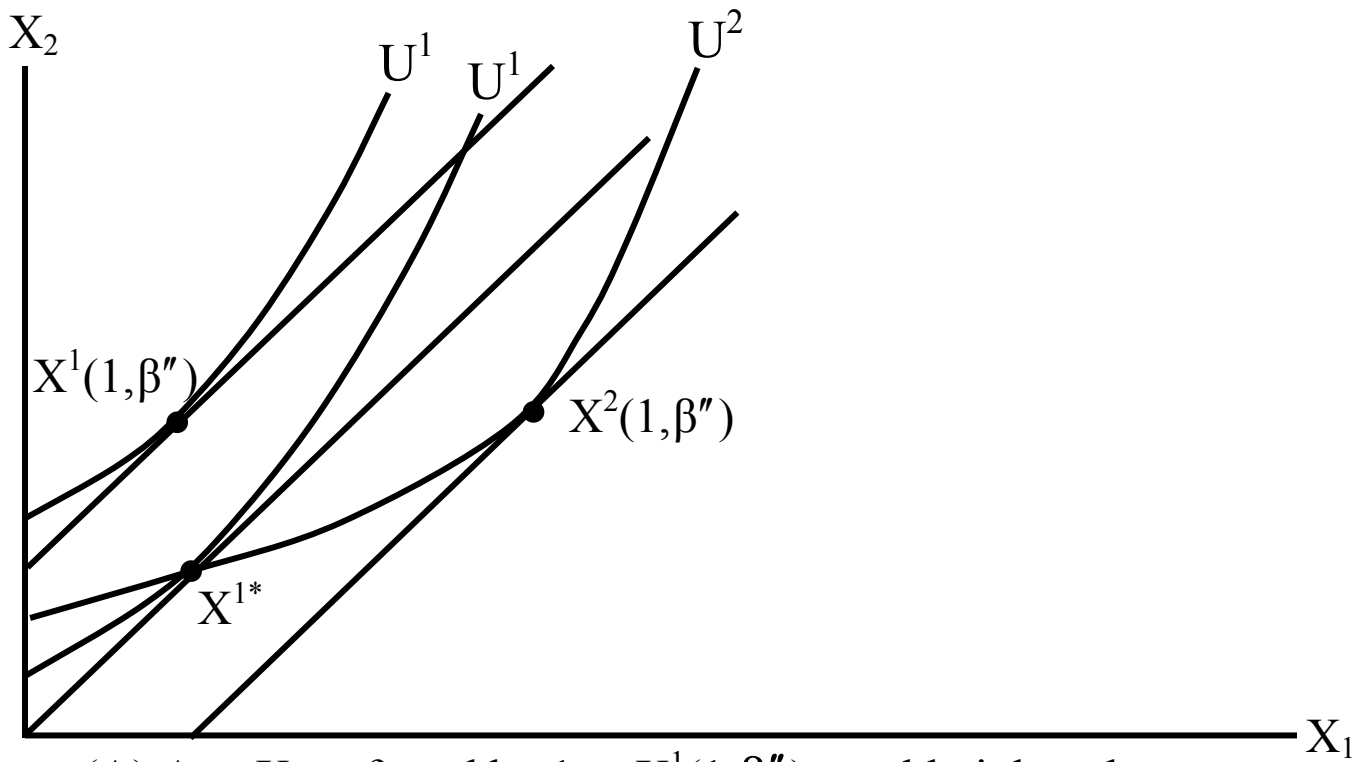
- \* All report truthfully
- \*\* Both misreveal type
- \*\*\* Only A misreveals
- \*\*\*\* Only B misreveals

**Figure 2**

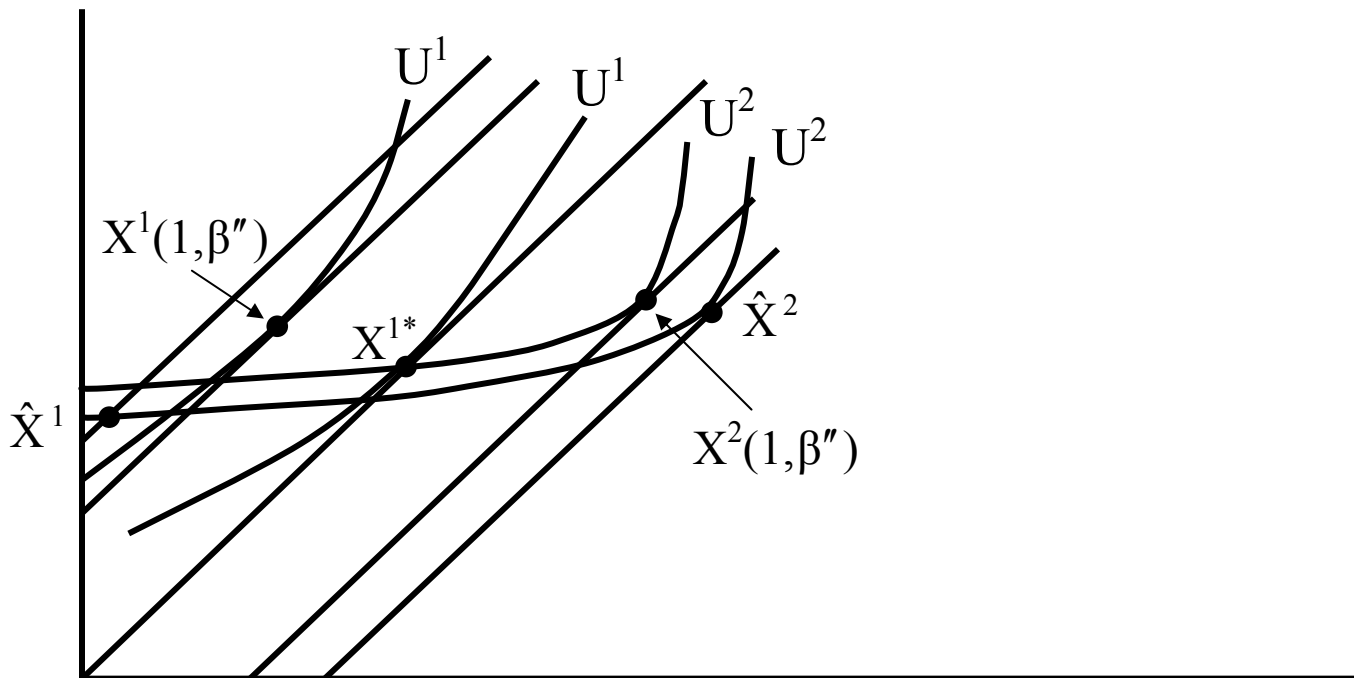


The utility possibility frontier with no commitment

**Figure 3**



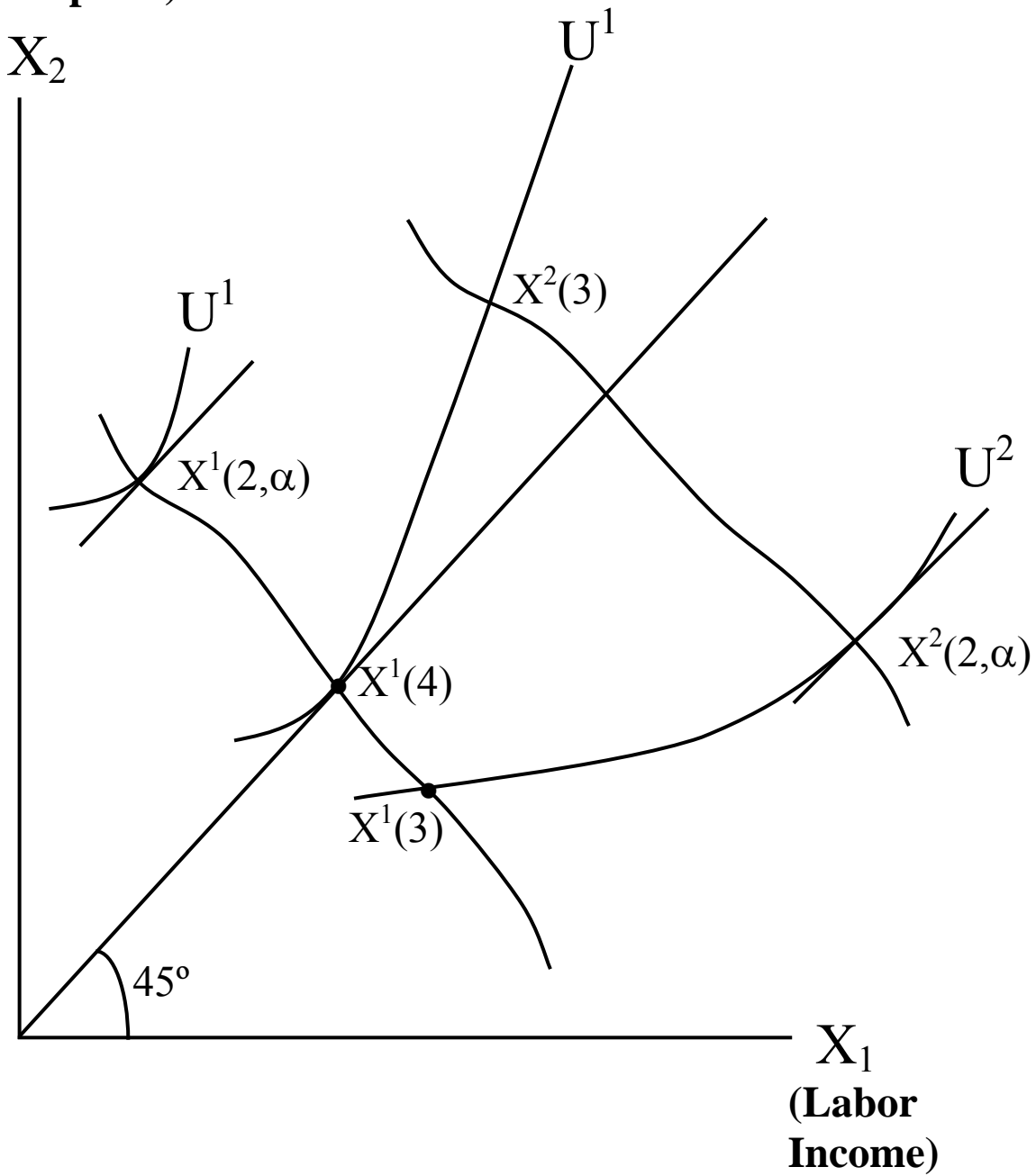
(A) Any  $X$  preferred by 1 to  $X^1(1, \beta'')$  would violate the standard self-selection constraint for 2. Hence,  $\beta'' > \gamma''$ .



(B)  $(\hat{X}^1, \hat{X}^2)$  satisfy standard self-selection constraints and do more redistribution than  $(X^1(1, \beta''), X^2(1, \beta''))$ . Hence,  $\gamma'' > \beta''$ .

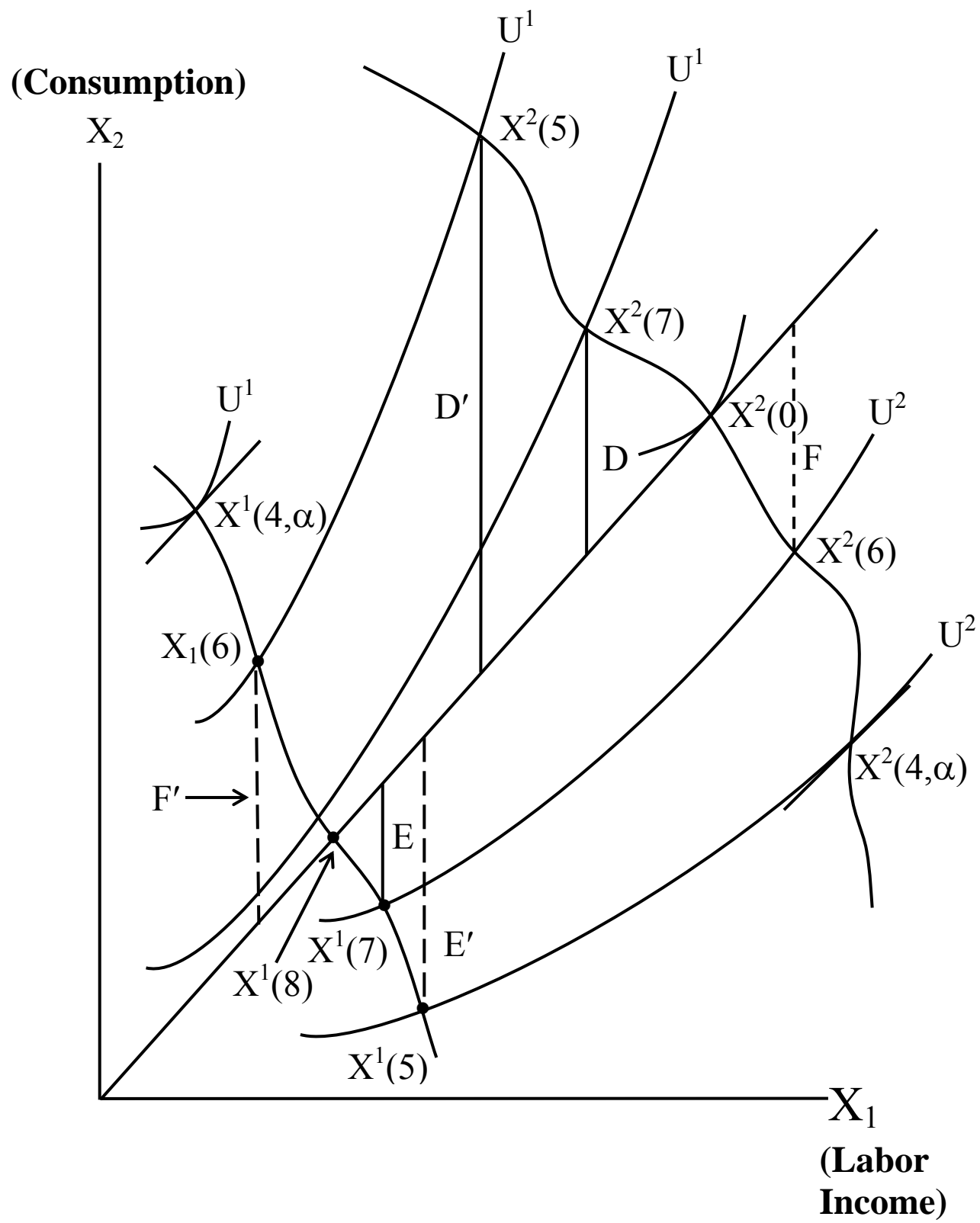
**Figure 4**

(Consumption)



$X^2(3)$  is chosen to use the maximum resources subject to (A1).  $X^1(3)$  is then constrained by budget balance.

**Figure 5**



**Figure 6**