Competition in fragmentation among political coalitions

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Abstract

This article proposes a game theoretical framework to explain the fragmentation of majority and opposition coalitions in governments. The model concludes that there exists a positive relationship between the concentration of two opposite coalitions. We test this finding in the case of French local jurisdictions. The empirical results lend some support to the theory: the less fragmented the right-wing (left-wing) majorities, the less fragmentated the left-wing (right-wing) oppositions — which points out some competition in fragmentation among political coalitions.

JEL-Classification: C35, D70, H40, H72
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1. Introduction

There are mainly two kinds of studies looking for the determinants of the number of parties that compete in a given polity (see Neto and Cox, 1997 for a survey). The first one examines the importance of preexisting social cleavages. Following the paradigm of citizen-candidates as in Osborne and Slivinski (1996) and Besley and Coate (1997), the policy preference differences would determine the number of candidates who will choose to run. The second stream of studies is interested in the role of institutions in structuring coalitional incentives (see Duverger, 1954; Lijphart, 1990; Ordeshook and Shvetsova, 1994, among others). For instance, according to Duverger's law, a plurality rule election system tends to favor a two-party system while a double ballot majority system and proportional representation tend to multipartism.

The present paper departs from the existing literature by focusing on the role of political fragmentation on the effective political power of a coalition. It starts from two intuitions. First, the more fragmented a political coalition, the lower its effective political power. This question has already been addressed both theoretically and empirically by Le Maux et al. (2011). Second, it is costly for a coalition to be concentrated. This is explained by the fact that moving a politician from a coalition party to another one will induce some costs for the politician and its coalition.

If the intuition of the paper is correct, there should be a trade-off within a coalition between its effective political power which is positively related to its concentration and the individual satisfaction of politicians which is decreasing with the concentration of the coalition. As a result there may exist an optimal fragmentation level. Since the coalitions are in competition, this optimal level may also be determined by the fragmentation of the opposite coalition. The first part of our paper put forward a game-theoretical model which presents this idea in a more formal way. From this model we deduce the proposition that the fragmentation of a majority coalition should be positively correlated to the fragmentation of the opposition coalition. We test this proposition in the case of the French *départements* on a set of panel data. We find empirical evidences which are in line with our theoretical analysis.

The outline of the paper is as follows. Section 2 presents the theoretical model. Section 3 discusses the estimation strategy and provides the empirical results.

2. Theoretical model

Consider a constituency where two players, labeled Coalition A and Coalition B, compete for political power. The political power a coalition has depends on the number of its members who take an active part in the coalition political activities. Our basic framework is that of a game-theoretical model where the leaders of coalitions A an B choose the number of politicians in each party of the coalition. The optimal choice results from the trade-off between increasing the effective political power of the coalition and reducing the satisfaction of its members who prefer to run their own party. Knowing the fragmentation of Coalitions A and B, the leader of each party in A and B chooses its participation level to the coalition activities, namely the number on politicians who will be active. This choice results from the maximization of the coalition's effective political power subject to the party cost of effort.

Subsection 2.1 solves the party leaders game and shows that the politicial power of the coalitions depends on how fragmented the coalitions are. Subsection 2.2 solves the coalition leaders game and concludes to a positive relationship between the coalitions' fragmentation.

2.1. The party leaders game

We denote s^A and s^B the number of politicians of coalition A and coalition B. Let $a \leq s^A$ and $b \leq s^B$ represent the number of coalition A's (respectively B's) politicians who take an active part in coalition A's (respectively B's) political activities. We will define the effective political power of each coalition to be:

$$\pi^A(a,b) = \frac{a}{a+b} \qquad \text{and} \qquad \pi^B(a,b) = 1 - \pi^A(a,b), \tag{1}$$

where $\pi^A(a,b)$ and $\pi^B(a,b)$ are *contest success functions* as in Tullock (1980) and Skaperdas (1996). They are used to translate each coalition's political activity into their influence on public policy. We assume that each coalition is respectively composed of n^A and n^B parties. The number of politicians of party p in Coalition A (respectively Coalition B), $p = 1 \ldots n^A$ (respectively $p = 1 \ldots n^B$), is denoted s^A_p (respectively s^B_p). Let $a_p \leq s^A_p$ (respectively $b_p \leq s^B_p$) define the number of politicians of party p in Coalition A (respectively in Coalition B) that participate in coalition A's(respectively coalition B's) political activities. The total amount of participation of each coalition is defined as $\sum_{p=1}^{n^A} a_p = a$ and $\sum_{p=1}^{n^B} b_p = b$. The effective political power, π^K, of Coalition K increases as the number of active politicians of the coalition increases. A politician can participate or not participate in its coalition's political activities.

The party leaders within each coalition are the players in this game. They are assumed to behave non-cooperatively. Party leader p in coalition A chooses its participation, namely the number of active party members, a_p so as to maximize its net welfare, the difference between its benefit and its cost of participation. On the one hand, the utility of party p in Coalition K depends on the effective political power of the entire coalition, π^K. On the other hand, the cost of participation of party p depends only on its own participation. The net welfare is consequently as follows:

$$U_p^A(a_p) = \frac{a}{a+b} - \left(\frac{a_p}{s_p^A}\right)^{\lambda}, \qquad p = 1 \ldots n^A, \tag{2}$$

with $\lambda > 1$ (to ensure that second order conditions are met). Note that the effective political power of coalition A is a pure public good that benefits all parties in the coalition while the cost of participation is limited to party p. While λ does not play much of a role in our analysis, a higher λ lowers the cost of participation (because the fraction $\left(\frac{a_p}{s_p^A}\right)$ is lower than 1), and its value may depend on the constitutional organization of the country.

The participation level of party p in Coalition A must satisfy the following first-order condition:

$$\frac{\partial U_p^A}{\partial a_p} = \frac{b}{(a+b)^2} - \frac{\lambda}{s_p^A}\left(\frac{a_p}{s_p^A}\right)^{\lambda-1} = 0, \qquad p = 1 \ldots n^A. \tag{3}$$

Equation 3 says that the marginal benefit of participation should be set equal to its marginal cost. The marginal benefit is a downward sloping function of party p's participation, while the marginal cost (with $\lambda > 1$) is an upward sloping function. Rewriting the first-order condition gives $a_p = (s_p^A)^{\frac{\lambda}{\lambda-1}} \left(\frac{b}{\lambda(a+b)^2}\right)^{\frac{1}{\lambda-1}}$, $p = 1 \ldots n^A$. Since $a = \sum_{p=1}^{n^A} a_p$, we have

$$a = \sum_{p=1}^{n^A} (s_p^A)^{\frac{\lambda}{\lambda-1}} \left(\frac{b}{\lambda(a+b)^2}\right)^{\frac{1}{\lambda-1}}, \tag{4}$$

which implicitly defines the optimal response $a = a(b)$ of Coalition A to any strategy b chosen by Coalition B. By symmetry, if the net welfare of party p in Coalition B is given by $U_p^B(b_p) =$

$\frac{b}{a+b} - \left(\frac{b_p}{s_p^A}\right)^{\lambda}$, we find that

$$b = \sum_{p=1}^{n^B} (s_p^B)^{\frac{\lambda}{\lambda-1}} \left(\frac{a}{\lambda(a+b)^2}\right)^{\frac{1}{\lambda-1}}, \tag{5}$$

which implicitly defines the reaction function of Coalition B, that is, $b = b(a)$. It follows from Equations 4 and 5 that

$$\frac{a}{b} = \left(\frac{X^A}{X^B}\right)^{\frac{\lambda-1}{\lambda}}, \tag{6}$$

where $X^K = \sum_{p=1}^{n^K} \left(s_p^K\right)^{\frac{\lambda}{\lambda-1}}$, $K = \{A, B\}$. Using Equation 6 with Equations 4 and 5 yields the Nash participation level equilibrium of Coalition A and Coalition B :

$$a^* = \left(X^A\right)^{\frac{\lambda-1}{\lambda}} \left(\frac{\left(X^A\right)^{\frac{\lambda-1}{\lambda}} (X^B)^{\frac{\lambda-1}{\lambda}}}{\lambda\left((X^A)^{\frac{\lambda-1}{\lambda}} + (X^B)^{\frac{\lambda-1}{\lambda}}\right)^2}\right)^{\frac{1}{\lambda}} \tag{7}$$

$$b^* = \left(X^B\right)^{\frac{\lambda-1}{\lambda}} \left(\frac{\left(X^A\right)^{\frac{\lambda-1}{\lambda}} (X^B)^{\frac{\lambda-1}{\lambda}}}{\lambda\left((X^A)^{\frac{\lambda-1}{\lambda}} + (X^B)^{\frac{\lambda-1}{\lambda}}\right)^2}\right)^{\frac{1}{\lambda}} \tag{8}$$

By using Equations 7 and 8 with Equation 1, we directly find the equilibrium values of the contest success functions:

$$\pi^A(a^*, b^*) = \frac{\left(X^A\right)^{\frac{\lambda-1}{\lambda}}}{(X^A)^{\frac{\lambda-1}{\lambda}} + (X^B)^{\frac{\lambda-1}{\lambda}}} \tag{9}$$

and $\pi^B(a^*, b^*) = 1 - \pi^A(a^*, b^*)$. With respect to Equation 9, it is easy to see that Coalition A's effective political power, π^A , is an increasing function of the ratio $\frac{X^A}{X^B}$.

We say that coalition K is less fragmented (similarly more concentrated) if there is a shift of politicians from a smaller to a bigger party within K. By considering this definition of fragmentation, it can be seen that X^K is an increasing function of the concentration of coalition

K. Function X^K may be written as:

$$X^K = \left(s_1^K\right)^{\frac{\lambda}{\lambda-1}} + \cdots + \left(s_i^K\right)^{\frac{\lambda}{\lambda-1}} + \cdots + \left(s_j^K\right)^{\frac{\lambda}{\lambda-1}} + \cdots + \left(s_{n^K}^K\right)^{\frac{\lambda}{\lambda-1}}, \quad \text{where} \quad s_j^K = s^K - \sum_{p\neq j}\left(s_p^K\right) \tag{10}$$

The change in X^K resulting from a shift of politicians from Party j to Party i is given by:

$$\frac{\partial X^K}{\partial s_i^K} = \frac{\lambda}{\lambda-1}\left(\left(s_i^K\right)^{\frac{1}{\lambda-1}} - \left(s_j^K\right)^{\frac{1}{\lambda-1}}\right) \tag{11}$$

The sign of partial derivative 11 is positive if $s_i^K > s_j^K$. It means that a shift of politicians from a smaller to a bigger party within coalition K always increases X^K. In other words the more concentrated a coalition is, the higher is the value of X^K.

The next section presents the coalition leaders game where the fragmention of each coalition is considered as endogenous.

2.2. *The coalition leaders game*

The leaders of Coalitions A an B choose their fragmentation level by knowing that a higher concentration increases their effective political power but reduces the satisfaction of their members who would prefer to belong to their own party. For this reason we assume that becoming more concentrated is costly. As a result being more concentrated brings some benefit to the coalition but also some cost to the members of the coalition. In order to measure the marginal benefit of concentration, we first compute the partial derivative of π^A with respect to s_i^A:

$$\frac{\partial \pi^A}{\partial s_i^A} = \frac{\lambda-1}{\lambda}\frac{\left(X^A\right)^{\frac{-1}{\lambda}}\left(X^B\right)^{\frac{\lambda-1}{\lambda}}}{\left(\left(X^A\right)^{\frac{\lambda-1}{\lambda}} + \left(X^B\right)^{\frac{\lambda-1}{\lambda}}\right)^2}\frac{\partial X^A}{\partial s_i^A} \tag{12}$$

Equation 12 shows that increasing the size of party i within coalition A improves coalition A's effective political power if $\frac{\partial X^A}{\partial s_i^A}$ is positive. As suggested by equation 11, this will be true if the change in s_i^A is the result of a shift of politicians from a smaller to a bigger party. We assume that the leader of coalition A maximizes the following programme:

$$
\begin{aligned}
&\underset{\{s_1^A,..,s_p^A,..,s_{n^A}^A\}}{Max} \quad \pi^A(s_1^A,..,s_p^A,..,s_{n^A}^A) - c^A\left(X^A\left(s_1^A,..,s_p^A,..,s_{n^A}^A\right)\right) \\
&\text{subject to} \quad s^A = \textstyle\sum_{p=1}^{n^A} s_p^A
\end{aligned}
\tag{13}
$$

where c^A is the concentration cost function. It captures the idea that increasing the size of a party brings some costs. The Lagrangian of the optimization problem may be written as:

$$
L^A = \pi^A(s_1^A,..,s_p^A,..,s_{n^A}^A) - c^A\left(X^A\left(s_1^A,..,s_p^A,..,s_{n^A}^A\right)\right) + \mu\left(s^A - \sum_{p=1}^{n^A} s_1^A\right) \tag{14}
$$

The partial derivatives of L with respect to s_p^A for $p = 1$ to n^A are given by:

$$
\frac{\partial \pi^A}{\partial s_p^A} - \frac{\partial c^A\left(X^A\right)}{\partial X^A}\frac{\partial X^A}{\partial s_p^A} - \mu = 0, p = 1 \text{ to } n^A \tag{15}
$$

By replacing $\frac{\partial \pi^A}{\partial s_p^A}$ by its value from equation 12 we obtain:

$$
\frac{\lambda-1}{\lambda}\frac{\left(X^A\right)^{\frac{-1}{\lambda}}\left(X^B\right)^{\frac{\lambda-1}{\lambda}}}{\left(\left(X^A\right)^{\frac{\lambda-1}{\lambda}} + \left(X^B\right)^{\frac{\lambda-1}{\lambda}}\right)^2} = \frac{\partial c^A\left(X^A\right)}{\partial X^A} \tag{16}
$$

By proceeding similarly for coalition B and by using equation 16 we compute an optimal condition that has to be met at the Nash equilibrium:

$$
X^A = \frac{\frac{\partial c^B\left(X^B\right)}{\partial X^B}}{\frac{\partial c^A\left(X^A\right)}{\partial X^A}} X^B \tag{17}
$$

Replacing X^A and X^B by their value and rearranging yields:

$$
\sum_{p=1}^{n^A}\left(\alpha_p^A\right)^{\frac{\lambda}{\lambda-1}} = \frac{\frac{\partial c^B\left(X^B\right)}{\partial X^B}}{\frac{\partial c^A\left(X^A\right)}{\partial X^A}}\left(\frac{s^B}{s^A}\right)^{\frac{\lambda}{\lambda-1}} \sum_{p=1}^{n^B}\left(\alpha_p^B\right)^{\frac{\lambda}{\lambda-1}} \tag{18}
$$

where $\alpha_p^K = \frac{s_p^K}{s^K}$, $K = \{A, B\}$, is the share of politicians of party p in Coalition K. Equation 18 shows that there exists a positive link between the concentration of coalition

A and that of coalition B. When $\lambda = 2$, $\sum_{p=1}^{n^K} \left(\alpha_p^K\right)^{\frac{\lambda}{\lambda-1}}$ is the Herfindahl-Hirschman index of coalition K. When $\lambda \to 1^+$, $\sum_{p=1}^{n^K} \left(\alpha_p^K\right)^{\frac{\lambda}{\lambda-1}}$ is the best shot index. Only the contribution of the biggest party of coalition K matters.

3. Empirical testing of the model

In this section we conduct an empirical investigation of the link between the concentration of two opposite coalitions a suggested by Equation 18. Subsection 3.1 introduces the empirical specification and provides a description of data. Subsection 3.2 discusses the estimation strategy and provides some preliminary tests.

3.1. Specification and data

We use a panel data of 90 French local jurisdictions called *départements* over 3 years (1994, 1998 and ????). The *départements* represent one of the three layers of local governments in France. Each *département* is administered by a General Council. For each constituency a General Councillor is directly elected for a term of six years. A constituency is a part of the *département* territory known as *canton*. The elections (referred to as *Elections Cantonales*) are held every three years. A General Council is generally made of a majority and an opposition coalition. Each coalition is composed of parties sharing the same ideology. Roughly speaking there are left-wing and right-wing coalitions. Over the 3 years of our study, the average number of parties forming a coalition is equal to ??? for the rigt-wing groups and to ???? for the left-wing groups. The concentration of coalitions as measured by the Herfindahl-Hirshman index is variable from coalition to coalition as depicted on figure ????. It amounts to ??? and to ??? for the right-wing and left-wing coalitions respectively.

According to our theoretical model (equation 18), there should be a positive relationship between the concentration of the majority and that of the opposition. We investigate this relationship in the case of the French General Councils. The empirical model tested in this paper is as follows:

$$\begin{aligned} FRAG_{i,t}^R &= \Phi_0 + \Phi_1 \left(FRAG_{i,t}^L\right) + \Phi_2 \left(OLD_{i,t}\right) + \Phi_3 \ln\left(POP_{i,t}\right) + \Phi_4 \ln\left(INCOME_{i,t}\right) + \\ &\quad \Phi_5 \ln\left(UNEMP_{i,t}\right) + Y_{94} + Y_{98} + u_{i,t}, \end{aligned} \tag{19}$$

where i and t stand for *département* i and year t, respectively. Variable OLD denotes the share of people over sixty, POP the population, $INCOME$ the mean taxable income, and $UNEMP$ the unemployment rate of the *département*. Variables $FRAG_{i,t}^R$ and $FRAG_{i,t}^L$ denote the

Table 1. *Geographical dummies.*

Area	Regions
West	Bretagne, Basse-Normandie, Pays de la Loire, Poitou-Charentes.
North	Nord-Pas-de-Calais, Haute Normandie, Picardie, Ile-de-France, Picardie.
East	Champagne-Ardenne, Lorraine, Franche-Comté.
Centre	Centre, Bourgogne, Auvergne.
South-West	Limousin, Aquitaine, Midi-Pyrénées.
South-East	Rhône-Alpes, Provence-Alpes-Côte dŠAzur, Languedoc-Roussillon.

fragmentation index of the Right-wing and Left-wing coalitions, respectively. Two approaches will be used to measure the fragmentation variables. The first approach consists in focusing on the usual Herfindahl-Hirschman index *HHI* ($\lambda = 2$ in Equation 18). However, this approach may lead to misleading empirical results since on the one hand this index is likely to increase as the coalition's number of politicians decreases (e.g., if there is only one politician, the index is equal to one). On the other hand, the index is likely to decrease as the coalition's number of politicians increases, even though the leading party has more politicians (statistically, the number of parties is likely to be higher, Lijphart 1990; Ordeshook and Shvetsova 1994). This is the reason why we also measure the fragmentation by the Best shot index *BSI*, namely the share of politicians held by the largest parties in the right-wing and left-wing coalitions ($\lambda -> 1^+$ in Equation 18). The description of variables and some summary statistics are provided in Table 2 and 3.

3.2. Estimation strategy and preliminary tests

We estimate Equation 19 by using the Pooled-OLS estimator with regional dummies. Here the individual fixed effects cannot be used since some *départements* may appear only once in a sample. On the other hand, using geographical dummies instead of individual effects offers a good compromise between the Pooled-OLS estimator and the Fixed-effects estimator. We have regrouped the 90 *départements* into six areas, as shown in Table 1.

The estimation results are given in Tables 4.

Table 2. *Description of the Variables.*

Variables	Content
HH^K	Herfindahl-Hirschman index of coalition K. $HH^K = \sum_{p=1}^{n^K} (SHARE_p^K)^2$ where n^K is the number of parties in coalition K and $SHARE_p^K$ is the share of representatives of party p of coalition K in the *département* council, (K=Right-wing,Left-wing).
BS^K	Best shot index of coalition K. $BS^K = Max(SHARE_1^K, \dots, SHARE_{n^K}^K)$.
$INCOME$	Mean household taxable income. Source: *Direction Genérale des collectivités locales* (DGCL).
POP	Local population. Source: *Institut National des la Statistique et des Etudes economiques* (INSEE).
OLD	Share of population beeing more than 60. Source: *Direction Genérale des collectivités locales* (DGCL).
$UNEMP$	Unemployment rate. Source: *Institut National des la Statistique et des Etudes economiques* (INSEE).

Table 3. *Summary Statistics.*[a]

Variables	Mean	Min	Max	SD
HH^R	1370	646	2467	283
HH^L	0.521	0.067	0.884	0.076
BS^R	1370	646	2467	283
BS^L	0.521	0.067	0.884	0.076
$INCOME$	42353	29460	80474	6332
POP	585984	72390	2555471	435502
OLD	0.2219	0.1231	0.3362	0.0430
$UNEMP$	2.722	0.5689	21	2.287

[a] Number of observations: 270. *INCOME* are in current Francs.

Table 4. *Estimations using the Herfindahl-Hirschman index of right-wing départements.*[a]

	Pooled OLS	Geographical	Plumber
Intercept	-4.139**	-3.227*	9.150***
	(-2.794)	(-2.083)	(9.509)
Herfindahl-Hirschman index of the left-wing: $HH_{i,t}^{R}$	0.129*	0.115*	0.302***
	(2.332)	(1.988)	(11.041)
Elderly people: $OLD_{i,t}$	-0.011	-0.089	-1.698***
	(-0.069)	(-0.512)	(-19.916)
Unemployment rate: $UNEMP_{i,t}$	0.214·	0.224*	1.153***
	(1.892)	(2.007)	(21.156)
Population: $\ln(POP_{i,t})$	-0.148*	-0.128*	-0.825***
	(-2.591)	(-2.276)	(-23.813)
Income: $\ln(INCOME_{i,t})$	0.429 **	0.334*	0.295***
	(2.914)	(2.096)	(4.176)
Year 1994: $\ln(Y_94)$	-0.058	-0.052	-0.041*
	(-1.292)	(-1.134)	(-2.085)
Year 1998: $\ln(Y_98)$	0.198***	0.215***	0.208***
	(4.039)	(4.350)	(8.213)
Adjusted R^2	0.266	0.293	0.805
Number of observations	270	270	270

[a] t value in parentheses.
***, **, *, and · indicate significance at 0.1%,1%, 5% and 10% level, respectively.

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Table 5. *Serial correlation and cross-sectional dependence tests.*

Estimator	Test for serial correlation	Test for cross-sectional dependence
Pooled-OLS	435.6963***	48.4422***
Geographical dummies	435.2672***	48.2871***
Plümper and Troeger procedure	143.4256***	29.0321***

*** indicates significance at the 0.1% level.