Electoral Competition, Ability of Political Parties and Platform Divergence

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Abstract

The standard Median Voter Theorem says that any political party announces the moderate policy. But, in the real world, this does not necessarily hold. In some cases, a party announces an extremist policy and wins the election.

In this paper, we try to explain such cases. We consider the case where there are two parties and the parties may be less able. The voters do not know the ability of each party. If the low ability party wins the election, the voters' utilities are decreased.

We assume that each party has his own ideological position. If a party announces the policy that is distant from his own ideological position, the party needs coordination in the process of implementation of the policy. This coordination imposes some cost. So the more able party can announce the policy that is distant from his own ideological position but the less able party cannot. That is, the extremist policy has signaling effect.

We will show that there are separating equilibria. In the separating equilibria, the more able party announces an extreme policy while the less able party announces the policy that the median voters prefer. Moreover, we will show that there are two types of separating equilibria. One is a symmetric separating equilibrium. In this equilibirum, each party becomes an extremist to the same degree in spite of difference of ideological positions. The other is an asymmetric separating equilibrium. In this, the degrees of two parties' extremeness are different. In addition, the separating equilibrium will vanish when the relative extreme party becomes more excessive extremist. We can say that extremeness in the ideological position leads to moderateness in the policy.

Moreover, we analyze the property of the equilibria. Some derived results are counterintuitive. For example, if the less able party's capability improves, the policy in the equilibrium goes to more extreme one and has negative effect on the voters' welfare.

1. Introduction

The standard Median Voter Theorem says that any political party's policy converges to the median voter's favorite policy. This theorem means that an extremist party announces a moderate policy. But, in the real world, this does not necessary hold. In some cases, a party announces an extremist policy and wins the election.

In this paper, we try to explain such cases. Several articles have been devoted to this problem. Almost of these assume that there is uncertainty with respect to the distribution of voters. On the contrary, we don't assume such uncertainty.

We consider the cases where there are two parties and the parties may be less able. The voters do not know the ability of each party. If the less able party wins the election, the voters' utility are decreased. So the voters have incentive to vote for the party that seems to be more able, even if the party announces the policy that is distant from their preferred policy.

We assume that each party has his own ideological position. If a party announces the policy that is distant from his own ideological position, the party needs coordination in the process of implementation of the policy. This coordination imposes some cost. The more able party's cost is low, but the less able party's cost is high. So the more able party can announce the policy that is distant from his own ideological position. That is, the extremist policy has signaling effect.

We analyze the equilibria of this election game among the two parties and the voters. Especially, considering perfect Bayesian equilibria, we focus on separating equilibria. That is, the policy of the party is different depending on his or her ability. In a separating equilibrium, we can distinguish the party's ability by looking at the announced policy.

We will show that there are separating equilibria. In the separating equilibria, the more able party announces an extreme policy while the less able party announces the policy that the median voters prefer. Moreover we will show that there are two types of separating equilibira. One is a symmetric separating equilibrium. In this, each party becomes an extremist to the same degree in spite of difference of ideological positions. The other is an asymmetric separating equilibrium. In this, the degrees of two parties' extremeness are different.

The separating equilibrium will vanish when the relatively extreme party became more excessive extremist. This implies that if one of parties is an extremist to an excess degree, each party announces the moderate policy. We can say that extremeness in the ideological position leads to moderateness in the policy.

Moreover, we analyze the property of the equilibria. Some derived results are counterintuitive. For example, if the less able party's capability improves, the policy in the equilibrium goes to more extreme one and has negative effect on the voters' welfare. In addition, if the probability of more able party increases, the policy goes to more extreme one and has negative effect.

2. Model

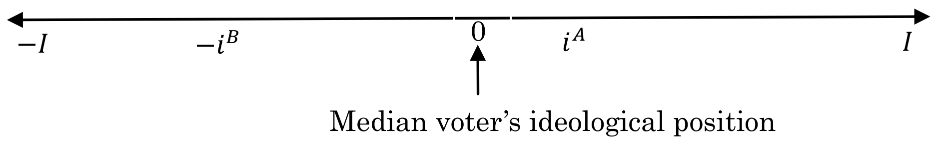
There are two political parties, Party A and Party B. Each party selects his policy from the policy space $[-I, I]$ where I is a positive constant. Party A has his own ideological position $i^A \in [0, I]$ and Party B has her own ideological position $i^B \in [-I, 0]$. Without loss of generalities, we assume that $i^A \leq i^B$. So Party B has the more extreme ideological position.

Each party may be more able or less able. The ability is denoted by a_t where $t \in \{H, L\}$. If $t = H$, the party has high ability and if $t = L$, the party has low ability. We assume that $a_L = a < a_H = 1$. Let $c = {}^1\!/_a$. This c represents the cost of the less able party. The cost of the more able party is 1 ($= {}^1\!/_{a_H} = {}^1\!/_1$).

Assume that Party A announces his own policy $x^H \in [-I, I]$. If Party A wins the election, his utility is $V - \frac{1}{a_t}|i^A - x^A|$. If he loses, his utility is zero. Similarly, if Party B wins the election with her announcing policy x^B, her utility is $V - \frac{1}{a_t}|-i^B - x^B|$. If she loses, her utility is zero.

The party knows his or her own ability but the other party and voters do not know the party's ability. We denote by p the probability of Party i's having high ability ($i \in \{A, B\}$). This probability is common knowledge.

Each voter has his preference $-|i - x| + ra_t$ where i is his ideological position, x is the policy that the winning party announces, a_t is the ability of the winning party and r is the weight attached to the wining party's ability. We assume that the median voters' ideological position is zero, that is the median voters' preferences are $-x + ra_t$. Each voter makes sincere voting. In the case of indifference, a voter is assumed to vote for each party with probability equal to 1/2.



The sequence of the events is as follows. At first, the nature chooses each party's ability with probability p. Each party knows his or her ability but the other party and the voters do not know the ability. Each party announces his or her policy. Election takes place. The party that wins the election implements the announcing policy. All agents' utilities are realized.

If the Median Voters Theorem holds, each party announces the policy $x^i = 0$ in the election. In this paper, we show that the Median Voter Theorem does not necessary hold. In the following section, we investigate perfect Bayesian equilibria. Especially, we focus on separating equilibria.

3. Equilibria

If there is a separating equilibrium, the parties' policies are different. We denote Party A with high ability by AH and Party A with low ability by AL. Similarly, we denote Party B with high ability by BH and Party B with low ability by BL. In addition, we denote by x_H^A the policy of AH and by x_L^A the policy of AL. Similarly, we denote by x_H^B the policy of BH and by x_L^B the policy of BL.

Assumption 1
We have $V - cI = V - \frac{1}{a}I > 0$.

This assumption means that even if the party wins the election with the policy imposing the highest cost, his or her utility is positive.

Assumption 2
We have $(1 - a)r \leq I$.

The meaning of this assumption will become clear in the proof of Propositions.

At first, we prove that there always exists a pooling equilibrium.

Proposition 1 (Pooling equilibrium)

There always exists a pooling equilibrium. In this equilibrium, Party A's policy is $(x_H^A, x_L^A) = (0,0)$ and Party B's policy is $(x_H^B, x_L^B) = (0,0)$.

Proof: Consider the following voters' beliefs: (1) after observing $x^A \in [-I, I]$, the probability of AH is p; (2) after observing $x^B \in [-I, I]$, the probability of BH is p. Then we have the given pooling equilibrium. Q.E.D.

In this equilibrium, the voters cannot distinguish parties' ability. So the policy that the median voters prefer is announced in the election.

Then we show there are separating equilibria in some cases. First, we prove the existence of a symmetric equilibrium. In this equilibrium, $x_H^A = |x_H^B| \neq 0$ holds.

Proposition 2 (Symmetric separating equilibrium)

If $i^B \leq \frac{(1-a)V}{3-2p} + i^A$ and $i^B < \frac{(1-a)(2-p)r - aV}{3-2p}$ hold, there is a symmetric separating equilibrium. In this equilibrium, Party A's policy is $(x_H^A, x_L^A) = \left(\frac{aV + (3-2p)i^B}{2-p}, 0\right)$ and Party B's policy is $(x_H^B, x_L^A) = \left(-\frac{aV + (3-2p)i^B}{2-p}, 0\right)$.

Proof: Let (x_H, x_L) and $(-x_H, -x_L)$ be policies in a symmetric separating equilibrium. Because the voters can distinguish the parties' ability, the median voter theorem holds if each party's ability is low. That is, we have $x_L = 0$.[1]

In a separating equilibrium, the voters will select the party announcing the policy that is not zero. At first, we investigate the incentive behind the voters' behavior. It is sufficient to calculate the median voters' utility. If the less able party wins the election, the voters' utility is $-|0 - 0| + ra$. If the more able party wins, the voters' utility is $-|0 - x_H| + r \cdot 1$. The median votes prefer the more able party, only if

[1] Let $x_L \neq 0$. If AL announces $x^A = 0$ instead of announcing $x_L \neq 0$, then the median voters vote for him if Party B announces $x^L \neq 0$. So AL can increase his utility. This is a contradiction.

$$-|0 - x_H| + r \cdot 1 \geq -|0 - 0| + ra$$

holds. So if there is a separating equilibrium,

$$x_H \leq (1 - a)r \quad (1)$$

should hold.

Next, we investigate BL's incentive to pretend BH. If she announces $x_L = 0$, her utility is

$$(1 - p)\frac{1}{2}(V - c|-i^B - 0|).$$ [2]

If she announces $-x_H$, her utility is

$$p\frac{1}{2}(V - c|-i^B - (-x_H)|) + (1 - p)(V - c|-i^B - (-x_H)|).$$ [3]

In a separating equilibrium, we have to get

$$(1 - p)\frac{1}{2}(V - c|-i^B - 0|) \geq p\frac{1}{2}(V - c|-i^B - (-x_H)|) + (1 - p)(V - c|-i^B - (-x_H)|).$$

From this, we have

$$x_H \geq \frac{aV + (3 - 2p)i^B}{2 - p}. \quad (2)$$

From (1) and (2), if $i^B < \frac{(1-a)(2-p)r - aV}{3-2p}$ holds, we have

$$\frac{aV + (3 - 2p)i^B}{2 - p} < (1 - a)r.$$

Then we can show that Party A announces $(x_H^A, x_L^A) = \left(\frac{aV + (3-2p)i^B}{2-p}, 0\right)$ [4] and Party B

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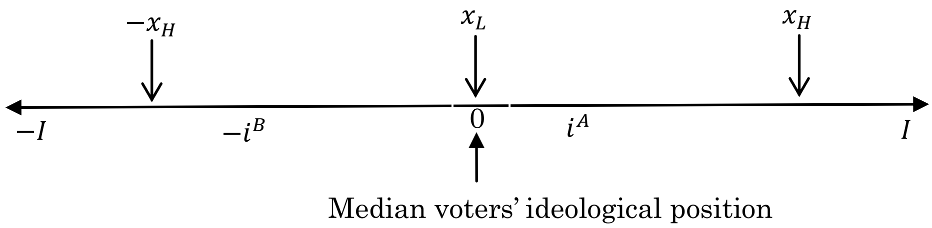
[2] If Party A announces $x^A = x_H$, the median voters prefer Party A; if Party A announces $x^A = 0$, the median voters are indifferent between Party A and Party B.

[3] If Party A announces $x^A = x_H$, the median voters are indifferent between Party A and Party B; if Party A announces $x^A = 0$, the median voters prefer Party B.

announces $(x_H^B, x_L^A) = \left(-\frac{aV+(3-2p)i^B}{2-p}, 0\right)$[5] if the voters' beliefs are as follows: (1) If $x^A \geq \frac{aV+(3-2p)i^B}{2-p}$, the probability of AH is one. (2) If $x^A < \frac{aV+(3-2p)i^B}{2-p}$ the provability of AH is zero. (3) If $x^B \leq -\frac{aV+(3-2p)i^B}{2-p}$, the probability of BH is one. (2) If $x^B > -\frac{aV+(3-2p)i^B}{2-p}$, the probability of BH is zero.

Moreover, we can show that these beliefs are rational. Q.E.D.



In the above equilibrium, the more able party is an extremist to the same degree, that is, $x_H^A = |x_H^B| \neq 0$ holds. The following proposition shows that there exists another type of a separating equilibrium in some cases.

Proposition 3 (Asymmetric separating equilibrium)

If $\frac{apV+(3-p)i^A}{3} < i^B < \frac{2(1-a)r-aV}{3}$ holds, there is an asymmetric separating equilibrium. In this equilibrium, Party A's policy is $(x_H^A, x_L^A) = \left(\frac{(1+p)aV+(3-p)i^A}{2}, 0\right)$ and Party B's policy is $(-x_H^B, x_L^A) = \left(-\frac{aV+3i^B}{2}, 0\right)$.

Proof: Because the voters can distinguish the ability of the parties, the median voter theorem holds if each party's ability is low. That is, we have $x_L = 0$. Moreover, if there is

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[4] From $i^B \leq \frac{(1-a)V}{3-2p} + i^A$, we get $\frac{aV+(3-2p)i^B}{2-p} \leq \frac{V+(3-2p)i^A}{2-p}$. If $x^H \leq \frac{V+(3-2p)i^A}{2-p}$ holds, we have $p\frac{1}{2}(V - |i^A - x_H|) + (1-p)(V - |i^A - x_H|) \geq (1-p)\frac{1}{2}(V - |i^A - 0|)$.

[5] From $-\frac{aV+(3-2p)i^B}{2-p} > -\frac{V+(3-2p)i^B}{2-p}$, $-x^H > -\frac{V+(3-2p)i^B}{2-p}$ holds. Then we have $p\frac{1}{2}(V - |-i^B - (-x_H)|) + (1-p)(V - |-i^B - (-x_H)|) > (1-p)\frac{1}{2}(V - |-i^B - 0|)$.

a separating equilibrium,

$$x_H^A < (1-a)r$$

and

$$-x_H^B > -(1-a)r$$

should hold.

Suppose that $x_H^A < x_H^B$ holds. We investigate AL's incentive to pretend AH. If he announces $x_L^A = 0$, his utility is

$$(1-p)\frac{1}{2}(V - c|i^A - 0|).$$ [6]

If he announces x_H^A, his utility is

$$(V - c|i^A - x_H^A|).$$ [7]

In a separating equilibrium, we have to get

$$((1-p)\frac{1}{2}(V - c|i^A - 0|) \geq (V - c|i^A - x_H^A|).$$

From this, we have

$$x_H^A \geq \frac{a(1+p)V + (3-p)i^A}{2}.$$

Next, we investigate BL's incentive to pretend BH. If she announces $x_L^B = 0$, her utility is

$$(1-p)\frac{1}{2}(V - c|-i^B - 0|).$$ [8]

[6] If Party B announces $x^B = -x_H^B$, the median voters prefer Party B; if Party B announces $x^B = 0$, the median voters are indifferent between Party A and Party B.
[7] Regardless of Party B policy, the median voters prefer Party A.
[8] If Party A announces $x^A = x_H^A$, the median voters prefer Party A; if Party A announces $x^A = 0$, the median voters are indifferent between Party A and Party B.

If she announces $-x_H^B$, her utility is

$$(1-p)(V-c|-i^B-(-x_H^B)|).$$[9]

In a separating equilibrium, we have to get

$$(1-p)\frac{1}{2}(V-c|-i^B-0|) \geq (1-p)(V-c|-i^B-(-x_H^B)|).$$

From this, we have

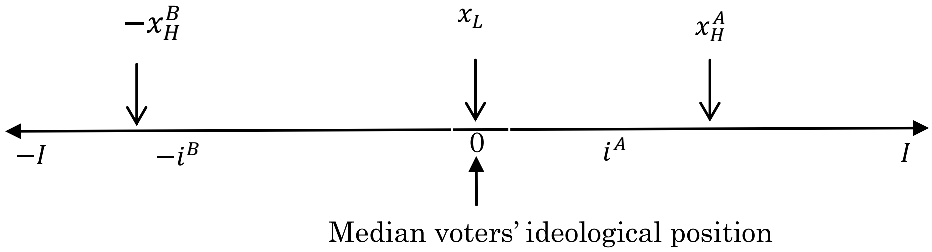
$$-x_H^B \leq -\frac{aV+(3-p)i^B}{2}.$$

If $\frac{apV+(3-p)i^A}{3} < i^B$ holds, we have $\frac{a(1+p)V+(3-p)i^A}{2} < \left|-\frac{aV+(3-p)i^B}{2}\right|$. Moreover, if $i^B < \frac{2(1-a)r-aV}{3}$ holds, we have $-\frac{aV+(3-p)i^B}{2} > -(1-a)r$.

Then we can show that Party A announces $(x_H^A, x_L^A) = \left(\frac{(1+p)aV+(3-p)i^A}{2}, 0\right)$ and Party B announces $(-x_H^B, x_L^A) = \left(-\frac{aV+3i^B}{2}, 0\right)$ if the voters' beliefs are as follows: (1) If $x^A \geq \frac{(1+p)aV+(3-p)i^A}{2}$, the probability of AH is one. (2) If $x^A < \frac{(1+p)aV+(3-p)i^A}{2}$, the probability of AH is zero. (3) If $x^B \leq -\frac{aV+3i^B}{2}$, the probability of BH is one. (2) If $x^B > -\frac{aV+3i^B}{2}$, the probability of BH is zero.

Moreover, we can show that these beliefs are rational. Q.E.D.

[9] If Party A announces $x^A = x_H^A$, the median voters prefer Party A; if Party A announces $x^A = 0$, the median voters prefer Party B.



In the asymmetric separating equilibrium, we have $x_H^A < |-x_H^B|$. In addition, we can show that there is no asymmetric separating equilibrium with $x_H^A > |-x_H^B|$.[10] Moreover, there is no hybrid equilibrium of a pooling equilibrium and a separating equilibrium in which a party announces (x_H, x_L) with $x_H \neq x_L$ and the other announces $(x_H{}', x_L{}')$ with $x_H{}' = x_L{}'$.[11]

From the proofs of Propositions 2 and 3, we can show there is no separating equilibrium if $i^B > \frac{2(1-a)r-aV}{3}$ holds. Then we have a pooling equilibrium (see Proposition 1). In other words, if the relatively extreme party's ideological position becomes more extreme, each party's announcing policy is moderate.

Figures 1, 2, 3 and 4 show the relation between the policy of the more able party in the separating equilibria and Party B's ideological position. (Notice that the vertical axis represents the absolute value of the policy and the horizontal axis represents the absolute value of the ideological position.)

Figure 1

Figure 2

Figure 3

Figure 4

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[10] If there is such an equilibrium, we have $x_H^A = \frac{aV+3i^A}{2}$ and $x_H^B = -\frac{(1+p)aV+(3-p)i^B}{2}$. From $aV - i^B > 0$ and $i^B > i^A$, we have $\frac{(1+p)aV+(3-p)i^B}{2} > \frac{aV+3i^A}{2}$. This is a contradiction.

[11] Let $x_H' = x_L' = x'$. If $|x'| = |x_L| + pr(1-a)$, the median voters are indifferent between the party announcing x' and the party announcing x_L. If $|x'| < |x_L| + pr(1-a)$, the median voters prefer the party announcing x'. So the party announces x' satisfying $|x'| < |x_L| + pr(1-a)$ and the party announcing x_L loses in the election. Therefore the utility announcing x_L is zero and he has incentive to pretend the more able party by announcing x_H.

Next, we analyze the property of the separating equilibria.

Proposition 4

With respect to $x_H = \frac{aV+(3-2p)i^B}{2-p}$, we have (1) $\frac{\partial x_H}{\partial V} > 0$, (2) $\frac{\partial x_H}{\partial i^B} > 0$, (3) $\frac{\partial x_H}{\partial a} > 0$ and (4) $\frac{\partial x_H}{\partial p} > 0$.

From this, if the party's utility from winning increases, each party becomes more an extremist. If the relative extreme party's ideological position becomes more extreme, announced policy of the relative moderate party as well as that of the relative extreme party becomes more extreme.

Increase of a implies that the less able party improve his or her capability. Then each party becomes more an extremist. Moreover, if the probability of more able party increases, each party becomes more an extremist.

Proposition 5

With respect to $x_H^A = \frac{a(1+p)V+(3-p)i^A}{2}$, we have (1) $\frac{\partial x_H^A}{\partial V} > 0$, (2) $\frac{\partial x_H^A}{\partial i^A} > 0$, (3) $\frac{\partial x_H^A}{\partial a} > 0$ and (4) $\frac{\partial x_H^A}{\partial p} > 0$.

Proposition 6

With respect to $x_H^B = -\frac{aV+3i^B}{2}$, we have (1) $\frac{\partial |x_H^B|}{\partial V} > 0$, (2) $\frac{\partial |x_H^B|}{\partial i^B} > 0$, (3) $\frac{\partial |x_H^B|}{\partial a} > 0$ and (4) $\frac{\partial |x_H^B|}{\partial p} = 0$.

We have similar results in an asymmetric separating equilibrium. That is, if the party's utility from winning increases, each party becomes more an extremist; if the party's ideological position becomes more extreme, his or her policy also becomes more extreme; improvement of less able party' capability results in the more extreme policy.

Next, we investigate the median voters' utility $U^M = -|x| + ra_t$. If there is a separating equilibrium, the median voters' expected utility $E(U^M)$ in the separating equilibrium is higher than that in the pooling equilibrium, from the equation (1). Moreover, in the following Proposition, we analyze the utility assuming that we observe the same type of the separating equilibrium when the parameter changes.

Proposition 7

We have (1) $\frac{\partial E(U^M)}{\partial V} < 0$, (2) $\frac{\partial E(U^M)}{\partial i^B} < 0$, (3) $\frac{\partial E(U^M)}{\partial \mathrm{a}} < 0$ and (4) $\frac{\partial E(U^M)}{\partial p} < 0$.

The interpretation of (1) and (2) is natural. On the other hand, (3) and (4) are counterintuitive. Improvement of less able party' capability is socially desirable. But, this decreases the median voters' utilities. Increase of the probability of more able party is also socially desirable. But this has also negative effect on the median voters' utilities.

4. Concluding remarks

Propositions 2 and 3 say that there is a separating equilibrium. That is, the party does not necessary announce the policy that coincides with the median voters' favorite policy. So there is platform divergence. Moreover, we show there are two types of separating equilibria. In the symmetric equilibrium, each party becomes an extremist to the same degree in spite of difference of ideological positions. In the asymmetric separating equilibrium, the degrees of two parties' extremeness are different.

In a pooling equilibrium, each party announces the policy that the median voters prefer. So, the standard Median Voter Theorem holds. Especially, if one of the parties is an extremist, we have the pooling equilibrium. So we can say that extremeness in the ideological position leads to moderateness in the policy. The reason of this is as follows. If the party is an excessive extremist, the party has to announce the exceedingly extreme policy in order to signal the ability. But such a policy is not preferred by the voters.

In addition, we analyze the property of the separating equilibria in Propositions 4, 5 and 6. These propositions say, for example, that if the less able party's capability improves, the policy of the moderate party, as well as that of the extreme party, becomes more extreme. Moreover, the probability of high ability increases, not only the policy of the extreme party but that of the moderate party becomes more extreme.

When the policies become more extreme, the utilities of the median voters and others are decreased. The improvement of less able party's capability is intuitively desirable. But this makes it possible for the less able party to pretend the more able party. So, the more able party has to announce the exceedingly extreme policy in order to signal his or her ability and such a policy has negative effect on social welfare.

Similarly, the high possibility of more able party is intuitively desirable but has negative effect. This high possibility decreases the probability of less able party's winning and the party's expected utility. So the party has strong incentive to pretend the more able party. So, the more able party has to announce the exceedingly extreme policy, resulting in decreased utilities of the voters.

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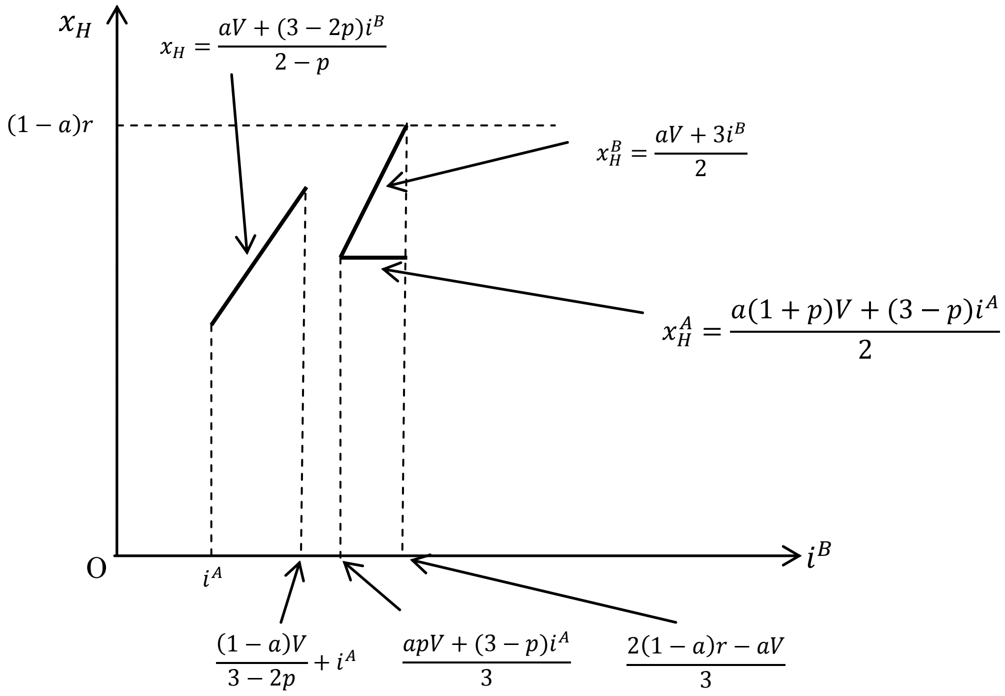

Figure 1 $\left(\frac{(1-a)V}{3-2p} + i^A < \frac{apV+(3-p)i^A}{3} \;\&\; \frac{(1-a)V}{3-2p} + i^A < \frac{(1-a)(2-p)r - aV}{3-2p}\right)$

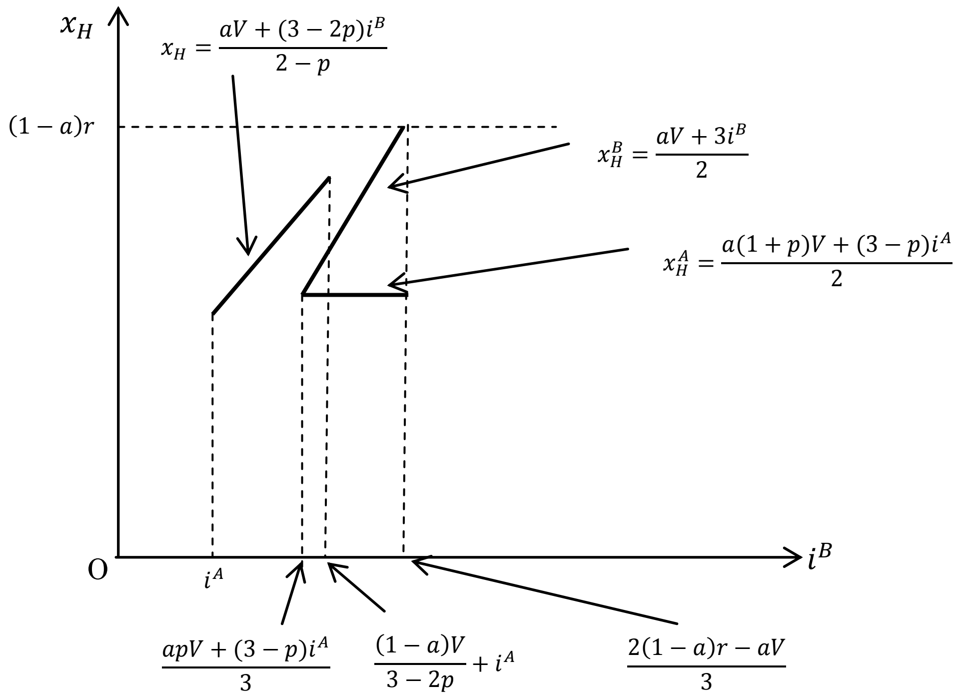

Figure 2 $\left(\frac{(1-a)V}{3-2p} + i^A > \frac{apV+(3-p)i^A}{3}\right)$ & $\frac{(1-a)V}{3-2p} + i^A < \frac{(1-a)(2-p)r-aV}{3-2p}$

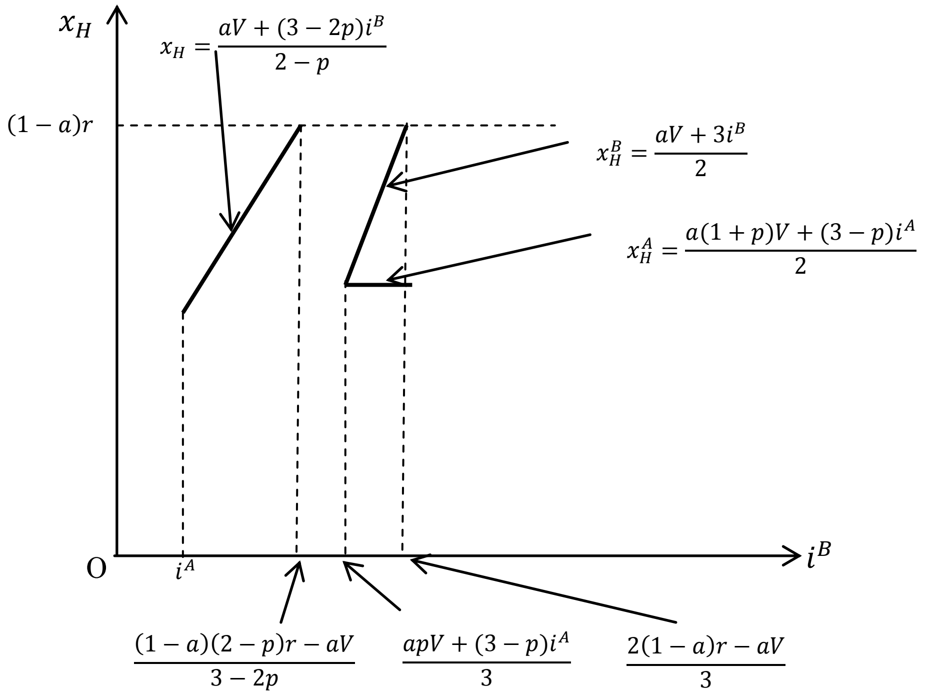

Figure 3 $\left(\frac{(1-a)(2-p)r-aV}{3-2p} < \frac{apV+(3-p)i^A}{3} \ \& \ \frac{(1-a)(2-p)r-aV}{3-2p} < \frac{(1-a)V}{3-2p} + i^A\right)$

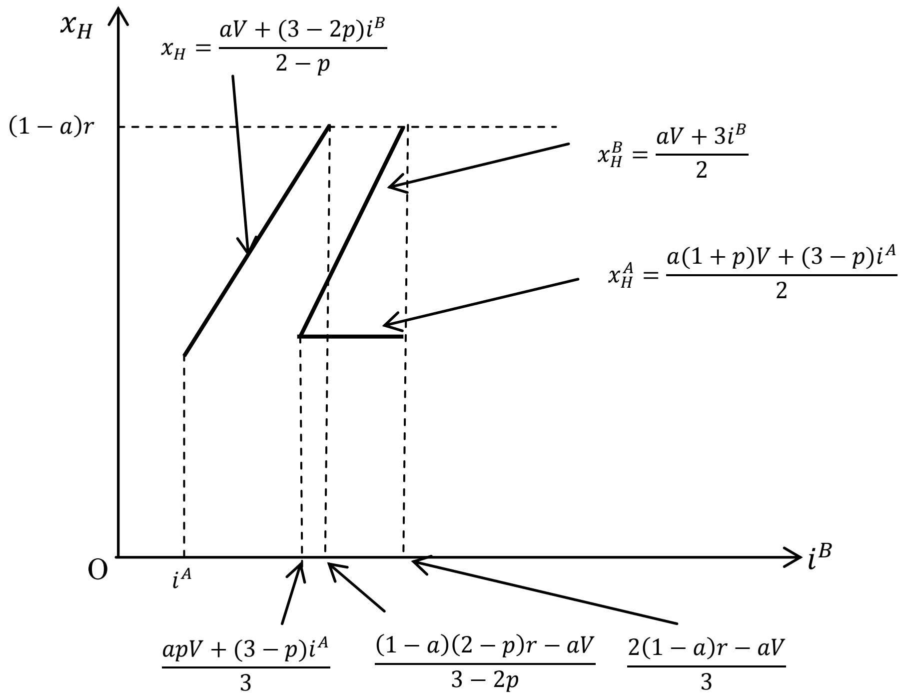

Figure 4 $\left(\frac{(1-a)(2-p)r-aV}{3-2p} > \frac{apV+(3-p)i^A}{3} \;\&\; \frac{(1-a)V}{3-2p} + i^A > \frac{(1-a)(2-p)r-aV}{3-2p}\right)$