Interest Group Formation and Competition

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I present a model of interest group formation in which individuals organize to compete for favorable public policies. The sizes, preferences and other characteristics of interest groups are determined endogenously in a strategic equilibrium. I show that competition between groups can ensure that interest groups remain cohesive, which represents an instrumental solution to the long standing free-rider problem. Moderate groups tend to be larger than more extreme groups whose members are more willing to spend for the right to select policy. The politically inactive and disorganized are the most moderate of all. If the costs of lobbying are sunk, then interest groups compete with one another by employing mixed strategies over their level of lobbying effort. Interest groups' mixing distributions follow a power law whose parameter is directly related to the number of active groups. I present support this prediction using US lobbying data.

1 Introduction

Public policies affect diverse sets of individuals and firms in profound ways, and there is a rich tradition in economics of analyzing and measuring these effects. But such policies do not arise in a vacuum, and individuals and firms can increase their ability to affect these policies through collective action that, broadly speaking, unfolds in two phases: organization and competition. In the first phase, agents organize themselves into coalitions of like minded individuals known as interest groups. By forming interest groups, agents gain the ability to influence public policy towards a common goal through

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coordinated effort. In the second phase, interest groups compete amongst themselves in order to shape public policy to their liking. This competition, in the form of lobbying government officials, may include the direct provision of information to policymakers and regulators regarding optimal policy, the indirect signaling of the preferences of interest groups, or even the direct influence of the actions of policymakers and regulators through political and financial pressure.[1]

In this paper, I formally model both phases of this process. I analyze the types of coalitions (interest groups) that individuals form and characterize stable configurations of interest groups. I define stability as a non-cooperative equilibrium concept in which no small set of individuals can be made better off by changing their decision to organize or affiliate with a particular group. Individuals face an inherent trade off in deciding whether to politically organize or not. Organization allows an individual to participate in the lobbying process by proxy, which may result in more favorable public policy; however, participation in the lobbying process is costly (Hansen (1985)). These costs to individuals may vary with the scale of organization, so even if individuals who comprise a group possess homogeneous preferences, the incentives for them to organize may be heterogeneous. I show that competition between groups ensures that even those members with the least incentive for political organization may choose collective action over free-riding because they are pivotal actors in the lobbying process.

I argue that individuals with moderate preferences are more likely to be politically disorganized and unrepresented by an interest group. To the extent that moderates do organize, they are represented by large interest groups with a broad spectrum of constituents. On the other hand, individuals with extreme policy preferences have stronger incentives to organize politically, and the interest groups that they form tend to attract small but active, active and likeminded constituencies.

Groups compete over the right to implement a preferred policy which may affect both their constituency and all other individuals. Following a long tradition in political economy (e.g., Hillman and Riley (1989), Baye et al. (1993)), I specify competition as an all-pay contest with the observation that lobbying efforts are largely sunk. Once interest groups make expenditures, they are unable to (fully) convert their lobbying investments back into money if unfavorable policies are implemented.[2]

I show that interest groups employ mixed strategies in spending, and if lobbying expenditures are sunk, then they will be distributed as a power law. This result is consistent

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[1] Grossman and Helpman (2002) provide a theoretical treatment of these three modes of interest group competition.

[2] The results generalize when lobbying costs are not sunk.

with the robust empirical finding that over the past decade, federal lobbying expenditures in the vast majority of industries in the United States are distributed as power laws. When lobbying expenditures are not fully sunk, their distribution no longer follows a power law. This result implies a simple statistical test of whether interest groups' lobbying efforts are sunk costs with minimal data requirements.

My key contribution is that I explicitly consider the strategic interactions between the individuals who form interest groups in a model of social interaction (Schelling (2006)). Instead of taking the size, preferences and other attributes of interest groups as exogenous, I provide microfoundations for these attributes of groups by characterizing the sizes and configuration of interest groups in policy space as a strategic equilibrium in a political organization game.

Previous attempts to model the formation of interest groups have relied on external mechanisms which discipline free-riding. Mitra (1999) extends the Grossman and Helpman (1996) common agency model by endogenizing lobby formation but relies heavily on fixed costs of political organization to ensure the existence of collective action in equilibrium. Barbieri and Mattozzi (2009) provide a model of dynamic collective action in which political organization relies on the excludability of political benefits and membership fees. Gordon and Hafer (2010) offer an informational model in which individuals who organize are able to credibly signal policymakers. Other self described models of endogenous lobbying (e.g., Felli and Merlo (2006) and Laussel (2006)) assume the existence of interest groups and only consider the extent to which groups lobby as truly endogenous.[3]

My model of interest group formation resembles models of party formation (Osborne and Tourky (2008)), citizen candidate models of electoral competition (Osborne and Slivinski (1996) and Besley and Coate (1997)) and models of the endogenous formation of national boundaries (Alesina and Spolaore (1997)) in that I utilize a similar equilibrium concept. However, this equilibrium arises from a fundamentally different source: group competition.

In contrast to interest group formation, competition has been studied extensively. The economic theory of collective action beginning with the seminal work of Olson (1965) has developed along a number of distinct lines. Peltzman (1976) and Becker (1983) argue that competition between groups minimizes inefficiency in public policy. Tullock

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[3] Caplin and Nalebuff (1997) offer an analytical framework for understanding the formation and competition of collective institutions, paying particular attention to the question of when collective action can exist in equilibrium. While this is a substantial and general result, their model does not allow individuals to affect policies. In this particular setting, I relax that assumption and offer a coalitional equilibrium in which individuals are pivotal.

(1980) conceptualizes interest group competition as a contest (particularly, an auction), and Baye et al. (1993) recasts the lobbying contest as an all-pay auction. Fang (2002) provides a theoretical comparison of the policy and welfare implications of these two specifications of the lobbying contest. The common agency framework of Bernheim and Whinston (1986) has been successfully adapted to model lobbying as a competition over control of a government agent (Dixit et al. (1997)). This stylized specification of interest group competition allows for predictions of the level of lobbying activity and policy outcomes. In a more narrowly focused approach, Groseclose and Snyder (1996) and Banks (2000) consider interest group competition over committees.

There have been empirical investigations into the motivations for and success of interest group formation.[4] Walker (1983) measures the role of financial support for the successful formation of citizens' groups in the United States. Kennelly and Murrell (1991) correlate the formation of interest groups with economic characteristics of the industries they represent. And at an institutional level, Coates et al. (2007) explore which national characteristics lead to the robust formation of interest groups.

2 The Model

I model the lobbying process as a two stage sequential game of full information. In the first stage heterogeneous, policy minded individuals have the option of creating and joining (or leaving) interest groups. In the second stage those interest groups that were formed participate in an all-pay auction for the right to choose policy. Each group's bid can be interpreted as their lobbying effort, and the all-pay analysis follows from Baye et al. (1996).

2.1 Individual Preferences and the Group Formation Stage

Consider a compact, connected policy space $Q \in \mathbb{R}^N$ that is represented, without loss of generality, by the unit hypercube $[0,1]^N$. For exposition, fix $N = 1$ (in section 2.4.3 I relax this simplifying assumption.) Policy minded individuals are indexed according to their ideal policy and their types , i, are distributed uniformly throughout the policy space.[5] Individuals have quasilinear utilities over policy, q, given by $v_i = u_i(q) - x_i$ where x_i is the cost to individual i of lobbying. An individual's utility is assumed to be single-peaked at their type/ideal policy, i.e. $i = \arg\max u_i(q)$. As the type space and

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[4] For example, see Moe (1980), Berry and Wilcox (1984) and Baumgartner and Leech (1998).

[5] The analysis below holds if the assumption of uniformly distributed individual types is replaced by any continuous density with full support.

policy space are identical, I use these terms interchangeably.

The continuous specification of the type space is substantively important. Olson (1965) notes that free-riding is more highly pronounced in large groups, and he argues that the lack of an external mechanism to overcome free riding will render collective action of sufficiently large numbers of individuals impossible. By specifying an individual and type space that contains an infinite number of individuals, I deliberately handicap the analysis and prove the stronger claim that interest group competition is a sufficiently strong mechanism to allow for collective action.[6]

Individuals may form *interest groups* that are generally defined as subsets of the type space that are countable unions of open intervals containing the types of all of the group's constituents (figure 1). The *size* of interest group j, denoted s_j, is given by its measure and is positive. Two interest groups are *neighbors* if they are separated at some point by a set of measure zero. A *distal* subgroup of group j is a subset of j that neighbors another group. The set of all groups is denoted by J. Individuals who belong to an interest group are *organized*, and individuals who do not belong to an interest group are *disorganized*. Individuals may belong to at most one interest group.

There is an intrinsic trade off to political organization. By belonging to an interest group, individuals may coordinate their efforts and lobby as one. However, participation in the lobbying subgame is costly, so members of a group must bear a share of this endogenous cost. Given a profile of N groups, the continuum of players (individuals) can each take $N + 1$ actions in the group formation stage: *join* group $j = 1 \dots N$ or remain unaffiliated with any group. The number of groups is itself endogenously defined in this stage.

In a Nash equilibrium, no player can posses a profitable deviation. But because an individual is but a point in the type space, its singular choice of which (if any) group to join does not affect the configuration of interest groups in a measurable way, hence all configurations of interest groups are trivially Nash equilibria.[7] As such, I consider the joint actions of arbitrarily small sets of individuals. If arbitrarily small sets of individuals do not possess profitable deviations, then the continuous analogy of a non-cooperative equilibrium is immediate.

A set of of individuals j' will join a group j if two criteria are satisfied: First, holding the configuration and membership of all other groups fixed, the expected utility of j' after lobbying must be at least as large as it would be if they joined a group $k \neq j$ or

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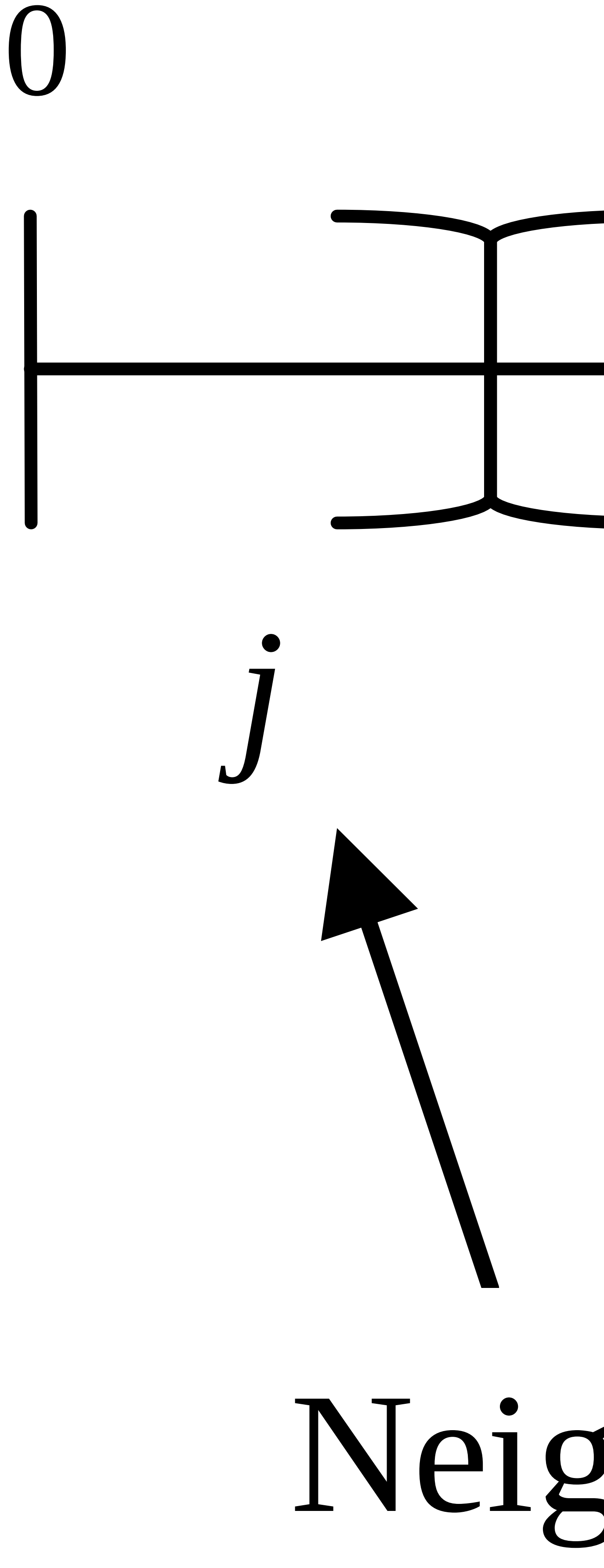
[6] The central result on the existence of an equilibrium configuration of interest groups is proved under the assumption that there are only a finite number of individuals in section 2.4.1.

[7] This is a trivial application of Caplin and Nalebuff (1997).

Figure 1:



remained disorganized. Second, a majority of j's members must be made better off by j' joining.[8] In short, j' will join j only if it can be made mutually advantageous to a majority in the receiving group. Meanwhile, a set of individuals will *secede* from a group if the defectors are better off out of the group than in it.

I define equilibrium in the group formation stage in terms of four stability concepts that are possessed by "small secession-proof" configurations of individuals into interest groups. This and other similar stability concepts have been used to analyze coalition structures in other settings (Alesina and Spolaore (1997), Dreze et al. (2007) and Osborne and Tourky (2008)).[9]

Definition 1. *An* equilibrium configuration *of interest groups is one in which there exists a* $z > 0$ *such that*

1. *No disorganized set of individuals of size* $\epsilon < z$ *would be made better off by joining a group (*α-stability*) .*
2. *No subgroups of size* $\epsilon < z$ *would be made better off seceding from their original group and joining another group (*β-stability*).*
3. *No subgroups of size* $\epsilon < z$ *would be made better off by seceding from their group and opting to stay disorganized (*γ-stability*).*
4. *No organized subgroups or sets of disorganized individuals of size* $\epsilon < z$ *would be made better off seceding from their group and forming a new group (*δ-stability*).*

My central result is that such an equilibrium exists. Each stability concept can be used to characterize various aspects of interest group configurations. Moreover, I show that γ-stability is in fact a sufficient condition for the existence of an equilibrium configuration.

α-stability ensures that all individuals that do not belong to a group are disorganized by choice. I show that α-stability implies that disorganized individuals tend to be moderates, and organized individuals tend to have more extreme ideal policy points. β-stability ensures that interest groups are not susceptible to small secessions of their constituencies to existing groups, and γ-stability ensures that interest groups are not susceptible to small portions of their constituencies seceding and free riding off of other groups' efforts. I show that these two types of stability imply that moderate groups tend

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[8] The second criteria can be generalized. The requirement that a majority of j's members must be in favor of acceptance can be replaced by the requirement that a fixed share α of j's members must be in favor of acceptance for any $\alpha < 1$.

[9] This equilibrium definition is a necessary (though clearly insufficient) condition for core stability as described in Aumann and Dreze (1974).

to be larger than extremist groups. Finally, δ-stability ensures that interest groups and the politically disorganized are not susceptible to small defections that result in entirely new groups. I show that δ-stability is equivalent to γ-stability, so it plays no further role in characterizing the equilibrium configuration of interest groups.

2.2 Interest Group Competition and the Lobbying Subgame

Interest groups compete in an all pay auction for the right to implement a single policy. Each group makes a non-negative lobbying effort of size x_j, and the (winning) group that makes the largest lobbying effort chooses policy. Ties are broken randomly. Payments are made regardless of the outcome of auction.

The aggregate utility that group j derives from policy q is

$$U_j(q) = \int_{i \in j} u_i(q)\, di \tag{1}$$

Conditional on winning the auction, group j implements the policy $q_j^\star = \arg\max\{U_j(q)\}$. The ex ante expected utility of a group j in the lobbying subgame can be written as

$$\mathrm{E}\left[\Pi_j(x_j)\right] = \sum_{k \in J} p_k U_j(q_k^\star) - x_j \tag{2}$$

where p_k is the (endogenous) probability that k makes the largest lobbying effort. Lobbying costs are distributed proportionally across constituents.

Because the lobbying subgame is an all-pay auction with bids of size x_j (albeit one where the value of the prize is endogenously determined), it does not posses a pure strategy Nash equilibrium.

Lemma 1. *In the lobbying subgame, there is no Nash equilibrium in pure strategies.*

The proofs of this claim and all others may be found in the appendix. The intuition behind lemma 1 is immediate. Any sure non-zero bid will be either too high or too low (if the bidding group loses the auction or wins the auction respectively) with probability 1. And any sure bid of zero is too low if all other groups submit a bid of zero. As such, groups bid according to mixed strategies. These strategies are defined by N mixing distributions $f_j(x)$ which denote the probability that group j makes a lobbying effort of size x. F_j is the cumulative density of j's mixing distribution, and $\overline{x}_j$ and $\underline{x}_j$ are the upper and lower bounds of the support of f_j.[10]

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[10] The mixing distribution may be degenerate. That is, there may exist an $x \in \left(\underline{x}_j, \overline{x}_j\right)$ such that $f_j(x) = 0$.

An *inactive* group j is one for whom $x_j = 0$ with probability 1. Similarly, an inactive set of individuals is one that makes no expenditures in the lobbying game with probability 1. Finally, groups can be described as extreme or moderate depending on where they are located relative to the expected implemented policy. Let $\bar{q}^\star = \sum_{j \text{ active}} p_j q_j^\star$. Then group j is more *extreme* than group k if $u_j\left(q_j^\star\right) - u_j\left(\bar{q}^\star\right) > u_k\left(q_k^\star\right) - u_k\left(\bar{q}^\star\right)$. Similarly, an individual is more extreme than another individual (group) if their type is farther from $\bar{q}^\star$ than the individual's type (group's favored policy) is.

2.3 Characterizing Equilibrium

The model above raises three questions: Does an equilibrium configuration of interest groups exist? If so, what are the features of the equilibrium? What are groups' lobbying strategies? I now characterize equilibrium in the lobbying game under a given configuration of interest groups and show that equilibrium configurations of interest groups do exist, though they are not unique. I complete the analysis by explicitly solving for equilibrium lobbying strategies.

2.3.1 The Lobbying Subgame

In the lobbying subgame, groups are fixed in policy space, and all information is common knowledge. The net *valuation* that group j places on winning the lobbying subgame is

$$V_j = U_j\left(q_j^\star\right) - \psi_j \sum_{k \neq j} p_k U_j\left(q_k^\star\right) \tag{3}$$

where $\psi_j = \left(\sum_{k \neq j} p_k\right)^{-1}$ conditions the probabilities p_k on j losing. Group j's valuation is the surplus group j enjoys from winning the auction relative to losing, which is the maximum that group j would be willing to spend lobbying. Although $U_j < 0$, V_j is appropriately normalized to be non-negative. This surplus is increasing in the aggregate utility the group gets from choosing its own policy and decreasing in the expected aggregate utility the group gets from other groups choosing policy. In addition, $V_j^\star = E\left(\Pi_j\left(x_j^\star\right)\right)$ denotes the expected equilibrium payoff to group j, which is a function of j's equilibrium action $x_j^\star$. The valuation of a subgroup is defined analgously. Hence, the costs of lobbying borne by subgroup k of group j are given by $x_k = \frac{V_k}{V_j} x_j$.

It is useful to rank ivide the interest groups by valuations into three disjoint equivalence classes as follows.

Definition 2. *Group j is in set* A *(an* A-group*) if $V_j = \underset{k \in \Gamma}{max} \{V_k\}$. Group j is set* B *(a* B-group*) if $V_j = \underset{k \in \Gamma, k \notin A}{max} \{V_k\}$. Otherwise, a group j is in set* C *(a* C-group*) if it is neither an A-group nor a B-group. Let $|X|$ signify the number of elements in set X.*

All A-groups value winning the most. If a single group values winning more than all other groups, then it is the sole A-group. Similarly, all B-groups value winning more than all other groups besides the A-groups. C-groups are simply a residual class consisting of all other groups. By construction, all A-groups possess a single valuation, and all B-groups possess a single valuation. This classification is useful because in the lobbying game, all groups of the same type take similar actions and enjoy identical equilibrium payoffs in expectation.

Proposition 1. *In the lobbying subgame:*

a. *All C-groups are inactive.*

b. *If $|A| > 1$, $V_j^\star = 0$ for all $j \in \Gamma$, and all B-groups are inactive.*

c. *$\underline{x}_j = 0$ for all j. Let $V = \underset{j \text{ } active}{max} \{V_j\}$. Then $\overline{x}_j = \overline{x}_k = V$ for all active groups j, k.*

In order to be competitive with B-groups, C-groups incur higher lobbying costs than they find worthwhile. Hence, C-groups are inactive because they do not value winning the lobbying subgame enough. In a similar vein, when there is competition between multiple A-groups, B-groups must incur higher costs than they find worthwhile to remain competitive. Finally, because costs are sunk, all active groups possess the same support for their mixing distributions. No group would ever choose to put forth needlessly large lobbying efforts.

This simplifies the analysis of the lobbying subgame by shrinking the set of active groups. Additionally, proposition 1 implies certain "discontinuities" in lobbying behavior that have implications for the existence of stable group configurations. Arbitrarily small changes in groups' valuations may result in changes to group classification. In turn, this may abruptly switch a group's status from active to inactive. Because arbitrarily small groups are necessarily (inactive) C-groups, their members are functionally equivalent to disorganized individuals, hence:

Corollary 1. *A configuration of interest groups is γ-stable if and only if it is δ-stable.*

2.3.2 Characterizing a Stable Configuration of Interest Groups

Although groups are generally defined, equilibrium conditions restrict their shapes, sizes and configurations in policy space. Because inactive interest groups are functionally equivalent to disorganized individuals, they are indistinguishable from disorganized individuals in equilibrium. This limitation is of little theoretical concern, since inactive groups do not affect the lobbying equilibrium. Furthermore, this limitation is of no empirical concern, since politically inactive groups are unobservable.

In equilibrium, groups must approximate connected intervals (i.e., they can only be punctured by single points), and α- and γ-stability ensure than inactive individuals, whether organized or not, must be more moderate than actively lobbying interest groups in equilibrium. I summarize these facts:

Proposition 2. *In an equilibrium configuration of interest groups,*

a. *All active groups are* connected almost everywhere. *That is, for a group j, if $\underline{j} = \inf j$ and $\overline{j} = \sup j$, then $\left[\underline{j}', \overline{j}'\right]$ is of measure 0.*

b. *Any set of individuals of positive measure whose types are more extreme than an active interest group must belong to an active interest group.*

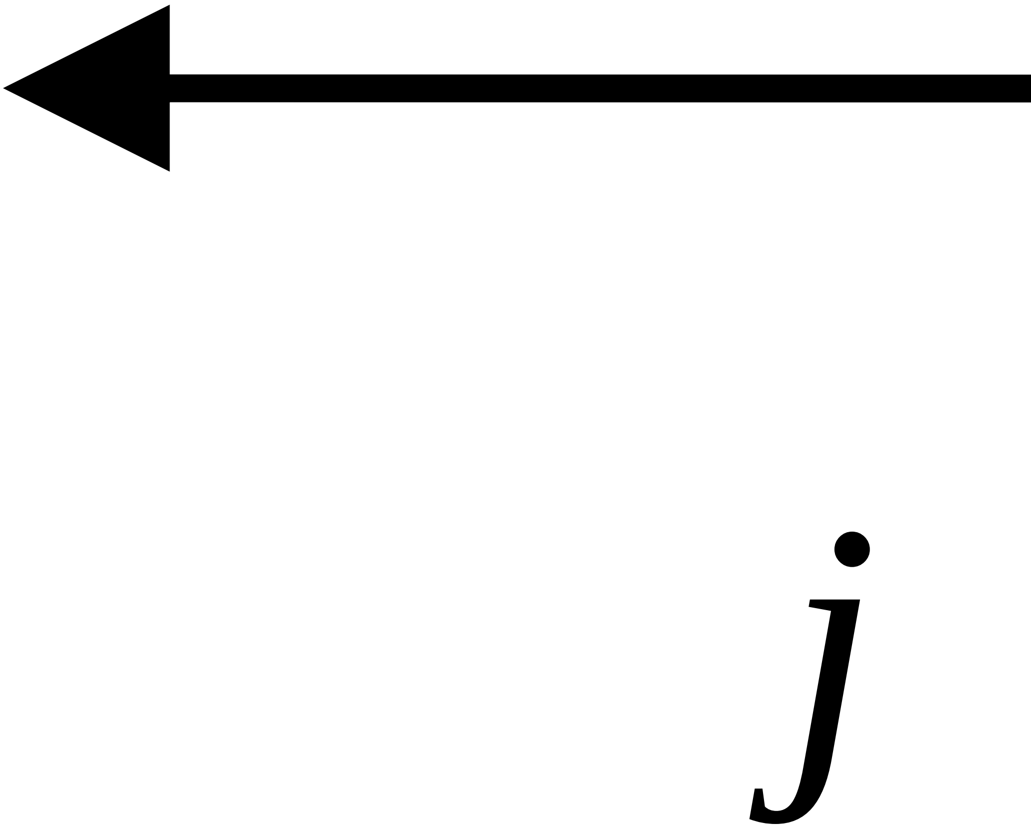
The proof of proposition 2 proceeds as follows. A group that is not connected everywhere is interrupted by whole intervals of non-members (see 2). Such an active group j is inherently unstable for the following reason. If the full membership of group j was better off remaining in j as implied by β- and γ-stability, then some individuals caught between the two disconnected subsets of j must also wish to join j. But this would be a violation of either α-stability. Put another way, a collection of individuals whose ideal policies fall between more extreme elements of an organized group are compelled to belong to that group. In general, individuals tend to organize with their like minded neighbors in type space.

A similar argument can be made for the second statement of proposition 2. If the full membership of a group was better off remaining organized (i.e., if the group was γ-stable), then more extreme, inactive individuals must also prefer to join j since these extreme individuals have more to gain from active organization than their moderate counterparts. But this would constitute a violation of α-stability. In general, moderates tend to be politically disorganized, whereas extremists self organize into active interest groups.

Competition in the lobbying subgame ensures the existence of an equilibrium in the group formation subgame and carries additional implications for the number, types, and thus sizes of groups in equilibrium.

Proposition 3. *An equilibrium configuration of interest groups exists.*

Figure 2:



I describe the proof. There are three distinct elements of proposition 3. The first element is a statement of the existence of equilibrium in the group configuration subgame. A configuration with several A-groups is α-stable because in joining an A-group, any set of disorganized individuals assume additional lobbying costs on the order of ϵ, but their expected utility from policy only increases on the order of ϵ^2. Hence, for small enough values of ϵ, joining the A-group is unfavorable. Similar cost-benefit arguments can be made to show that this configuration is β-stable and, if the number of A-groups is sufficiently small, γ-stable.

It is important to view the existence of equilibrium in the context of the free rider problem. In most models of collective action, the existence of groups is simply taken as an assumption. When groups compete over a common, non-excludable prize, it is difficult to see from an individual's perspective why she would prefer to belong to a group. By leaving the group, she would still enjoy the fruits of the group's efforts, yet she would bear none of the costs. As such, groups would not exist in equilibrium.

In this model, despite a common, non-excludable prize, competition between groups solves the free rider problem. The defection of even an arbitrarily small fraction of individuals jeopardizes the entire group's efforts by turning them inactive. In short, every subgroup, irrespective of size, is pivotal. The collective costs of lobbying are well counterbalanced by its collective benefits.

The second element of proposition 3 is that $|A| > 2$. More than two A-groups must exist, or else the configuration is not γ-stable. If there is a lone A-group, some small ϵ sized subset of the group could secede, saving lobbying costs on the order of ϵ. However, as long as the defectors are sufficiently small in number, the A-group will retain it's classification. At worst, the loss in benefits to the defectors scales on the order of ϵ^2. For small enough values of ϵ, this makes defection an attractive option. If there are exactly two A-groups, a small ϵ sized secession will reclassify the group as the lone B-group, leaving a single A-group. Since B-groups are active when $|A| = 1$, then the loss in benefits still scales on the order of ϵ^2. The only way to avoid this situation is to force any defection from the A-group to change its status from active to inactive; this occurs only if there are multiple A-groups. In short, with a fewer than three A-groups, competition is insufficient to overcome free-riding.

Competition provides pressure for groups to maintain equal valuations. Despite the positive correlation between group size and valuation, this does not imply that groups will be of equal sizes. Although U_j and s_j have a one-to-one relationship, in general V_j and s_j do not. Losing the lobbying game is more costly to extremist individuals than moderates because the policy space is, on average, farther from their blisspoints, so groups comprised of extremists will have greater per capita valuations for choosing policy. Since aggregate group valuations must be equal in equilibrium, this implies that extremist groups will be smaller than moderate groups.

Corollary 2. *In any equilibrium configuration, extremist groups will be active and relatively small, and moderate, active groups will be relatively large.*

2.3.3 The Equilibrium Actions of Interest Groups

I now extend the characterization of interest group actions by solving for the equilibrium mixed strategies of each group in a stable configuration. There are two types of mixed strategies: a symmetric mixed strategy and a continuum of asymmetric mixed strategies. In a symmetric case, all groups randomize continuously over the entire interval $[0, V]$. In a continuum of asymmetric cases, some groups randomize continuously over arbitrary intervals $[\alpha_i, V]$ and spend 0 with positive probability.

Proposition 4. *In equilibrium, all active groups possess common valuation V. be the number of active groups. Then the cumulative density function of the symmetric mixed strategy that groups use in equilibrium is*

$$F(x) = \left(\frac{x}{V}\right)^{\frac{1}{N-1}} \tag{4}$$

for all $x \in [0, V]$.[11]

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[11] Baye et al. (1996) also identify a continuum of asymmetric equilibrium mixed strategies. The c.d.f. of these mixed strategies is formulated as follows: Divide up the support of F into horizontal bins of arbitrary width. There can be up to $N-2$ of these bins. In each bin, the mixed strategy c.d.f. is a simple polynomial of the form $F(x) \propto x^{\frac{1}{1-k}}$ where k is the number of groups that pace bids from the k^{th} bin with positive probability. Bins that are further to the right have larger k's, and the c.d.f. in each bin is scaled in such a way as to connect to the c.d.f.'s in the bins to the left and to the right.

More formally, let α_n, $n = 1 \ldots N$, represent a weakly decreasing sequence of arbitrary constants, and let $w \geq 2$ equal the largest n such that $\alpha_n = 0$. Then the c.d.f. of the mixed strategy that group j uses in equilibrium is

$$F_j(x) = \begin{cases} \left(\frac{x}{V}\right)^{\frac{1}{N-1}} & x \in [\alpha_N, V] \\ c_n \left(\frac{x}{V}\right)^{\frac{1}{j-1}} & x \in [\alpha_n, \alpha_{n+1}),\ n = 1 \ldots j,\ n \in \{w+1, \ldots, N-1\} \\ c_n \left(\frac{\alpha_j}{V}\right)^{\frac{1}{(j-1)}} & x \in [\alpha_n, \alpha_{n+1}),\ j = n+1 \ldots N \\ d_n \left(\frac{x}{V}\right)^{\frac{1}{z-1}} & x \in [0, \alpha_{n+1}),\ j = 1 \ldots w \\ d_n \left(\frac{\alpha_j}{V}\right)^{\frac{1}{(j-1)}} & x \in [0, \alpha_{n+1}),\ j = w+1 \ldots N \end{cases}$$

where constants $c_n = \left[\prod_{m>n} F_m(\alpha_m)\right]^{\frac{1}{n-1}}$ and $d_n = \left[\prod_{m>w} F_m(\alpha_m)\right]^{-\frac{1}{w-1}}$. It is apparent that the symmetric mixed strategy in (4) is a special case of this where $\alpha_j = 0$ for all j.

The key to proposition 4 is that no group receives net positive payoffs in expectation, so mixed strategies can be computed directly from the equilibrium condition.

Lobbying expenditures are defined only by the number of active groups and the importance of policy to these groups. Equilibrium in group formation systematically aggregates the heterogeneity of individual preferences into a smaller set of players (groups) with identical preferences with respect to setting policy. The symmetry in valuation, or lobbying intensity, between active interest groups ensures that lobbying expenditures across all groups will take on a power law distribution.

Corollary 3. *The rightmost tail of the distribution of lobbying expenditures of interest groups is a power law with exponent* $\frac{N-2}{N-1}$.[12] *For large* N*, this exponent converges to* 1.

Corollary 3 implies a simple empirical strategy for identifying the effective number of interest groups lobbying on a given issue. In particular, there is a one-to-one correspondence between the exponent of the rightmost polynomial with N.

2.4 Robustness of the Model

The preceding results follow from three modelling choices: there is a continuum of individuals, groups' lobbying costs are sunk, and interest group competition is fundamentally unidimensional. In the formulation above, the entirety of a group's lobbying effort is forfeited irrespective of the outcome of the competition. I discuss the implications of relaxing these modelling assumptions and show that the qualitative results of the model surive.

2.4.1 Finite Number of Individuals

Olson (1965) points out that sufficiently small groups are capable of withstanding free riding. This suggests that the existence of equilibria when I is infinite may imply the existence of equilibria when I is finite. Although this implication does not hold strictly speaking, there is a sense in which the set of equilibria when I is finite is "larger" than the set of equilibria when I is infinite.

To make this statement more concrete, I first define an equilibrium configuration of interest groups for finite I.

Definition 3. *A* finite equilibrium configuration *of interest groups is one in which*

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[12] Recall that a power law possesses a p.d.f. of the form $f(x) \propto x^{-\gamma}$, so the exponent is treated as positive by convention.

1. *No disorganized individual would be made better off by joining a group (*α-finite stability*) .*
2. *No individual would be made better off seceding from their original group and joining another group (*β-finite stability*).*
3. *No individual would be made better off by seceding from their group and opting to stay disorganized (*γ-finite stability*).*
4. *No individual would be made better off seceding from their group (if previously organized) and forming a new group (*δ-finite stability*).*

This is the obvious analogue to the definition of an equilibrium configuration given earlier, and can be correctly interpreted as the definition of a subgame perfect Nash equilibrium in the interest group formation subgame with a finite number of players.

Let X_∞ represent the set of equilibrium configurations of groups when there are an infinite number of individuals on a continuum (as given by definition 1). Similarly, let X_I represent the set of finite equilibrium configurations of groups for a given number of individuals $I \geq 2$, at least two of whom are of different types. Each element of X_∞ and X_I is a tuple containing the number of groups and expected policy outcome.

Every equilibrium configuration with a continuum of individuals can be supported by an equilibrium configuration with a finite number of individuals, though the converse is not true. Hence, the set of equilibrium configurations of groups is smaller than the set of finite equilibrium configurations, so proposition 3 is the strongest possible proof of existence of equilibrium in the group formation stage.

Proposition 5. *For all $I \geq 2$ with at least two individuals of different types*

a. *X_I is non-empty..*

b. *$X_I \subset X_{I+1}$ for all I..*

c. *There exists a well defined correspondence $h : X_\infty \to \bigcup X_I$.*

d. *There exists an element $x \in X_I$ such that $x \notin h\left(X_\infty\right)$.*

Part 5 of proposition 5 is the finite analog to proposition 3, and it is established with a trivial example: the configuration of two groups each of which contains as a single member the most extreme individuals in the type space. Part5 states that adding individuals can only increase the set of finite equilibrium configurations. Part 5 states that every equilibrium configuration on a continuum of individuals can be represented as a finite equilibrium configuration. Put another way, given sufficiently many individuals, any equilibrium configuration can be perfectly mimicked by a finite equilibrium configuration. Finally, part 5 states that there are "more" finite equilibrium configurations of individuals than equilibrium configurations of individuals. The trivial example clearly shows this to be the case.

In sum, proposition 5 suggests that the main result featuring an infinite number of individuals is a stronger result than the existence of finite equilibrium configurations of interest groups. This comports well with the Olsonian view that free riding is a threat to large scale organization. The existence result in in the infinite case implies that this threat can always be dealt with. The potential for organization rests not on small numbers but rather heterogeneity in individual policy preferences.

2.4.2 General Lobbying Costs

Suppose that a group's lobbying efforts is not entirely forfeited when the group fails to win the lobbying subgame. A group's payoffs can be recast as

$$\Pi_j\left(x_j\right) = \begin{cases} U_j\left(q_j^{\star}\right) - x_j & \text{if } x_j > x_k \text{for all } k \neq j \\ U_j\left(q_k^{\star}\right) - C\left(x_j\right) & \text{if } x_k > x_l \text{for all } k \neq l \end{cases} \tag{5}$$

The lobbying cost function C captures the extent to which the lobbying effort is sunk and embeds many familiar specifications of interest group competition. For example, when $C\left(x\right) = 0$, the lobbying subgame is a standard first price auction, and when $C\left(x\right) = x$ as above, the lobbying subgame is a standard all-pay auction.

Proposition 6. *If $C\left(x\right)$ is a non-negative and non-decreasing function, propositions 1-3 still hold. Let V be the common valuation for the $N > 2$ active groups.*

If $C\left(x\right) > 0$ for some $x \in \left(0, V\right)$, then there is no pure strategy equilibrium in the lobbying subgame. For the N active groups that possess a common valuation V, the cumulative density function of the symmetric mixed strategy that groups use in equilibrium is given by

$$F\left(x\right) = \left(\frac{C\left(x\right)}{V}\right)^{\frac{1}{N-1}} \tag{6}$$

for all $x \in \left[z, C^{-1}\left(V\right)\right]$ where z is the largest value for which $C\left(z\right) = 0$.

If $C\left(x\right) = 0$ for all $x \in \left(0, V\right)$, there is a unique symmetric pure strategy equilibrium *in the lobbying subgame. All active groups possess and bid their identical valuation* V *and win with probability* $1/N$.

Proposition 6 generalizes the characterization of the configuration of interest groups to broader types of lobbying competition. In particular, the existence of an equilibrium configuration of groups is not simply an artifact of the all pay specification.

By simply observing data on groups' lobbying expenditures, it is possible to estimate non-parametrically the actual lobbying cost function C that interest groups face (up to a constant) for a given value of N. If N is unknown, non-parametric lower and upper bounds on C can be obtained by setting $N = 3$ and $\gamma = \frac{N-2}{N-1} \approx 1$ respectively. This represents an implicit statistical test of whether lobbying competition is similar to an all-pay auction, a standard first price common value auction, or some intermediate variant with partial sunk costs.

2.4.3 Multidimensional Policy Space

Now suppose that the policy space is represented by $Q \in \mathbb{R}^N$, $N > 1$. As this generalization of the policy space does not affect the lobbying subgame, the ability of group competition to mitigate free-riding is unabated, and all results persist.

Proposition 7. *If the policy space is multidimensional and individuals have single peaked preferences,*

a. *All results from the lobbying subgame hold (propositions 1, 4 and 6).*

b. *All active groups are connected almost everywhere (proposition 2a).*

c. *Any set of individuals of positive measure whose types are more extreme than an active interest group must belong to an active interest group. (proposition 2b).*

d. *An equilibrium configuration of groups exists (proposition 3).*

The arguments presented in the proofs of the propositions when $N = 1$ are easily extended to the more general case in proposition 7. This greatly expands the applicability of the model to observed situations.

There is an important caveat to this extension. If policy in one dimension is lobbied for and/or determined before policy in another dimension, then the results break down. In particular, active groups need not be connected everywhere (they may consist of disconnected subgroups of individuals with strong ideological preferences for policy in different dimensions), and groups of extremists of positive measure may exist in equilibrium.

3 Some Empirical Results

Proposition 4 does not make a precise prediction of a particular group's actual lobbying effort since the effort is derived only up to a mixed strategy. Nevertheless, this prediction can be tested by analyzing the *distribution* of lobbying efforts over a particular policy. For a single group, I observe only a single action drawn from their mixed strategy distribution. However, if interest groups are using symmetric mixed strategies, then the multiple actions observed by groups lobbying on a particular issue can be used to estimate their mixing distribution and thereby test proposition 4 .[13]

According to the Lobbying Disclosure Act of 1995, all federal lobbyists in the United States are required to register with both chambers of Congress and must disclose all lobbying activity by their clients (interest groups) to the Department of Treasury. The Center for Responsive Politics (CRP), a non-partisan watchdog group, has collated all federal lobbying activity since 1998 and categorized these efforts by industry (or policy area when relevant).

In 2009, interest groups hired over 13 thousand lobbyists and spent roughly $3.5 billion to shape public policy at the federal level. These interest groups include privately owned and publicly held firms of all sizes, industry trade groups, labor unions, non-profit organizations, and other collections of private individuals. According to the model above, interest groups lobbying over related policies utilize mixed strategies from a common distribution. Estimates of these cumulative mixing distributions for six selected industries are provided in table 3.

The estimates of the mixing distributions for each industry (represented by the solid lines) are constructed non-parametrically through a locally weighted regression of the empirical cumulative density evaluated at each observation in the sample (represented by the shaded circles). The sizes of the circles vary depending on how many distinct observations they represent. Because lobbying efforts may be rounded in disclosures to the IRS, there is some clumping at increments of $10 thousand, and larger circles correspond to more observations. The x-axis of each plot is scaled logarithmically. Proposition 4 predicts that on such a plot, the cumulative density of lobbying expenditures should be a straight line.

For the banking, telecommunications services and tobacco industries, the empirical

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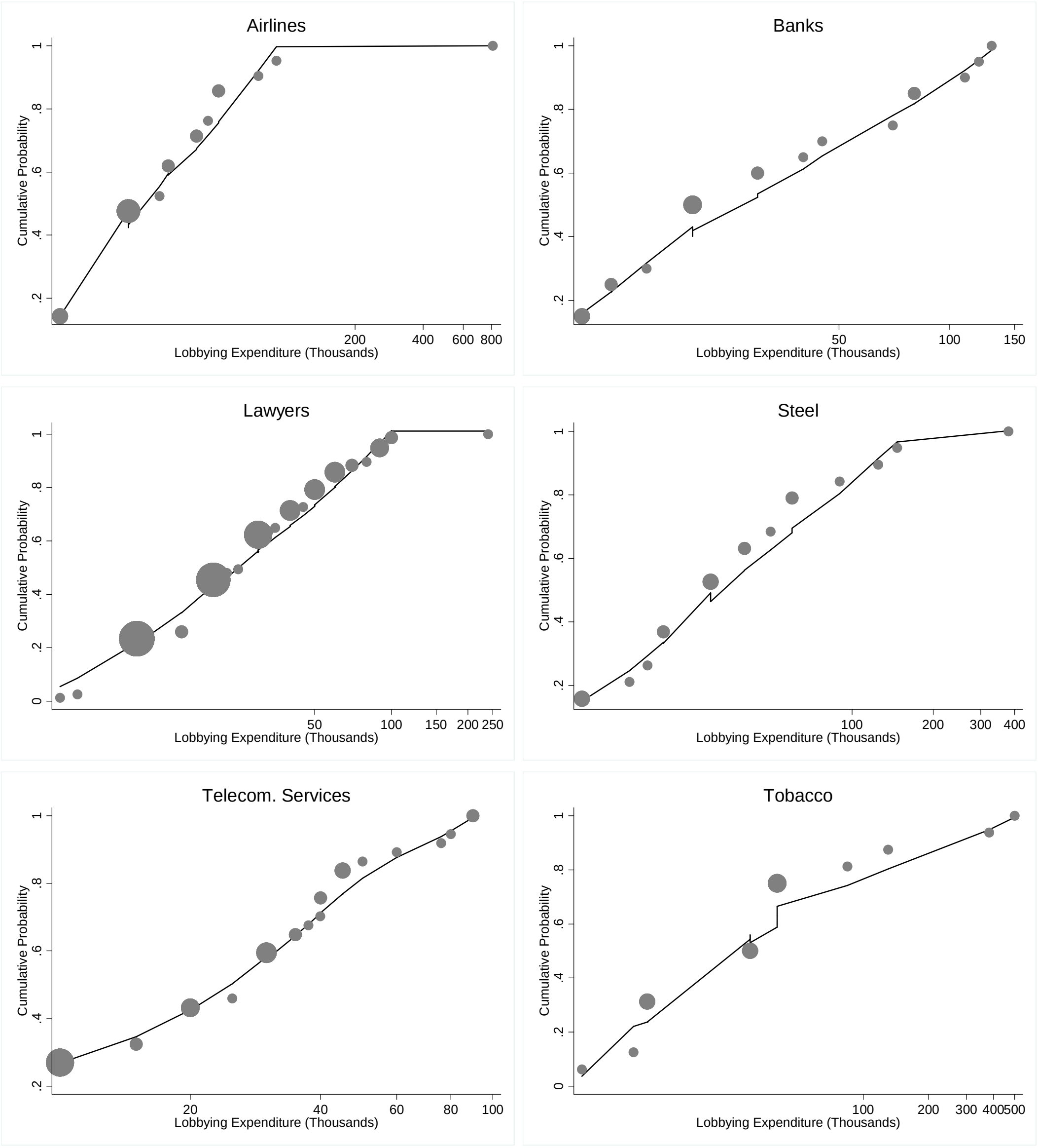
[13] In fact, the empirical test of proposition 4 does not rely upon the assumption that groups utilized symmetric strategies. Even if groups utilized asymmetric strategies, the probability that group j takes an action x is equivalent to the probability that a group k takes the same action x conditional on both actions being observed. For a precise definition of the asymmetric mixed strategies, see footnote 11.

Figure 3: Locally Weighted Regression Estimates of the Empirical Cumulative Distribution Functions for Lobbying in Selected Industries, 2009



distributions are well approximated by straight lines of fixed slopes. Additionally, in the airline, legal and steel industries, the empirical distributions are well approximated by straight lines of fixed slopes with the exception of a prominent outlier at the top. These outliers represent the Air Transport Association, American Bar Association and US Steel respectively. These powerful interests have strong, well established, external abilities to prevent free riding in the form of licensing and membership requirements that are outside of the assumptions of the model above. Nevertheless, the empirical distributions are consistent with a configuration of a single large and protected A-group along with a collection of B-groups. Each of these six cases provides strong empirical support for proposition 4.

I now turn to the prediction of corollary 3. While the common relative valuation of each group (V) is not empirically identifiable since it merely represents a scaling of observed expenditures (x), the effective number of groups (N) is identifiable by the functional form of the mixing distribution. I estimate power law exponents $\gamma = \frac{N-2}{N-1}$ for each industry using the rank-minus-half method of Gabaix and Ibragimov (Forthcoming). Results for selected industries are provided in table 1, and robust standard errors are provided alongside. Given the estimates of the power law exponents, I am able to compute N, the effective number of active interest groups within the industry. Note that for several industries, $\hat{\gamma} > 1$. While this is not possible according to the model above, $\gamma = 1$ falls within a 95% confidence interval of the estimate. In effect, I can interpret the number of active interest groups in these industries to be very large. Each estimate $\hat{\gamma}$ is highly precise and statistically significant from zero. Indeed, the ranking of industries by the effective number of active interests is plausible and consistent with general portrayals of political organization.

In the last four columns of table 1, I present four market concentration ratios for each industry from the 2002 Economic Census of the United States. The ratio C_n represents the share of revenues that are accounted for by the n largest firms in the industry. Those industries that are more consolidated tend to have lower values of $\hat{\gamma}$ and hence lower associated effective numbers of active interest groups. On the other hand, less consolidated industries tend to have higher values of $\hat{\gamma}$ and higher associated effective numbers of active interest groups. This suggests that there is a positive relationship between economic structure and political structure. Moreover, this relationship would not be uncovered by simply computing political concentration ratios since groups can rename themselves and distribute lobbying efforts over several lobbyists or organizations.

Suppose that there are fixed costs, λ, associated with political organization. In this case, no inactive groups remain organized in equilibrium, and the costs of lobbying can

Table 1: Estimated Power Law Exponents, Computed N and Industry Concentration Ratios for Selected Industries

Industry Name	$\hat{\gamma}$	SE	N	C_4	C_8	C_{20}	C_{80}
Tobacco	0.66	0.16	3.8	86.7	93.2	98.8	99.9
Auto Manufacturing	0.67	0.07	4.0	81.2	91.4	98.4	99.6
Petroleum Refining	0.74	0.07	4.8	41.2	63.5	89.3	99.3
Aircraft Manufacturing	0.76	0.09	5.2	80.7	93.6	98.2	99.7
Railroads	0.78	0.04	5.5	–	–	–	–
Cable and Satellite TV	0.79	0.08	5.8	63.9	77.7	91.9	98.4
Insurance Companies	0.8	0.06	6	24.0	38.8	68.6	88.9
Concrete, Cement, Stone	0.84	0.08	7.3	11.9	20.3	32.0	43.6
Pharmaceutical Manufacturing	0.86	0.03	8.1	34.0	49.1	70.5	83.7
Industrial/Commercial Construction	0.91	0.07	12	–	–	–	–
Electric Utilities	0.92	0.11	13.5	16.1	29.3	53.3	77.8
Commercial TV and Radio	0.93	0.09	15	39.1	53.6	66.6	78.5
Airlines	0.95	0.08	21	22.3	34.4	48.8	65.0
Dairy	0.95	0.1	21	24.9	36.1	55.2	74.6
Hospitals, Nursing Homes	0.96	0.03	26	9.0	12.3	18.9	28.5
Computer Software	1.04	0.06	*	–	–	–	–
Security and Investment Companies	1.04	0.06	*	23.6	34.0	50.0	63.3
Physicians	1.04	0.07	*	3.4	4.3	6.2	9.0
Law Firms	1.12	0.1	*	12.1	16.7	24.8	37.4
Advertising	1.16	0.15	*	16.3	21.3	28.2	33.7
Residential Construction	1.35	0.28	*	–	–	–	–

Power law exponents are estimated using 2009 lobbying data from the Center for Responsive Politics. Concentration ratios are from the most recently available US Economic Census, 2002. Ratios are reported at the highest appropriate NAICS level. Concentration ratios for the railroad, software development and construction industries are unavailable.

be represented by $C(x) = \lambda + x$. The distribution of lobbying expenditures within industries should still follow a power law in the right tail, and all groups with valuations $V < \lambda$ disband. However, the upper bound on the number of groups in equilibrium, $\overline{N}$, is decreasing in λ, as defection becomes a more attractive option for subsets of smaller groups. Hence, fixed costs to political organization generate political concentration. This is in some ways a political analog of the theory of endogenous sunk costs and their relationship to market structure (Sutton (1992)) and the robust finding that industries with greater fixed costs feature larger lower bounds on concentration ratios. The positive empirical relationship between political concentration and market concentration is revealed to the extent that politically concentrated interests are able to protect exogenous (and endogenous) barriers to economic entry.

4 Conclusion

Free riding is the central existential obstacle to collective action and hence formation of interest groups. I show that when information is public and groups' valuations for setting policy are common knowledge, competition between groups is sufficient to mitigate completely the issue of free riding. The common pool problem that plagues collective action – an individual's marginal cost of participating in collective action is usually in excess of their marginal benefit and leads them to secede – is avoided because when lobbying is competitive, the marginal benefit to political organization for arbitrarily small collections of individuals is fixed, whereas the marginal cost to political organization varies in the size of a potential defection.

This simple mechanism through which groups coexist and compete in a secession-proof equilibrium has several orthogonal implications which, on their face, are reasonable and empirically testable. First, I argue that in equilibrium, competitive forces will lead interest groups to be comprised of like-minded individuals – neighbors in policy space. Second, I argue that the politically organized tend to be more extreme than the politically inactive and disorganized. Third, I argue that active interest groups utilize mixed strategies when lobbying and make a strong prediction of the distribution of lobbying expenditures which is empirically supported. The configuration of groups in policy space may affect the interaction of groups in a competitive setting. In this paper, I show that it does in a characteristic manner.

The model presented here is general in many aspects, but requires admittedly strong assumptions on the timing of the lobbying process. Further inquiry into generalization of these assumptions is warranted. Moreover, while the empirical evidence provided in support of the model is suggestive, presentation of systematic empirical evidence in support of the qualitative results on interest group configuration is in order.

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Appendix: Proofs

Proof. Lemma 1. Suppose that for group j, bidding $x_j \in (0, V_j]$ was a pure strategy Nash equilibrium (with all other groups playing x_k). Then if $x_j > x_k$ for all k or $x_j < x_k$ for some k, group j would be better off shading their bid down. If $x_j = x_k$ for all k, then group j would either be better off shading their bid down if $x_j > \frac{1}{N} V_j$ and shading their bid up otherwise.

Finally, $x_j = x_k = 0$ is not a Nash equilibrium since any group could benefit by shading their bids upward. ☐

Proof. Proposition 1. Before turning to the statements of the proposition, I prove two intermediate results.

Lemma 2. $\underline{x}_j = 0$ *for all* j.

Proof. Lemma 2 states that all groups include zero in the support of their mixing distributions. This is not surprising, since a group that never spends below a positive amount always runs the risk of overbidding. Let

$$V_j^{\star} = V_j^{\star}(x) = \Pi_j(x) \tag{7}$$

denote the expected payoff to group j from taking action x in the support of its mixed strategy.

Say $\underline{x}_j > 0$ for all j. Then $\underline{x}_j = \underline{x}_k = \underline{x}$ for all j, k, since no group would ever spend a positive amount with no chance of winning the lobbying subgame. In addition, no group would ever play $\underline{x}_j$ with positive probability, since then all other groups k could stand to gain by increasing $\underline{x}_k$ by $\epsilon > 0$. But if all groups possess the same $\underline{x}$ and spend that amount with zero probability, group j could benefit by shifting mass of their density F_j from $\underline{x}_j$ to $\underline{x}_j - \epsilon$. This is a contradiction.

Say $\underline{x}_j > 0$ and $\underline{x}_k$=0 for some k. Then group j could benefit from shifting mass of their density F_j from $\underline{x}_j$ to $\underline{x}_j - \epsilon$ since group k plays $\underline{x}_j$with probability zero. This is also a contradiction. ☐

Lemma 3. $V_j^{\star} = 0$ *for all* $j \in B, C$.

Proof. Let $V_a^\star$ be the A-group(s)' (common) valuation for winning the auction, $V_b^\star$ be the B-group(s)' (common) valuation for winning the auction and $V_c^\star$ be the largest valuation of a C-group for winning the auction. By construction, $V_a^\star \geq V_b^\star \geq V_c^\star$.

Say $|A| = 1$. Then $V_a^\star > 0$, since the A-group could guarantee itself a positive payoff by spending $\frac{V_a+V_b}{2}$. Hence, for every action $x \in (\underline{x}_a, \overline{x}_a]$ the A-group must outbid all other groups with probability strictly greater than zero. Since we know that $\underline{x}_a = 0$ from lemma 2, all B- and C-groups must spend 0 with positive probability.

Now, say $|A| > 1$. Suppose $V_j^\star > 0$ for all $j \in A$. Then $V_j^\star(0) > 0$, so all other groups must spend 0 with positive probability. Let $k \neq j$ denote another group in A. Then k spends 0 with positive probability, so $V_k^\star = 0$. But this is a contradiction, as both j and k are A-groups but $V_j^\star \neq V_k^\star$. Thus $V_a^\star = V_j^\star = 0$ and the claim follows. ☐

a. Let $a \in A$ be an A group, $b \in B$ be a B-group and $c \in C$ be a C-group. For any group j, we can write define $p_j(x)$, the probability that j wins the lobbying subgame with an expenditure of x, as

$$p_j(x) = \prod_{k \neq j} F_k(x), \tag{8}$$

Suppose $\overline{x}_c > 0$. From lemma 3, for all $x \leq \overline{x}_c$, $V_c p_c(x) - x = 0$, or $V_c = \frac{x}{p_c(x)}$. Since $\overline{x}_c > 0$, $\overline{x}_b > \overline{x}_c$ so there exists a b with $V_b = \frac{y}{p_b(y)}$ for all $\overline{x}_c < y \leq \overline{x}_b$. This implies that

$$p_b(y) > \prod_{k \neq b} F_k(\overline{x}_c) > \prod_{k \neq c} F_k(\overline{x}_c) = p_c(\overline{x}_c) \tag{9}$$

or equivalently,

$$y > \frac{V_b}{V_c}\overline{x}_c \tag{10}$$

is always true. Since $V_b > V_c$, but y can be chosen to be arbitrarily close to $\overline{x}_c$, (10) is a contradiction.

b. From lemma 3, we know that when $|A| > 1$, all A-groups have $V_a^\star = 0$. Substitute an A-group for the B-group in the proof of part a., and substitute a B-group for the C-group in the proof of part a. The claim follows immediately.

c. The first part was proved in lemma 2. Suppose $|A| = 1$. Then the A-group will never spend more than V_b, since a could profitably deviate by reducing $\overline{x}_a$ towards V_b. Furthermore, no B-group would choose $\overline{x}_b < V_b$ since they could then increase $\overline{x}_b$ and increase their payoffs in expectation. As a result, $\overline{x}_a = \overline{x}_b = V_b$, and $\overline{x}_c = 0$ from part b. If $|A| > 1$, by similar logic $\overline{x}_a = V_a$, and $\overline{x}_b = \overline{x}_c = 0$ from parts a. and b. Hence, there is a common $\overline{x} = V$ for all groups that make expenditures with positive probability in equilibrium.

□

Proof. Proposition 2. I first prove that sufficiently small sets of inactive individuals hoping to join active groups will be welcomed by those active groups and then prove the two statements of the proposition.

Lemma 4. *Suppose there exists a set of (organized or disorganized) individuals of size z that would be better off in a different, active group j, and all subsets of size $\epsilon < z$ would also be better off in that group j. Then there exists an $z' \leq z$ such that a majority of the existing membership of j would be made weakly better off by any subset of size $\epsilon < z'$ joining the group.*

Proof. NEED TO FIX THIS, ONLY SHOWN IN CASE WHERE $u_i(q) = -|i-q|$. Denote the size of group j by s_j and the number of active groups by N_A. I assume that the set of individuals joining j is previously inactive in the lobbying game. (If these individuals were previous active, the argument below still holds, as this weakens the lemma.) There are two cases to consider. In the first case j is an A-group, and in the second case, $|A| = 1$ and j is a B-group.

Case 1. j is an A-group. I assume that $|A| > 1$ (the argument below holds if j is the sole A-group). It suffices to show that for small enough ϵ, the majority of the membership of j would be weakly better off by adding a distal subgroup of size ϵ. The enlargement of j affects the expected costs and benefits to j's membership. Consider a subgroup of j of size ϵ. The change in total costs to this subgroup due to enlargement is given by

$$\Delta C = \frac{\epsilon}{s_j}\left(\frac{x'}{s_j+\epsilon} - \frac{x}{s_j}\right) \tag{11}$$

where x is the the total expected expenditure by j before the enlargement and x' is the total expected expenditure by j after the enlargement. $x' \leq x$ since the enlargement can at worst reduce the valuation of the second highest group and reduce the total number of actively lobbying groups. Thus,

$$\Delta C \leq \frac{\epsilon^2 x}{s_j^2\left(s_j+\epsilon\right)} \tag{12}$$

To assess the change in expected benefits to j's membership, I need only consider how the enlargement affects $u_i\left(q_j^\star\right)$ for individuals $i \in j$ (see figure 4). For some individuals (a fraction $\frac{1}{2}-\frac{\epsilon}{4s_j}$ of j), $q_j^\star$ moves closer to their bliss-points, so they are unambiguously better off, and for the remainder, represented by the lightly shaded region in figure 4 , $q_j^\star$ moves farther from their bliss-points. For the "pivotal" fraction $\frac{\epsilon}{4s_j}$, represented by the darkly shaded region, the enlargement reduces their expected benefits by less than $\frac{\epsilon^2}{8(N_A-1)}$ (since they experience this reduction only when they win which now occurs with probability strictly greater than $\frac{1}{N_A-1}$. Of course, enlargement could increase their expected benefits.) Since the enlargement reduces their total costs by at least $\Delta C \leq \frac{\epsilon^2 x}{4s_j^2(s_j+\epsilon)}$ from (12) (note that $\Delta C < 0$), the total change in surplus of the pivotal group is

$$\Delta B - \Delta C > \left(\frac{\epsilon^2}{4}\right)\left(-\frac{1}{2\left(N_A-1\right)}+\frac{x}{s_j^2\left(s_j+\epsilon\right)}\right) \tag{13}$$

Ignoring the leading factor, $\Delta B - \Delta C > 0$ for small enough values of ϵ if

$$2x\left(N_A-1\right) > s_j^3 \tag{14}$$

where x, the expected amount that j would have spent on lobbying before enlargement is $x = V_j^2 \frac{N_A-1}{2N_A-1}$ as calculated from proposition 4. Also, $V > s_j - \frac{s_j^2}{2}$, which is the total utility that j's membership would receive from picking their favored policy, $U_j\left(q_j^\star\right)$, minus the best possible alternative policy being chosen (i.e., the policy at an endpoint of j's interval). Using these, results, (14), holds for the pivotal fraction of j, generating a (weak) majority that is in favor of enlargement.

Case 2. j is an active B-group, and $|A| = 1$. After enlargement, j will become the sole B-group, and there will only be two active groups. The cost reduction to a subgroup of j of size σ must still satisfy inequality (12). But now the "pivotal" fraction of j as defined in case 1 actually enjoys an *increase* in benefits from enlargement because j's probability of victory rises discontinuously as even the smallest enlargement forces all other previously classified B-groups to become inactive C-groups. Hence, there exists a majority of j in favor of enlargement.

□

a. I proceed by contradiction. Suppose that there is an equilibrium configuration of interest groups where one group is not connected almost everywhere. Then there must exist some region of the type space of the form in figure 5 where k is of positive measure.

Figure 4:

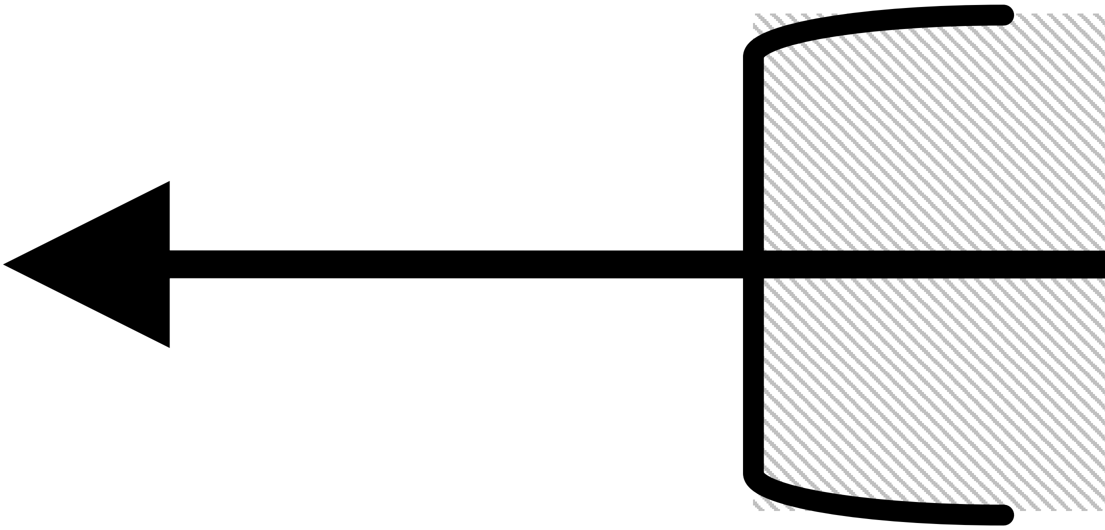
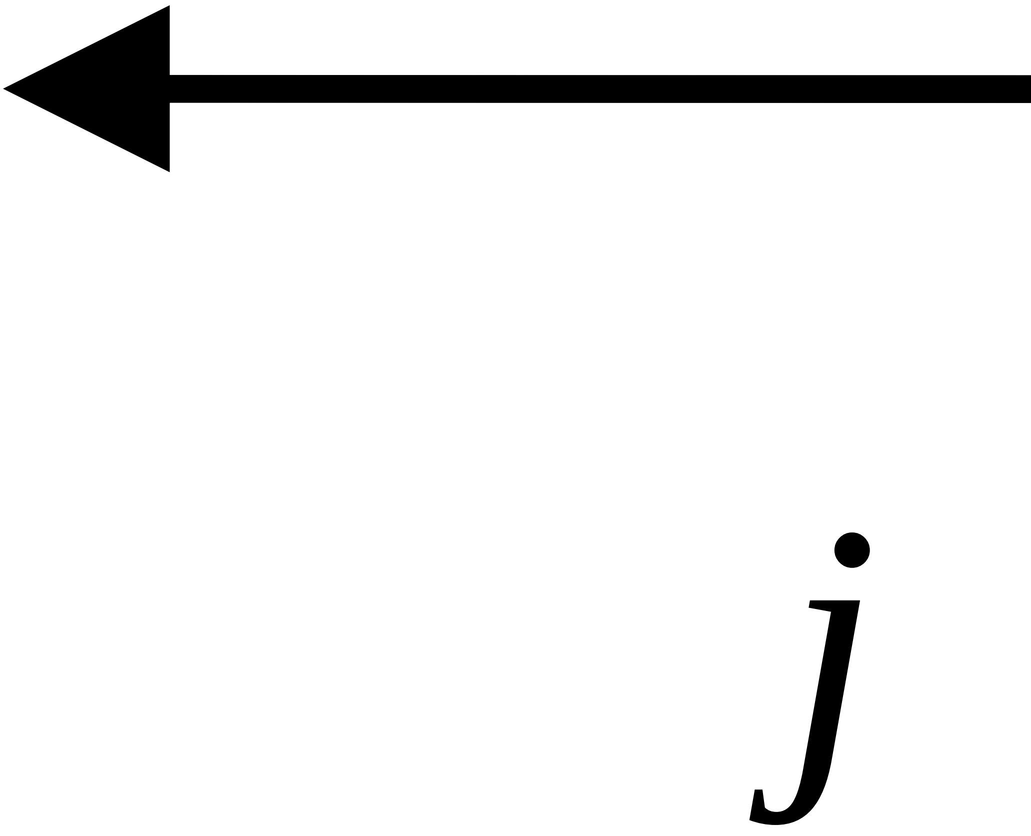


Figure 5:



Because the configuration of interest groups is an equilibrium, it follows that no distal subset of group j would be better off seceding and being disorganized. Define ΔB_g to be the difference in benefits to a small, distal subset of a group g between belonging to the group versus seceding from the group (and ΔC_g to be the analogous difference in costs). Consider the distal subgroups of j and distal subsets of k as indicated by the darkly and lightly shaded regions of figure 5 respectively. These subgroups can assumed to be of equal and arbitrarily small size $\epsilon > 0$ (since k is of positive measure). For the distal subgroups of j,

$$\Delta B_j - \Delta C_j > 0 \tag{15}$$

Would the distal subgroups of k prefer to join j? For all four combinations of the shaded distal subgroups of j and subsets of k, $\Delta C_k < \Delta C_j$. Increasing the group's size would spread lobbying costs over a larger constituency. For both distal subgroups of j, secession would entail a shifting $q_j^\star$ away from the subgroup's midpoint by $\frac{\epsilon}{2}$, and for at least one of the distal subgroups of j, secession would also (weakly) decrease their expected benefits in the event that j lost in the lobbying subgame. For that subgroup, $\Delta B_j \leq \frac{\epsilon}{2}$. A similar argument could be made that for a distal subgroup of k, $\Delta B_k \geq \frac{\epsilon}{2}$.

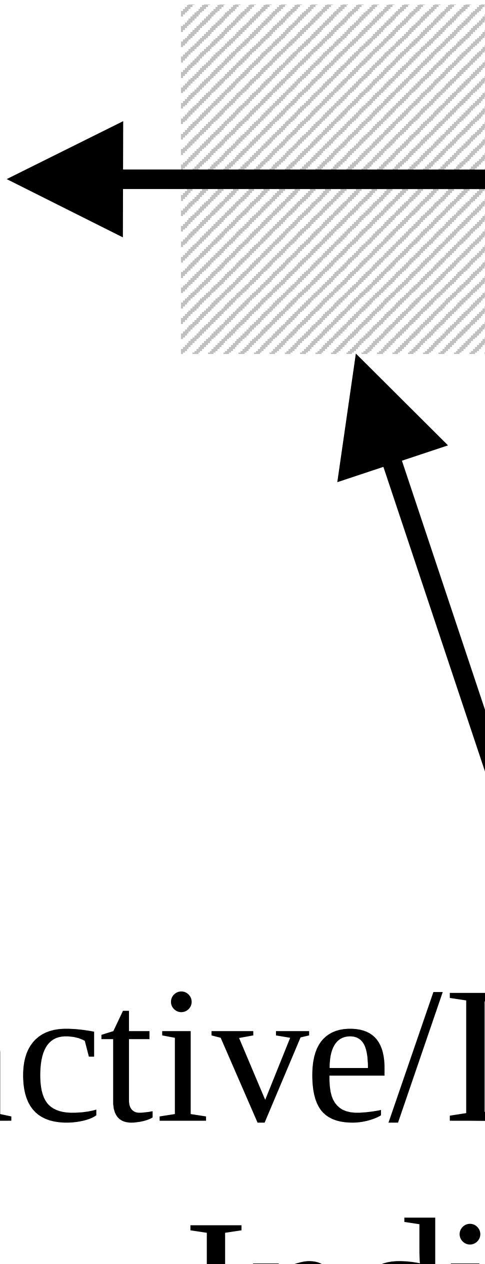
Since $\Delta C_k < \Delta C_j$ and $\Delta B_k > \Delta B_j$ for at least one combination of distal subgroups of j and subsets of k, equation 15 implies that $\Delta B_k - \Delta C_k > 0$. That is, one of the lightly shaded subgroups of k would be better off joining j. But this is cannot be the case in equilibrium. If the distal portions of k are comprised of disorganized individuals, then this is a violation of α- stability, and if the distal subsets of k are comprised of organized individuals, then this is a violation of β-stability. Hence, k must be of measure 0. This completes the proof.

b. This proof is identical in spirit to the proof of part a. Suppose in an equilibrium configuration of groups, we observe an interval of inactive individuals that is more extreme than active individuals (see figure 6). Consider an ϵ sized subgroup of inactive individuals just to the left of x. Let ΔB_ϵ be the excess benefits that individuals in this subgroup enjoy from being disorganized relative to being part of group j, and let ΔC_ϵ be the analogous excess costs.α-stability ensures that

$$\Delta B_\epsilon - \Delta C_\epsilon < 0 \tag{16}$$

Now consider an ϵ sized distal subgroup of j. Let ΔB_j be the excess benefits that indi-

Figure 6:



viduals in this subgroup would enjoy from becoming disorganized relative to being part of group j, and let ΔC_j be the analogous excess costs. Because the inactive individuals are more extreme than j, they place greater value on winning the lobbying auction (the alternatives are less appealing to them), hence $\Delta B_j < \Delta B_\epsilon$. Also, since lobbying costs are spread only over members of group j and not inactive individuals, $\Delta C_j > \Delta C_\epsilon$. These two facts coupled with inequality 16 imply

$$\Delta B_\epsilon - \Delta C_\epsilon < 0 \tag{17}$$

but (17) stands at odds with the fact that the configuration is γ-stable, which proves the claim.

□

Proof. Proposition 3. The proof proceeds as follows. First, I show that a configuration of all A-groups is α- and β-stable. I also show that γ-stability imply that there must exist several A-groups ($|A| > 2$). I then show that there exists a γ-stable configuration of all A-groups if $|A|$ is sufficiently small . This proves existence of equilibria. Without loss of generality, I assume $B = C = \emptyset$.

Claim 1. A configuration of groups is α-stable if all active interest groups are in set A.

Proof. Consider a configuration of N A-groups indexed by j of size s_j. Suppose a disorganized subgroup of size ϵ considers joining group 1. By joining the group, this subgroup will see its lobbying costs increase by

$$\Delta C_\epsilon = \frac{V_\epsilon}{V_1 + V_\epsilon} x_1 \tag{18}$$

where V_ϵ is the valuation the ϵ-subgroup places on winning, V_1 is the valuation that the remaining members of group 1 place on winning, and x_1 is the amount that group $_1$ expects to spend lobbying (all quantities computed *after* group 1 absorbs the ϵ-subgroup.) In big-Oh notation, $\Delta C_\epsilon > 0$ is of the order $\frac{O(\epsilon)}{O(\epsilon)+O(1)}$. The ϵ-subgroup will experience a change in benefits as well. By joining the group, j_1 will become the sole A-group, the group(s) that are farthest away from the ϵ-subgroup will become B-groups (since their utility from losing the lobbying game has decreased the most with j_1's primacy, their valuation for winning increases the most). The change in the ϵ-subgroup's benefits $\Delta B_\epsilon > 0$ will scale on the order of $O(\epsilon)$. Put together, $\Delta B_\epsilon - \Delta C_\epsilon < \frac{O(\epsilon^2)-O(\epsilon)}{O(\epsilon)+O(1)}$. For small enough ϵ, $\Delta B_\epsilon - \Delta C_\epsilon < 0$, hence this configuration is α-stable. □

Claim 2. A configuration of groups is β-stable if all active interest groups are in set A.

Proof. The logic of the proof this claim is identical to the proof of the previous claim. ϵ sized deviations from one group ("old") to a neighbor ("new") result in cost increases of $\Delta C_\epsilon = \frac{V_\epsilon}{V_{new}+V_\epsilon} x_{new} - \frac{V_\epsilon}{V_{old}} x_{old} = \frac{O(\epsilon)}{O(\epsilon)+O(1)} \min\left(x_{new}, x_{old}\right)$. Benefits to the ϵ-subgroup again scale on the order of $O\left(\epsilon\right)$, so for small enough ϵ, $\Delta B_\epsilon - \Delta C_\epsilon < 0$. ☐

Claim 3. In a γ-stable configuration, $|A| > 2$.

Proof. Suppose $|A| = 1$, and call this group a. It is of size s_a and spends x_a on lobbying in expectation. Consider a secession from the a of size ϵ that is small enough to maintain the classification of all groups. If this seceding subgroup becomes disorganized, they will experience a change in expected cost of $\Delta C_\epsilon = -\frac{V_\epsilon}{V_a} x_a$ (the negative sign indicates savings) which scales on the order of $\frac{O(\epsilon)}{O(\epsilon)+O(1)}$. However, they will also experience a decrease in expected benefits, since $q_a^\star$ will now move away from the subgroup by a distance proportional to ϵ. Multiplying this by the size of the seceding subgroup, the loss in benefits is of order $O\left(\epsilon^2\right)$. Hence there is a small enough ϵ to make secession optimal ($\Delta B_\epsilon - \Delta C_\epsilon > 0$). If $|A| = 2$, then equation the group experiencing the secession becomes the sole (active) B-group. The cost savings to the seceding group remain the same, and although the change in benefits to the seceding group change, they still are of order $O\left(\epsilon^2\right)$. ☐

Claim 4. A γ-stable configuration of groups exists.

Proof. Take a configuration of 3 A-groups with no disorganized members. Number the groups from left to right $i = 1 \ldots 3$. It suffices to consider whether a profitable deviation of an ϵ sized subgroup from the middle group (2) exists. Let u_i, $i = 1 \ldots 3$ represent the utility this subgroup enjoys if group i wins the lobbying game. Following from proposition 1, $V_i = V_j$ for all i, j, and each group spends $\frac{V_i}{3}$ on lobbying.
The ϵ subgroup will remain organized if

$$u_2 - \frac{1}{2}\left(u_1 + u_3\right) - \frac{V_\epsilon}{V_2} \cdot \frac{V_2}{3} \geq \frac{u_1 + u_3}{2} \tag{19}$$

where the expression of the left hand side is the benefits less the costs of remaining organized, and the expression on the right hand side is the benefits of being disorganized. Substituting in $V_\epsilon = u_2 - \frac{1}{2}\left(u_1 + u_3\right)$, the claim follows. ☐

☐

Proof. Proposition 4.
First, note that on any interval where F_j and F_k are increasing, $F_j = F_k$. This is the case because groups derive the same expected net payoffs from spending in this interval and their net valuations for winning are equal, i.e.

$$p_j(x) = \frac{x}{V_j} = \frac{x}{V_k} = p_k(x) \tag{20}$$

for all x in the interval. This symmetry result provides a simple method for computing groups' mixed strategies. For any active group j ,

$$p_j(x)\, V_j - x_j = 0 \tag{21}$$

at all points x. That is, the payoff to group j from spending x is equal to zero in expectation. p_j, the probability that x is the winning bid for group j, is equal to the probability that all other $N-1$ active groups spend less than x, or

$$p_j(x) = \prod_{k \neq j} F_k(x) \tag{22}$$

The claim follows from a substitution of (22) into (21). □

Proof. Proposition 5.
a. It suffices to show that X_2 is non-empty and part **b.** of the proposition holds. I postpone the proof of part **b.** and show that X_2 is non-empty. Denote the types of the two individuals L and R where $L < R$ by assumption. Without loss of generality, let $L - R = 1$. Consider the trivial configuration of interest groups where L is a group of size 1 and R is a group of size 1. It suffices to show that this configuration is β- and γ-finite stable.

Claim 1. This configuration is β-finite stable.

Proof. Suppose L defected and opted to join R. R would not accept this enlargement, because by rejecting it, R would be the sole organized individual. More concretely, R's total payoff would decrease from 0 (selecting policy at 1, R's blisspoint, and not spending on lobbying) to $-1/2$ (selecting policy at $1/2$, a compromise, and not spending on lobbying). □

Claim 2. This configuration is γ-finite stable.

Proof. If L became inactive in the lobbying subgame, then this would be equivalent to playing the pure strategy $x_L = 0$. But L is already playing a Nash equilibrium strategy in the lobbying subgame, hence, this cannot represent a profitable deviation. More concretely, the total payoff to L by being active is equal to -1 (L and R each win with probability $1/2$, generating benefits to L of 0 and -1 respectively, and L bears an expected lobbying cost of $1/2$, while the total payoff to L from being inactive is also equal to -1 (L receives a payoff of -1 with certainty and spends 0 on lobbying.) ☐

b. Take an equilibrium $x \in X_I$, and add an individual i. If this configuration is still α- and δ-finite stable, then $x \in X_{I+1}$, which proves the claim.
First, note that if there are any disorganized individuals or inactive groups in x before adding i, then the claim is trivially satisfied. Second, note that the greatest potential net benefit that individual i could enjoy must occur in the case where there are only two organized groups, and each group is of minimal size (i.e., singleton). Consider the trivial configuration of groups in part **a.** along with a third disorganized individual. It suffices to show that this configuration is α- and δ-stable.

Claim 1. This configuration is α-finite stable.

Proof. It suffices to show that an individual of type $i = L$ will not want to join L. Before enlargement, i's utility is equal to $-1/2$ (L and R each win with probability $1/2$, generating benefits to i of 0 and -1 respectively). After enlargement, i's utility is equal to $-1/2$ (After enlargement, L wins with probability $3/4$ and R wins with probability $1/4$, generating an expected benefit of $-1/4$, and i bears an expected lobbying cost of $-1/4$). Hence, i is not made better off by joining L. For any $i \neq L$, i actually experiences a decrease in utility from joining L. ☐

Claim 2. This configuration is δ-finite stable.

Proof. Take any individual $i \in (L, R)$ who is considering forming a new group. After forming a new group, i's valuation for winning will be lower than $V_L = V_R$ since losing is less costly to her (she is relatively moderate). Hence, i will be inactive, and this is not a profitable deviation.
Now suppose $i = L$. Before forming a new group, i's utility is equal to $-1/2$, and after forming a new group, i's utility is still equal to $-1/2$ (this situation is arithmetically equivalent to the enlargement in claim 1). This does not represent a profitable deviation. ☐

c. Pick an $x \in X_\infty$. By construction, there exists a z that satisfies the four conditions of definition 1 for this x. Consider all distal subgroups of size z. Because of the density of the rational numbers, a setΛ can be constructed which contains two rational numbers that from each distal subgroup. Let λ be the least common multiple of the denominators of the elements of Λ, and choose I equal to the smallest integer greater than $1/\lambda$. Then $x \in X_I$. Intuitively, I uniformly spaced individuals form a sufficiently fine representation of the continuum of individuals.
d. Parts **a.** and **b.** imply that the trivial configuration of two extreme singleton groups and $I-2$ disorganized individuals is an equilibrium for all I. But this equilibrium cannot be an element of X_∞, since proposition 3 implies $N > 2$ for all $x \in X_\infty$. □

Proof. Proposition 6.
The proof of the first claim is analogous to the proofs of 1-3.

When $C\left(\bar{V}\right) = 0$, the lobbying subgame is equivalent to a first price auction with public valuations. In such an auction with $N > 2$ A-groups, each with valuation V, there is a unique Nash equilibrium. All groups bid V and win with equal probability. Clearly, no group has an incentive to deviate.

It is easy to show that there are no symmetric mixed strategy Nash equilibria in this game. Let $\underline{x}_j$ and $\bar{x}_j$ represent the lower and upper bounds of the support of group j 's mixed strategy. Then $\bar{x}_j$ must equal V. If not, this strategy would be strictly dominated by group k spending $\bar{x}_j$ with probability 1. Since the payoff to j when bidding $\bar{x}_j$ is always zero, it follow that j gets a payoff of zero at every action in the support of his mixed strategy. Symmetry requires $\underline{x}_j = \bar{x}_j = V$. □

Proof. Proposition 7.
The proofs of parts a. and d. are identical to the proofs presented above where $D = 1$. Obvious modifications to the proof of proposition 2 yield parts b. and c. □