

# **Social Cooperation under Endogenous Time Preference**

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*This paper provides a theoretical basis for the commonly observed cleft between societies which display social cooperation and non-corrupt institutions and those that do not. Adjoining the empirical reality of endogenous time preference as a function of wealth to the repeated prisoner's dilemma reinforces both joint cooperation and joint defection. Results are the same when applied to a large group of players randomly paired. Aside from a uniquely stable ratio of "defectors" and "cooperators", the group as a whole will gradually shift toward a "cooperative" or "non-cooperative" society.*

## **I. Introduction**

The sources of sustained poverty remain a pressing issue both in the developing world, where poverty is widespread, and within pockets of economically advanced countries. While some researchers still seek a broad cognitive cause for persistent poverty (Swoon et. al. 2011), lower permanent income has repeatedly correlated with higher discounting of the future (Hausman 1979, Lawrance 1991). A marked relationship also exists at the macro level between corrupt institutions and a lack of economic development (Serra 2006). This paper merges these joint empirical phenomena within the unified, game theoretic framework of a repeated prisoner's dilemma. The (non)persistence of poverty emerges as an adaptation of both behavior *and impatience itself* to prevailing societal circumstances, thereby reinforcing prosperity and honesty as well as poverty and corruption.

The relatively recent shift of focus in fostering economic development, from direct subsidy to microcredit, belies a paradigmatic reassessment of the root causes of long-term poverty, from a lack of immediate income to the lack of a means for saving

and borrowing. A present-oriented society with minimal savings is often perpetuated by the corruption of political institutions and unavailability of formal financial intermediaries (Morduch and Rutherford 2003). Long-term alleviation of poverty therefore begins with the creation and maintenance of reliable institutions and secure transactions, a two-way relationship between an honest lender/seller and honest borrower/buyer. The widespread coordination of individuals between honesty and economic development versus corruption and economic stagnation suggests a game theoretic treatment of the issue as a prisoner's dilemma (MacRae 1982).

Previous models of cooperation within the prisoner's dilemma framework have, however, neglected the empirical link between income and time preference, regarding discount rates as immutable and exogenous (notably, Axelrod 2006). An exogenous treatment of time preference divides players into tentative "cooperators" and "defectors" which results in a "tit for tat" strategy proving successful on average, even when a small proportion of the population is like-minded (Axelrod 2006, Ch. 3). The games I present in the subsequent section, in contrast, use payoffs as a surrogate for income which thereby affects time preference. Outcomes of mutual cooperation and unilateral defection make the player more future-oriented, while unilateral cooperation and joint defection have the opposite effect. Within the context of a large society engaged in multiple interactions, I additionally employ the realistic assumption that where one or both players defect, games do not continue for multiple rounds. This collapses each interaction into a quasi-repeated game, with three one-shot outcomes where one or more parties defect and one outcome of mutual repeated cooperation discounted to the present period.

Endogenizing time preference in the repeated prisoner's dilemma extends the basic prediction of the Folk Theorem. Where both players are initially cooperative and sufficiently patient, sustained cooperation occurs as before. The cooperative interaction makes both players more future-oriented, which further solidifies their commitment to cooperation. Thus, the jointly cooperative outcomes is not only sustained but also reinforced by additional rounds of play. Conversely, a joint defection outcome makes both players more impatient and subsequently reinforces a sustained, non-cooperative outcome. Taken less literally, this result implies a separating equilibrium between non-

corrupt groups of increasingly patient individuals and groups of hardened defectors driven increasingly toward a hand-to-mouth existence.

Extending the model to a large group of players (with the previously described stipulation regarding game termination) extends the two-player result to an entire society. Where a small number of individuals begin as “defectors”, they amass income via defection which raises their level of patience and thereby changes their type, not as a response to the ratio of “cooperators” to “defectors”, but as an *adaptive response* to greater income.<sup>1</sup> Conversely, when a small number of cooperators exists, repeated defection, including the possibility of their own defection due to a sufficiently low percentage of cooperators, will raise their level of impatience to the point where they *become* defectors regardless of the societal ratio. Accordingly, a unique and fragile equilibrium exists where society is composed of both cooperators and defectors. In every other circumstance, societies and their institutions will be pushed toward corrupt and impoverished or non-corrupt and prosperous.

Where some signaling is possible, the model predicts an increasing stratification between the relatively rich and future-oriented and the relatively poor and present-oriented. Increasing income inequality in the United States may be attributable to such a schism, in which the proliferation of poverty is not genetically inevitable but rather an adaptive response to a present-oriented social environment. The model’s results also underscore the importance of outlets for fostering patience in the developing world, where existing financial institutions may be untrustworthy. Bauer et. al. (2010) and Ashraf et. al. (2005) find the advantage of microcredit to be precisely such a commitment mechanism to long-term savings, unsurprisingly employed by those who are especially future-oriented.

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<sup>1</sup> This result ignores the distinct possibility that defectors would view their high payoffs as a signal of successful game play and never resort to cooperation. However, the payoff of a jointly cooperative outcome is only lower because the rewards are long-term rather than short-term, and must necessarily become greater when discounting of the future is sufficiently low. For the sake of generality, I also ignore the possibility that defectors may be identified and shunned by cooperators which would ultimately lead to a straightforward societal cleavage between two increasingly uniform groups. Lastly, this result overlooks the likelihood that present-oriented individuals are more likely to consume their entire payoff leaving their wealth and, therefore, their rate of time preference unaffected.

## 2. A Repeated Prisoner's Dilemma with Endogenous Discounting

Consider an infinitely repeated version of the simple Prisoner's Dilemma game illustrated in Figure 1, where two players choose between some form of cooperation and defection.<sup>2</sup> Payoffs are generically written as in Gintis (2009), with  $R$  for "reward" when both players cooperate,  $P$  for "punishment" when both defect,  $T$  for "temptation" to the lone defector, and  $S$  for "sucker" to the lone cooperator.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	R,R	S,T
	Defect	T,S	P,P

**Figure 1 – The Prisoner's Dilemma**

The ordering of payoffs is familiar, with  $T > R > P > S$ . To rule out the possibility that alternating defection is superior to sustained cooperation, also assume that  $S+T < 2R$ .<sup>3</sup> While many scenarios are possible, subsequent analysis is uninteresting unless a meaningful difference exists in the effect of various payoffs on a player's rate of time preference. More specifically, assume that payoffs of  $T$  and  $R$  increase a player's level of patience and that payoffs of  $P$  and  $S$  decrease a player's level of patience.

Consider the standard problem of Player 1 in whether to defect or cooperate under the assumption that Player 2 has adopted a tit-for-tat strategy. Defecting amounts to the payoff  $T$  in the current period and  $P$  in each subsequent period. Sustained cooperation accrues a payoff of  $R$  into the infinite future. Thus, the decision to cooperate hinges on satisfying the inequality

<sup>2</sup> Naturally, the analysis also applies to a prisoner's dilemma that is repeated each period with some given probability.

<sup>3</sup> Note that this is a simplified version of the restriction that the discounted payoff to alternating defection (to the initial defector) is less than that of discounted cooperation for all feasible discount factors. Thus, planning to "eventually resume cooperation" is never rational.

$$R + \delta R + \delta^2 R + \dots > T + \delta P + \delta^2 P + \dots,$$

where  $\delta$  is the period-specific discount factor of Player 1, which collapses to

$$\delta > (T - R) / (T - P). \tag{1}$$

Having played the initial game, players will reap their respective payoffs and their relative levels of impatience will be adjusted accordingly. First, consider the situation where both players choose the same strategy.

**Proposition 1:** Joint defection in the initial period ensures joint defection in each subsequent period.

**Proof:** Follows immediately from the assumption that  $\Delta\delta(P) < 0$ .

**Proposition 2:** Joint cooperation in the initial period ensures joint cooperation in each subsequent period.

**Proof:** Follows immediately from the assumption that  $\Delta\delta(R) > 0$ .

The case where players choose dissimilar strategies is less straightforward and hinges on period-specific effects of payoffs on respective discount factors. Aside from the plausible possibility that a mutual defection strategy continues thereafter as perpetuated by (consistently) impatient and patient players, three alternative scenarios exist: (I) the lone defector lowers the discount factor of the lone cooperator such that both become defectors regardless of the circumstances (II) the lone cooperator increases the discount factor of the lone defector such that both adopt a tit-for-tat strategy regardless of the circumstances, and (III) the lone cooperator becomes sufficiently

impatient and the lone defector becomes sufficiently patient that both alternate these roles in the subsequent round of play.<sup>4</sup>

### **Scenario I**

Scenario I implies a “contagion effect” with respect to the patient cooperator. In addition, accruing a temporarily high payoff does not, by assumption, fundamentally modify the time preference of the defector. Thus, the outcome is defection into the infinite future.

### **Scenario II**

In Scenario II, several outcomes are possible. Given the original assumption that Player 1 suspects Player 2 has adopted a tit-for-tat strategy, Player 1 should realize that in order to sustain cooperation, he must endure a “sucker” payoff for one period. Player 1 may do so and cooperate thereafter, creating a “contagion effect” as in Scenario I toward joint cooperation. However, accruing a one-period sucker payoff has the potentially perverse consequence of returning the Player 1 to his original state of an impatient defector by diminishing his discount factor. Thus, the outcome may be the sustained alternation of  $T$  and  $S$  payoffs to both players. Although such a sustained “oscillating equilibrium” is possible, given that  $S+T < 2R$ , a gradual decrease in the time preference of both players is likely to eventually create an outcome of perpetual joint defection.

### **Scenario III**

Scenario III can largely be regarded as a mathematical curiosity, whereby an alternation occurs between cooperators and defectors. However, as previously noted, a sustained alternation of payoffs is likely to collapse into joint defection.

## **3. The Evolutionary Stability of Cooperation**

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<sup>4</sup> One would naturally wonder, however, why a consistent cooperator would choose to voluntarily associate in future with a consistent defector. I address a scenario where the association ends in the subsequent section.

The previous section presents several possible outcomes to a repeated Prisoner's Dilemma where the rate of time preference is based on payoffs. Though self-reinforcing equilibria of joint defection and joint cooperation are illuminating, they are uninteresting from an evolutionary perspective. This section examines whether a society comprised of both impatient defectors and patient cooperators can feasibly exist.

Because the notion of evolutionary stability requires random pairings of agents, the permanent association assumed in the previous section is no longer directly applicable. However, the basic scenario can be modified to what becomes a “quasi-repeated game”. More specifically, I can embed the game's potential trajectories in the initial outcomes by making a “grim trigger” termination assumption. That is, if one or both players defect, the game between them simply ends. Under this assumption, joint cooperation in the first round implies continued cooperation in subsequent rounds because, as per Proposition 1, such cooperation will be reinforced by bolstering both players' discount factors. In all other scenarios the game ends. Thus, cooperators who suspect they are encountering other cooperators still have an incentive to cooperate.

Figure 2 illustrates the modified game, where  $\tilde{R}$  is the utility to sustained cooperation, discounted by players' present time preferences. However, unlike the standard prisoner's dilemma, the value of  $\tilde{R}$  relative to  $T$  may vary amongst players. Cooperators, although patient, are not saps, and therefore prefer to match their opponent's strategy, implying that, for them,  $\tilde{R} > T > P > S$ . By definition, defectors must view the defection strategy as dominant, thus, for them,  $T > \tilde{R} > P > S$ .

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	$\tilde{R}_1(\delta_1), \tilde{R}_2(\delta_2)$	S,T
	Defect	T,S	P,P

**Figure 2 – The Prisoner's Dilemma with “Grim Trigger Termination”**

Consider a society composed of a percentage,  $q$ , of cooperators.

**Proposition 3:** There exists some minimum value of  $q$  below which cooperation never occurs.<sup>5</sup>

**Proof:** The payoff to a cooperator of defection and cooperation are, respectively,

$$qT + (1 - q)P$$

and

$$q\tilde{R} + (1 - q)S.$$

Thus, the net expected benefit of defection is

$$qT + (1 - q)P - q\tilde{R} - (1 - q)S. \quad (2)$$

By assumption of the payoff ordering for a cooperator, Equation 2 is strictly decreasing in  $q$ . ■

As per the assumption that  $\Delta\delta(P) < 0$ , sustained defection due to a low percentage of fellow cooperators will actually turn cooperators into defectors, thereby creating a self-reinforcing trajectory which drives  $q$  toward zero.

Consider the opposite scenario, where a lone defector is introduced to a society entirely comprised of cooperators.<sup>6</sup> To generalize the situation, suppose that, absent contact with the lone defector, all cooperators have the same rate of time preference,  $\delta_c$ . As in the previous section, many outcomes exist when players choose opposite strategies.

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<sup>5</sup> This proof is identical to Proposition 2 in Chapter 3 of Axelrod (2006).

<sup>6</sup> The introduction of a small number of defectors within a large population of cooperators would yield the same results given that defectors were sufficiently unlikely to be paired against each other.

**Scenario I:**  $\Delta\delta(S) > -\Delta\delta(T)$

The previous inequality implies a greater effect on the time preference of the cooperator than the defector. Thus, the defector is likely to continue defecting in future, while the cooperator's rate of time preference is greatly diminished. However, the cooperator may continue to cooperate in future interactions with others, i.e. his optimal strategy may not change. In this case, interaction with the "mutant" defector will temporarily lower the discount factor of those with whom he interacts.<sup>7</sup> Alternatively, the cooperator may actually become a defector, implying a contagion process which will create additional defectors at an increasing rate. More formally,

**Proposition 4:** If  $\Delta\delta(S)$  is sufficiently large in absolute value and  $\Delta\delta(T)$  is sufficiently small, a society comprised of both cooperators and defectors is not evolutionarily stable.

**Proof:** There must exist some sufficiently large  $\Delta\delta(S)$  in absolute terms such that a former cooperator will subsequently value  $T$  more than  $\tilde{R}$  after receiving  $S$ . As  $\Delta\delta(T)$  approaches zero, the percentage of defectors will never decrease. Thus, if both conditions hold, the value of  $q$  will never increase and almost always decrease over time.<sup>8</sup>

**Scenario II:**  $\Delta\delta(S) < -\Delta\delta(T)$

The previous inequality implies that an episodic encounter between a cooperator and a defector is far more likely to change the basic strategy of the defector than that of the cooperator. Thus, the higher the value of  $\Delta\delta(T)$  and the lower the value of  $\Delta\delta(S)$ , the more likely that interactions with cooperators will create a "contagion effect" amongst the lone defector that pushes the value of  $q$  back to one.

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<sup>7</sup> Note that subsequent interaction with other cooperators will increase the time preference of the cooperator who received the "sucker payoff" due to interaction with the lone defector.

<sup>8</sup> In the highly unlikely scenario that all defectors are paired against each other, there will be no change in  $q$ .

**Scenario III:**  $\Delta\delta(S) = -\Delta\delta(T)$

If the absolute value of both terms in the previous equality is low, the outcome is likely to be similar to Scenario II, where repeated “temptation payoffs” eventually bolster the discount factor of the repeated defector such that he becomes a cooperator. If, however, the value of both terms is large, the two players may reverse their roles in subsequent interactions, implying an individually temporary but socially permanent contagion, whereby one member of the society adopts defector behavior for one round of play due to prior interaction with the ever-changing defector.

### **A Mixed Society**

Consider the more realistic case of a society comprised of many defectors and many cooperators. Assume that the change in time preference due to various payoffs is as follows (maintaining maximum and minimum values to  $\delta$  of zero and one, respectively):  $\Delta\delta(\tilde{R}) = r$ ,  $\Delta\delta(S) = s$ ,  $\Delta\delta(P) = p$ , and  $\Delta\delta(T) = t$ . Within a large group, a stable population comprised of both cooperators and defectors is assured if the average discount factor of both sub-groups remains unchanged. In any given period, the expected change in discount factor to a cooperator is  $qr + (1 - q)s$ . The expected change to a defector is  $qt + (1 - q)p$ . Setting both equal to zero implies a unique value of  $q$  which assures the stability of a mixed society,

$$q^* = \frac{p - s}{r + p - s - t}.$$

If  $q$  lies above  $q^*$ , the average discount factors of both groups must, on average, increase each period, given that  $r$  and  $t$  are positive and that  $s$  and  $p$  are negative, implying an ongoing net increase in the number of cooperators. Conversely, if  $q$  lies below  $q^*$ , both groups will collectively become more impatient each period, implying an ongoing net increase in the number of defectors. Just as in the original prisoner’s dilemma present in Section 2, players are pushed toward joint cooperation and joint defection, with a unique, fragile equilibrium where a mixed society is feasible.

### 3. Discussion and Conclusions

The previous model illustrates how societies can separate into two categories, those which are overwhelmingly future-oriented and prosperous where institutions are largely trustworthy, and those which are present-oriented and impoverished where institutions are largely corrupt. A trend toward homogeneity of time preference and corresponding strategy in (potentially) repeated prisoner's dilemmas is an adaptive response to prevailing social circumstances above and beyond a simple calculation of expected payoffs. Repeated exposure to non-cooperative and pure "sucker" payoffs impoverishes those who exhibit cooperation due to sufficient patience such that they become impatient and non-cooperative. Conversely, a "temptation" payoff builds wealth such that the long-term benefits of cooperation become more appealing due to the less-pressing need for present income via defection.

The previous result, particularly the latter transformation of defectors, is subject to several caveats. As in most iterated models of social interaction, players in the  $n$ -person context have or employ no means by which to distinguish likely defectors from likely cooperators. Discerning amongst potential partners would dampen the interaction of cooperators with defectors, thereby creating an increasingly bifurcated society. In this context, the separating equilibrium is not between two different homogeneous societies but two subsections of the same society (insofar as such stratifications does not cause a de facto or de jure division between the two groups).

Second, the closest proxy for permanent income in the game theoretic context is accumulated payoff. To avoid complicating the scenario, the formal model makes no allowance for how this income is saved or consumed by players between rounds. In a more realistic scenario, present-oriented defectors would likely consume most if not all of their "temptation" payoff, leaving them with little increase in permanent income. Future-oriented cooperators, in contrast, can not save what they don't accrue. Incorporating this reality into the model simply makes the transition to an all-defector society more likely and the transition to an all-cooperator society less likely.

Despite the potentially grim predictions of the model, the prescription for alleviating poverty is straightforward: the introduction of a means for accumulating wealth such that the infectious, adaptive tendency toward patience is made possible. A trustworthy mechanism for savings allows those wishing to escape a hand-to-mouth existence a vehicle to act on their desire patience, often as a pre-commitment (Ashraf et al. 2006). Absent such opportunities, those who would otherwise buck the cultural norm are forced into an undesirable common denominator of having potential savings dissipated by cultural norm amongst neighbors and relatives (Boudreaux and Cowen 2008).

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