Natural Catastrophe Insurance: How Should Government Intervene?*

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Abstract

The present paper develops a new theoretical framework for analyzing the decision to provide or buy insurance when natural risks are in play. In contrast with conventional models of insurance, the insurer has a non-zero probability of default which depends on the distribution of the risks, the premium rate, and the amount of capital in the company. Among several results, we show that risk-averse policyholders will accept to pay higher rates for an insurance with unlimited guarantee from the government. However, depending on the correlation between and within the regional risks, a governmental program can be more attractive to high-correlation areas than it is to low-correlation areas, which may lead to inefficiencies if the insurance ratings are not chosen appropriately.

JEL-Classification: G22, G28

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1. Introduction

According to many observers of the environment, both the frequency and strength of natural catastrophes such as hurricanes, floods and droughts have raised during the past few years (IPCC, 2007). Among other consequences, this trend in the number of hazardous events could endanger the viability of the insurance and reinsurance industry. Not only global weather-related insurance losses from large events have escalated from a negligible level in the 1950s to an average of $9.2 billion per year in the 1990s (Mills et al., 2001), but there have been also an increasing number of insolvencies: between 1969 and 1998, 36 US insurers became insolvent primarily as a result of catastrophe losses. Of these companies, 20 became insolvent between 1989 and 1993, the same time period as Hurricane Hugo (Matthews, 1999). The present paper aims to investigate these failures by developing a model of natural catastrophe insurance market and evaluating the need for government intervention.

Examples of public intervention can be found in several countries (e.g., US, France, Italy, Spain, Switzerland, Japan, New Zealand, Canada, Finland, Norway). In the US, the federal government and some States have developed governmental programs that supplement or substitute for private natural catastrophe insurance. These programs were successful at first, but they ultimately ended in large accumulated deficits, as exemplified by the $810 million deficit seen in the US flood insurance program in the mid-1990s (Mills et al., 2001; GAO, 2007). Another example is the French CAT-NAT system where private insurance companies have the possibility to cover for themselves from a public-policy reinsurer, the CCR. If it is not possible for the CCR to pay for all the damages reinsured, the CCR has to call for the guarantee of the government. It happened once in year 2000 for a total amount of 3 billion Francs (457 million Euros), because of severe flooding in the South, and the two storms of December 1999.[1]

One common characteristic of public programs is that government officials are faced with balancing the goals of ensuring a zero default risk and the demand for a limited tax exposure. This generally creates a discussion based on one single question: should taxpayers have a preference for government intervention? On the one hand, with a purely private market, only policyholders at risk have to deal with their insurer's insolvency. On the other hand, with a public program, policyholders participate to a collective sharing practice based on solidarity from the taxpayers. To our knowledge, no formal study has ever been undertaken to compare these two possible alternatives. The reason is that several assumptions have to be relaxed if we want to apply the existing models of insurance to natural disasters. In particular, natural risks are correlated, i.e., may imply a considerable number of claims at the same time. As a result, an insurer may have a non-zero probability of ruin depending on (1) the distribution of the risks (Kunreuther, 2001), (2) the premium rate (Tapiero et al., 1986), and (3) the amount of capital in the company (Cummins, 2006; Charpentier, 2007). The present article addresses this issue through an original theoretical framework taking into account all these dimensions.

Another issue with respect to natural disasters is the importance of risk diversification. Large insurance companies can pool the risks with independent risks from other regions,

[1] A survey about the different natural catastrophe insurance programs can be found in von Ungern-Sternberg (2004) and GAO (2005).

which can significantly reduce the probability of ruin (Cummins, 2006; Charpentier, 2007). According to this view, because it is the largest entity in a given jurisdiction, the government would be the most effective mechanism for spreading risks and losses broadly (Priest, 1996). A question that arises, however, is whether taxpayers from less risky regions are willing to show solidarity with taxpayers from riskier regions. This question is of high interest if we want to build a federal insurance program that is politically viable in the long run. To answer this, we extend the framework by focusing on a simultaneous non-cooperative game combining two regions with heterogeneous natural risks. The idea is to introduce pecuniary externalities in the model, via the default risk, so that the willingness to pay of one region will be affected favorably or unfavorably by the participation of the other.

Our results and policy implications are twofold. First, we show that the provision of natural catastrophe insurance by a free market is not necessarily the efficient solution: an insurance with unlimited guarantee from the government proves to be a mean preserving spread of a limited liability insurance. Put simply, public programs allow to spread the risks equally among the policyholders and, therefore, prove to be less risky and more attractive in terms of expected utility. Second, we show that the viability of a federal program strongly depends on the correlation between and within the regional risks. In particular, a federal program can be more attractive to correlated regions than it is to non-correlated regions, leading to inefficiencies if the insurance ratings are not chosen appropriately.

The outline of the research is as follows. Section 2 relates the main assumptions of the article to those of the existing literature. Section 3 develops the one-region model. Section 4 extends the theoretical framework to a two-region economy. Section 5 tests the robustness of the models with respect to nonindependence modeling. Section 6 discusses in detail the policy implications of the models. Section 7 concludes.

2. Literature review

Conventional models of insurance do not apply to natural risks for many reasons. Among them, the following four seem particularly important:

1. Natural risks invalidate the Law of Large Numbers. Standard insurance theories are generally based on the fundamental theorem of probability known as the Law of Large Numbers. This theorem suggests that, given a sufficiently large number of policyholders, the share of people claiming a loss should converge toward a predictable average. Unfortunately, catastrophes are generally not considered an insurable risk because the frequency and the magnitude of the losses cannot be forecasted accurately using this fundamental theorem (Berliner, 1982). Consider for instance an insurance company whose claims are partitioned into (1) claims due to isolated events, and (2) claims due to natural disasters. In that case, several possible states of nature could be observed depending on the occurrence or not of a natural catastrophe. This idea of a range of possible states will be formally developed in the present paper.

2. Natural risks imply a default risk. In practice, natural catastrophes may jeopardize not only the insurer's insolvency, but also financially threaten the policyholders whose claims could be not fully paid. To our knowledge, only a few papers directly consider the effects of a default risk on the demand and supply for insurance (see Tapiero et al., 1986; Doherty and Garven, 1986; Schlesinger and Schulenburg, 1987; Johnson and Stulz, 1987 and Doherty and Schlesinger, 1990). The main idea of these models is to incorporate a default risk into the insurance-pricing decision and the expected utility of purchasers. For example, Doherty and Schlesinger (1990) assume that the insurer will be solvent or insolvent with some probability, conditional on the occurrence of a loss. Unfortunately, this solvency probability is assumed to be exogenous in the model and does not depend on the level of the insurance premiums or the capital of the company. In Tapiero et al. (1986), the default risk depends on the premium rate, but claims occurrence is driven by a Poisson process (as in standard actuarial models) which makes impossible the occurrence of two or more claims at the same time. Our article seeks to fill these voids by considering an endogenous probability of ruin in the context of correlated risks.

3. The natural catastrophe insurance market is not perfectly competitive. When the risks are uncorrelated, the economic capital per policy required by an insurance company approaches zero as the number of insured risks approaches infinity. This result implies that insurance companies are more competitive and propose lower premiums as their number of risks increases. Because of these decreasing returns to scale, the insurance industry can be said to be a *natural monopoly* (Emons, 2001) or at least, can be characterized as having an oligopolistic market structure (Cummins and Zi, 1998; Sonnenholzner and Wambach, 2004; Chiappori et al., 2006). This is all the more true when the losses are correlated. In that case, some amount of capital per policy is required even if the number of risks approaches infinity (Cummins, 2006; Charpentier, 2007). Hence, only large companies might offer catastrophe coverage, because they have an easier access to capital and can pool the risk with independent risks from other regions. This is why the present research will also deal with monopolistic behaviors (i.e., price-maker insurers), by opposition to Rothschild and Stiglitz (1976) for instance who focus only on a competitive insurance market.

4. Adverse selection theories do not apply to natural risks. In Rothschild and Stiglitz's (1976) model of adverse selection, the assumption is made that the insurer cannot distinguish between low-risk and high-risk individuals. Hence, if insurance is available at the same price to all customers, individuals with the greatest risks are more likely to buy insurance than low-risk individuals. If this literature has found support in many empirical studies (see for instance Wang et al., 2009 for an updated survey about insurance markets), several authors recognize that adverse selection theories may not be suited to the analysis of natural catastrophe insurance. The reason is that information asymmetry could be all the way around (Kunreuther, 1984; Jaffee and Russell, 1997). While insurance companies have better access to information using predicting and risk-spreading techniques to rate the catastrophe exposures, the way citizens construct their probabilities when faced with high-loss/low-probability events may be distorted. Therefore, we will not include theoretical aspects of adverse selection in the present study.

3. The one-region model

Consider a region with a population of n inhabitants. This region is exposed to natural events which can cause a loss l to N individuals. Following the idea that the Law of Large Numbers is invalidated and that several states of nature can be observed (Feature 1 of the literature review), the focus is on the random variable $X = \frac{N}{n}$ defined on the interval $[0,1]$. Two elements will shape the distribution of X: first, the probability p for each individual to claim a loss and, second, the correlation δ between the individual risks. The coefficient δ can be said to determine the magnitude of natural catastrophes — i.e., the total number of people that will be claiming a loss at the same time — and will influence in return the chances for the insurance industry to go bankrupt. On the other hand, the probability p will represent the odds for each individual to be one of the victims.[2]

More specifically, the occurrence of X is based on the following increasing continuous probability distribution:

$$F = F(x|p,\delta) = F(x) = \int_0^x f(t)dt \in [0;1], \tag{1}$$

with:

$$\begin{array}{llllll} (i) & \int_0^1 xf(x)dx = p, & (ii) & \frac{\partial F}{\partial p} < 0 \quad \forall x \in [0;1], & (iii) & \frac{\partial^2 F}{\partial p^2} > 0 \; \forall x > x^*, \\ (iv) & \frac{\partial F}{\partial \delta} < 0 \quad \forall x > x^*, & (v) & \frac{\partial^2 F}{\partial \delta^2} > 0 \quad \forall x > x^*, & (vi) & \frac{dp}{d\delta} = 0, \end{array} \tag{2}$$

where any $x > x^*$ characterizes an extreme event, e.g., earthquakes, floods, droughts. The above assumptions may be interpreted as follows. (i) My chances of claiming a loss are directly related to the number N of losses in the population. Consequently, the probability p is also the expectation of X. (ii and iv) The higher the probability p and the correlation δ, the higher the occurrence of extreme events.[3] (iii and v) The probability of an event x increases with p and δ if $x > x^*$. (vi) A variation in δ has no impact on p.

For simplicity of exposition, the inhabitants will be strictly identical with same preference $U = U(Y)$, where Y denotes a negative random loss that depends on the state of nature. The function U is assumed to be differentiable and increasing, with $U(0) = 0$. The inhabitants of the region will decide simultaneously whether or not to pay full insurance coverage so as to maximize their expected utility.

3.1. Supply of insurance

Insurance coverage is provided by a single company. Following Einav et al. (2010), we take the characteristics of the insurance contracts as given: only the pricing of the contracts are

[2] A simple way to understand these concepts is to consider a nationwide coin-flipping game. Let X denote the share of tails in the economy. If all inhabitants play with their own coin, the probability of getting tail will be $p = 50\%$ for each individual, and we will expect to see an equal distribution of heads and tails in the country. In contrast, if we all bet on the toss of a single and unique coin, the nationwide outcome will be either only tails or heads, while the individual probability of having tail will still be $p = 50\%$. The number of coins in such a game is an inverse proxy for how the risks are correlated.

[3] A high correlation between risks should be also associated with a higher occurrence of small events, i.e., $\frac{\partial F}{\partial \delta} > 0$ for low values of x. This assumption is however not necessary for our analysis.

determined endogenously, not the offered coverage. The value of the premium inhabitants have to pay is denoted by α. The economic capital per policy held by the company is denoted by c. The insurer becomes insolvent when it is not possible to pay the full coverage l to the victims anymore, i.e., when the total losses (Nl) become higher than the total revenue ($n\alpha$) and the total economic capital (nc). The probability of ruin is:

$$\mathbb{P}(Nl > n\alpha + nc) = \mathbb{P}\left(X > \frac{\alpha + c}{l}\right) = 1 - F(\bar{x}), \tag{3}$$

where $\bar{x} = (\alpha + c)/l$ denotes the largest possible event without default. In comparison with Doherty and Schlesinger (1990), this probability of ruin is not exogenous (Feature 2 of the literature review). The higher the premium and the capital per head, the higher $\bar{x}$ and the lower the probability of ruin.

The insurance company is assumed to choose the premium α so as to maximize expected profit. If ruin does not occur, the profit is equal to the total revenue ($n\alpha$) minus the total losses claimed by the insured ($Nl = xnl$). In that case, the profit can be positive or negative depending on the amount of losses. On the other hand, if ruin does occur, the company is assumed to equally distribute the capital to the victims, i.e., the profit is always negative and equal to $-cn$. The expected profit can be written as:

$$\Pi(c, \alpha, p, \delta) = \int_0^{\bar{x}} [n\alpha - xnl] f(x)dx - [1 - F(\bar{x})]cn. \tag{4}$$

The company will provide insurance if and only if $\Pi(c, \alpha, p, \delta) > 0$. It can be shown that an increase in the economic capital (c) will lead to a decrease in the expected profit (Π) (see Proposition 1). For a given premium, the insurance company is better off without any capital because only the total revenue ($n\alpha$) can be lost in that case. On the other hand, the higher the capital per head, the more the shareholders are exposed to industry failure. In contrast, an increase in α will lead to an increase in Π. As a result, maximizing the expected profit with respect to α is equivalent to minimizing the probability of ruin.[4] Therefore, we can focus indifferently on a risk-neutral insurance company that maximizes expected profit or a risk-averse company that minimizes the default risk.

Proposition 1. *From the expected profit* Π*, we obtain the following comparative static derivatives (for* $\bar{x} \in [0;1]$*):*

$$\frac{\partial \Pi}{\partial c} < 0, \qquad \frac{\partial \Pi}{\partial \alpha} > 0.$$

Proof. See Appendix. □

3.2. Demand for insurance

The inhabitants' payoffs Y are provided in Table 1. Their value depends on whether insurance coverage is provided by an insurance company with limited liability or with unlimited guarantee from the government.

[4]in Equation 3, an increase in α leads to an increase in $\bar{x}$, and to a decrease in $1 - F(\bar{x})$

Table 1. *Comparison of final losses Y.*

	Individual claiming no loss (with a probability $1-p$)		**Individual claiming a loss** (with a probability p)	
	Insurance	No insurance	Insurance	No Insurance
Limited liability	$-\alpha$	0	$-\alpha-l+I(X)$	$-l$
Unlimited guarantee	$-\alpha-T(X)$	0	$-\alpha-T(X)$	$-l$
No default risk	$-\alpha$	0	$-\alpha$	$-l$

Scenario with limited liability. The payoffs are determined by both the eventuality of a loss and the decision to buy insurance. If an individual does not claim a loss, the payoff is $-\alpha$ with insurance, and 0 without. If an individual is victim of a loss, the payoff is $-\alpha-l+I(X)$ with insurance and $-l$ without, where $I(X)$ represents the indemnity. Figure 1 provides an illustration. If the insurer is not insolvent ($X \leq \bar{x}$), the indemnity fully covers the loss, i.e., $I(X)=l$. On the other hand, in case of ruin ($X > \bar{x}$), the indemnity received by each victim is equal to the total economic capital (nc) plus the company's revenue ($n\alpha$), divided by the number of policyholders claiming a loss, i.e., $I(X)=\frac{c+\alpha}{X}$. The expected utility of a policyholder can be written as:

$$V(c,\alpha,p,\delta)=\int_0^1 xU(-\alpha-l+I(x))f(x)dx+\int_0^1(1-x)U(-\alpha)f(x)dx, \tag{5}$$

The first integral represents the expected utility of a policyholder conditional to the probability x of being one of the victims. In that case, the risk of a reduced coverage, i.e., $-l+I(x)$, has to appear in the computation. The second integral represents the expected utility conditional to the probability $(1-x)$ of no loss. Given the properties of F (Equation 2), this integral is equal to $(1-p)U(-\alpha)$, i.e., a policyholder will loose only α with a probability $1-p$.

Scenario with unlimited guarantee. If ruin occurs ($X > \bar{x}$), the government will ask all the policyholders to pay an additional tax $T(X)$ in order to cover the default of payment, i.e., $T(X)=X[l-I(X)]=Xl-\alpha-c$. If the insurer is not insolvent ($X \leq \bar{x}$), the policyholders will not pay any taxes, i.e., $T(X)=0$. Figure 1 provides an illustration. The final payoff Y is always equal to $-\alpha-T(X)$. The expected utility becomes:

$$V(c,\alpha,p,\delta)=\int_0^1 U(-\alpha-T(x))f(x)dx. \tag{6}$$

Standard insurance theories presuppose a zero default risk. In our settings, this corresponds to the situation where $I(X)$ always covers the loss l. In that case, a policyholder will reach an expected utility equal to $pU(-\alpha)+(1-p)U(-\alpha)$ which is necessarily higher than $V(c,\alpha,p,\delta)$ in Equations 5 and 6 (see the third row of Table 1 for a comparison of the payoffs). The presence of a default risk is consequently detrimental to our agents.

Formally, it can be shown that a decrease in c and an increase in δ and p will diminish V (See Proposition 2). In addition, the premium yields a negative impact on V with one exception, however, in the limited liability scenario: if U is concave (i.e., inhabitants are risk-averse), an inflection point may exist when α is low and the probability of ruin is high

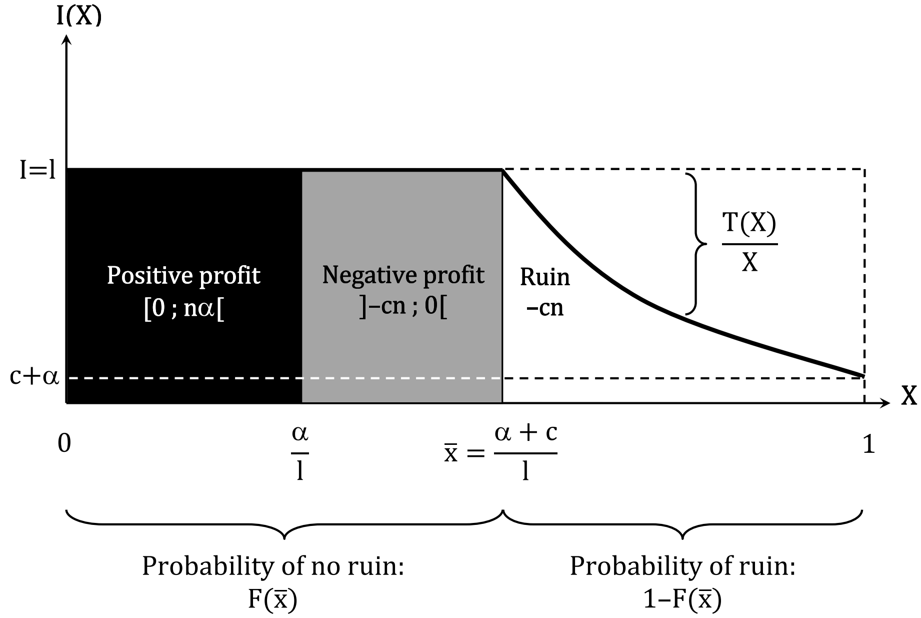

Figure 1. *Relationship between indemnity I, profit* Π *and tax* T.

($\bar{x} < \hat{x}$, where $\hat{x}$ denotes the inflection point). In that case, the policyholders can benefit from an increase in the premium since it will significantly reduce the probability of ruin.

Proposition 2. *From the expected utilities* V*, the one-region model leads to the following comparative static derivatives:*

$$\begin{aligned}
&\textit{Both scenarios:} && \frac{\partial V}{\partial c} > 0 \ \textit{for} \ \bar{x} \in [0;1], \ \frac{\partial V}{\partial \delta} < 0 \ \textit{and} \ \frac{\partial V}{\partial p} < 0 \ \textit{for} \ \bar{x} > x^*, \\
&\textit{Limited liability:} && \frac{\partial V}{\partial \alpha} \lesseqgtr 0 \ \textit{for} \ \bar{x} \gtrless \hat{x}, \ \textit{if} \ U \ \textit{concave}, \\
&\textit{Unlimited guarantee:} && \frac{\partial V}{\partial \alpha} < 0 \ \textit{for} \ \bar{x} \in [0;1].
\end{aligned}$$

Proof. See Appendix. □

Without insurance, the expected utility depends only on the probability p of loss and is defined as $pU(-l) + (1-p)U(0) = pU(-l)$. Consequently, an agent will buy insurance if and only if $V(c, \alpha, p, \delta) \geq pU(-l)$. Let denote α^* the maximum premium the inhabitants will be willing to pay for an insurance contract, i.e., such that $V = pU(-l)$. It can be shown that α^* increases with the capital (c) and decreases with the correlation (δ). The reason is that an increase in c, and a decrease in δ, will decrease the probability of ruin (from Equations 2 and 3) and increases the expected utility V (Proposition 2). This result gives support to Tapiero et al. (1986) who also evidenced a negative relationship between default risk and the premium a policyholder is willing to pay.

Proposition 3. *From the willingness to pay α^*, the one-region model leads to the following comparative static derivatives:*

$$\frac{\partial \alpha^*}{\partial c} > 0 \text{ for } \bar{x} \in [0;1], \qquad \frac{\partial \alpha^*}{\partial \delta} < 0 \text{ for } \bar{x} > x^*.$$

Proof. See Appendix. □

Notice that the difference between the scenarios comes from the expected utility V (Equations 5 and 6). Consider a given value of X, say x. With limited liability, a policyholder will have a x percent chance of being one of the victims with a payoff $Y = -\alpha - l + I(x)$, and a $(1-x)$ percent chance of getting $Y = -\alpha$. Let $A = \{-\alpha - l + I(x), x; -\alpha\}$ denote this lottery. With unlimited guarantee, a policyholder will always receive a payoff $Y = -\alpha - T(x)$ no matter what. The lottery can be defined as $B = \{-\alpha - T(x), 1\}$. Because A and B have the same mean, A can be said to be the mean-preserving spread of B. As a result, B will be preferred by all EU maximizers having concave utility (Rothschild and Stiglitz, 1970). Notice now that V is derived from a combination of A, or B, with a third lottery f. From the *Independence Axiom* of the von Neumann-Morgenstern Expected Utility Theory, the preference between A and B will be unaffected. The policyholders are consequently better off with government intervention, which implies that their willingness to pay will be higher too, as shown in Proposition 4.

Proposition 4. *Ceteris paribus, the expected utility V is higher in the unlimited-guarantee scenario if the policyholders are risk-averse. As a result, the unlimited-guarantee scenario leads to a higher willingness to pay (α^*) than the limited-liability scenario.*

Proof. See Appendix. □

3.3. Market equilibriums

Let us first consider the possibility of a non perfectly competitive market (Feature 3 of the literature review). If the company is a price-maker, the equilibrium will be characterized by a premium such that (1) the insurance company offers the catastrophe coverage to maximize expected profit (or equivalently to minimize the probability of ruin), and (2) inhabitants choose insurance to maximize expected utility.

Since $\partial \Pi / \partial \alpha > 0$ (from Proposition 1), the insurance company will choose the highest possible premium such that inhabitants buy insurance, i.e., such that $V \geq pU(-l)$, subject to the constraint that $\Pi \geq 0$. In other words, the insurer will set the premium equal to the policyholders' willingness to pay. The equilibrium outcome in terms of expected utility will be the same in both scenarios: each individual will reach a utility level equal to $pU(-l)$, with and without insurance coverage. Given Proposition 4, if the inhabitants are risk-averse, the insurer will be better off with unlimited guarantee from the government.

What are the impacts of δ and c in that case? As for δ, a decrease in the correlation will generate a higher expected utility (from Proposition 2), which will allow the company to increase the price of the contract (Proposition 3) and to reach higher expected profits (Proposition 1). The impact of the capital is more ambiguous. On the one hand, an increase in

Table 2. *Pecuniary externalities in the two-region model.*[a]

Region 1's strategy	Region 2's strategy	Probability of ruin	Indemnity in case of ruin: $I(X)$	Additional tax in case of ruin $T(X) = X[l - I(X)]$
Insure	Insure	$1 - F_0(\bar{x}_0)$	$\frac{n_1(c+\alpha_1)+n_2(c+\alpha_2)}{N_0}$	$X_0 l - \frac{n_1(c+\alpha_1)+n_2(c+\alpha_2)}{n_0}$
Insure	Don't	$1 - F_1(\bar{x}_1)$	$\frac{c+\alpha_1}{X_1}$	$X_1 l - c - \alpha_1$
Don't	Insure	$1 - F_2(\bar{x}_2)$	$\frac{c+\alpha_2}{X_2}$	$X_2 l - c - \alpha_2$
Don't	Don't	0	0	0

[a] with $\bar{x}_0 = \frac{n_1(c+\alpha_1)+n_2(c+\alpha_2)}{(n_1+n_2)l}$, $\bar{x}_1 = \frac{c+p_1}{l}$ and $\bar{x}_2 = \frac{c+\alpha_2}{l}$.

c will lead to an increase in the exposition of shareholders to industry failure (Proposition 1). On the other hand, according to Proposition 3, the capital per head also has an indirect positive impact on Π. An increase in c will lead to an increase in the policyholders' willingness to pay and, as such, will generate an increase in the expected profit via the premium rate ($\partial\Pi/\partial\alpha > 0$ in Proposition 1). Therefore, the decision for a company to provide insurance coverage for natural risks depends on the demand sensitivity to insurers' financial-strength. If the demand is inelastic, insurance companies are better off without any capital. On the other hand, if the demand is sensitive, a large company may reach a higher expected profit as the capital increases.

Our results can be easily extended to a price-taker insurance company. The optimal choice from the point of view of the policyholders would be the lowest possible premium such that the expected utility V is maximized subject to the constraint that the industry faces at least a positive expected profit (or a probability of ruin sufficiently small in the case of a risk-averse company). This optimum can be reached either through market regulation (if the policy-makers have sufficient information) or by the market itself (if the market is sufficiently competitive). In any case, the policyholders will be better off with (1) government intervention (if the inhabitants are risk-averse), (2) higher capital requirements (if the demand is sensitive to insurers' financial-strength) and (3) a lower correlation between risks.

4. Extension to a two-region economy

4.1. New theoretical framework

Consider now an economy composed of two populations n_1 and n_2 living in two different jurisdictions, labeled Region 1 and Region 2, respectively. Each region is exposed to natural events which can cause a loss l to N_i inhabitants in Region i, $i = 1,2$. We are interested in the stochastic process (X_1, X_2), with $X_i = N_i/n_i \in [0;1]$. The share of victims in the total population is denoted by $X_0 = \frac{N_1+N_2}{n_1+n_2}$. The states of nature are based on the following

		Region 2	
		Insure	Don't
Region 1	Insure	$V_1(c,\alpha_1,\alpha_2,p,\delta_1,\delta_2,\theta), V_2(c,\alpha_1,\alpha_2,p,\delta_1,\delta_2,\theta)$	$V_1(c,\alpha_1,p,\delta_1), pU(-l)$
	Don't	$pU(-l), V_2(c,\alpha_2,p,\delta_2)$	$pU(-l), pU(-l)$

Table 3. *Payoffs matrix.*

probability distributions:

$$\begin{aligned} X_1 &\sim F_1(x_1|p,\delta_1) = F_1(x_1), && (7)\\ X_2 &\sim F_2(x_2|p,\delta_2) = F_2(x_2), && (8)\\ X_0 &\sim F_0(x_0|F_1,F_2,\theta) = F_0(x_0|p,\delta_1,\delta_2,\theta) = F_0(x_0), && (9) \end{aligned}$$

where F_1 and F_2 are assumed to satisfy the properties described in Equation 2. For simplicity, the probability p to claim a loss is the same in both regions. The correlation between risks (referred to as *within-correlation*, hereafter), δ_1 for Region 1 and δ_2 for Region 2, are allowed to be different. The function F_0 is directly derived from F_1, F_2 and depends on a new parameter, the *between-correlation* (θ) which defines how the risks are correlated between the regions. We have:

$$\begin{array}{llll} (i)\ \frac{\partial F_0}{\partial p} < 0 \quad \forall x \in [0;1], & (ii)\ \frac{\partial^2 F_0}{\partial p^2} > 0 \quad \forall x > x^* & (iii)\ \frac{\partial F_0}{\partial \delta_i} < 0 \quad \forall x > x^*, \\ (iv)\ \frac{\partial^2 F_0}{\partial \delta_i^2} > 0 \quad \forall x > x^*, & (v)\ \frac{\partial F_0}{\partial \theta} < 0 \quad \forall x > x^*, & (vi)\ \frac{\partial^2 F_0}{\partial \theta^2} > 0 \quad \forall x > x^*, \end{array} \qquad (10)$$

for $i = 1,2$. The above assumptions may be interpreted as follows. (i, iii and v) The occurrence of extreme events in the economy increases with p, δ_1, δ_2 and θ. (ii, iv and vi) The probability of an event x increases with p, δ_1, δ_2 and θ if $x > x^*$.

There is no mobility between the regions, i.e., the natural events are sufficiently rare that people do not change their place of residency so as to reduce their expected losses. Instead, the inhabitants of each region have to decide simultaneously whether or not to buy insurance coverage. The citizens only differ in their location and, therefore, in their probability distributions (F_1 or F_2), which implies by symmetry that the decision to insure or not will be identical for the individuals of the same region. In other words, it is as if only two agents were in play. We will be referring to these agents as Region 1 and Region 2. The premiums they will pay are symbolized by α_1 and α_2, respectively.

The settings can be written in terms of a two-player non-cooperative game. The four possible outcomes are presented in Table 2. The set of actions available to Regions 1 and 2 is {Insure, Don't}. The expected utilities are similar to those of the one-region model (Equations 5 and 6). Consequently, the limited-liability scenario is still a mean preserving spread of the unlimited-guarantee case. The question that arises, however, is whether the pooling of the regions (a federal program for instance) is preferable than no pooling at all (two separated public programs). When one region chooses to insure, it may influence the probability of ruin to the other region, as well as the indemnity received and the additional tax. The change in the indemnity or tax represents a pecuniary externality to the other region. As a result, the willingness to pay of one region can be affected favorably or unfavorably by the participation of the other.

The final payoffs matrix is presented in Table 3. The first entry in each box is Region 1's expected utility for the corresponding strategy profile; the second is Region 2's. If Region j chooses not to have insurance, Region i's expected utility will depend only on its characteristics, as in the one-region model. Region i will buy insurance if and only if $V_i(c, \alpha_i, p, \delta_i) \geq pU(-l)$. In contrast, if Region j chooses to buy insurance, Region i's expected utility depends on both regions' characteristics: $V_i = V_i(c, \alpha_1, \alpha_2, p, \delta_1, \delta_2, \theta)$. The correlations δ_j and θ have an influence on V_i because the probability of ruin depends on F_0. The premium α_j has an impact on V_i since it plays a role in the probability of ruin, the indemnity and the tax (see Table 2).

The model can be applied indifferently to an insurance with limited liability or unlimited guarantee. Proposition 2 about the derivatives of V still holds if only one region buys insurance. The results can also be extended to the two-region case: when both regions buy insurance, we have a negative relationship between V and each correlation (see Proposition 5).

Proposition 5. *When both regions decide to buy insurance, the two-region model leads to the following comparative static derivatives (for $i = 1,2$, $i \neq j$, and for $\bar{x} > x^*$):*

$$\frac{\partial V_i}{\partial \delta_i} < 0, \qquad \frac{\partial V_i}{\partial \delta_j} < 0, \qquad \frac{\partial V_i}{\partial \theta} < 0.$$

Proof. See Appendix. □

Let α_i^* denote the willingness to pay of Region i when only Region i chooses to insure, $i = 1,2$. In that case, the results are strictly identical to the one-region model (Proposition 3). In contrast, if we denote by α_1^{**} and α_2^{**} the willingnesses to pay when both regions choose to insure, we have:

Proposition 6. *When both regions decide to purchase insurance, the two-region model of natural catastrophe insurance leads to the following comparative static derivatives (for $i = 1,2$ and $j \neq i$):*

$$\text{For} \quad \bar{x} \in [0;1]: \quad \frac{\partial V_i}{\partial \alpha_j} > 0, \qquad \frac{\partial \alpha_i^{**}}{\partial \alpha_j} > 0.$$

$$\text{For} \quad \bar{x} > x^*: \quad \frac{\partial \alpha_i^{**}}{\partial \delta_i} < 0, \qquad \frac{\partial \alpha_i^{**}}{\partial \delta_j} < 0, \qquad \frac{\partial \alpha_i^{**}}{\partial \theta} < 0.$$

Proof. See Appendix. □

Figure 2 provides an illustration of Proposition 6. The left panel displays the case where Region j decides not to buy insurance: only the premium α_i is determinant in the choice of Region i to insure. In contrast, the right panel displays the case where Region j decides to purchase insurance. The higher the premium of Region j, the lower the probability of ruin, and the higher the expected utility of Region i. As a result, Region i is willing to pay a higher rate if Region j pays a higher rate too.

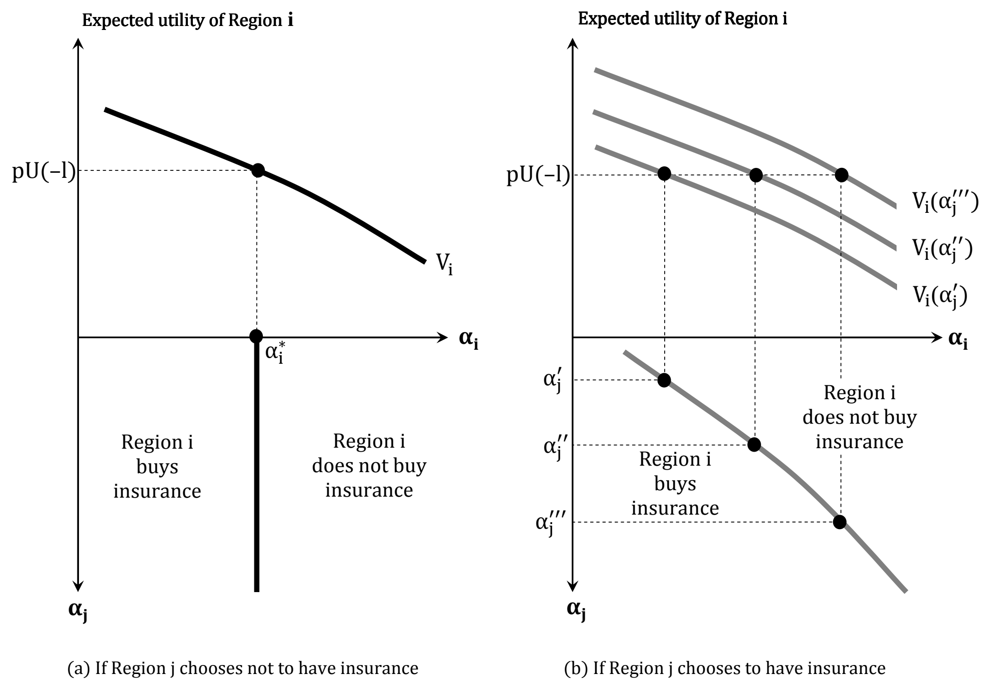

Figure 2. *Illustration of Proposition 6*

4.2. Nash equilibria

The set of Nash equilibria can be obtained by mixing the bottom-left and bottom-right panels of Figure 2 together. Figure 3 provides an illustration. The dotted line represents the 45-degree line where the premiums in Region 1 and Region 2 are identical. The black lines represent the willingness to pay when only one region is insured, i.e., α_1^* and α_2^*. Above the α_2^*-lines, Region 2 will refuse to purchase insurance, so will Region 1 on the right-hand side of the α_1^*-lines. In other words, the intersection point (P) between the black lines represents the willingnesses to pay when the regions are insured via two separated programs.

The grey curves represent the willingnesses to pay when both regions are insured, i.e., α_1^{**} and α_2^{**}. Above the α_2^{**}-curves, Region 2 will refuse to purchase insurance, so will Region 1 on the right-hand side of the α_1^{**}-curves. These curves are increasing with the other region's premium because of Proposition 6. Hence, the intersection point (Q) between the grey curves represents the willingnesses to pay when the regions are insured via the same program.

Let us assume that there exists a value of θ such that Point P is equal to Point Q. This starting situation is represented in Panel a of Figure 3 where the regions are assumed to be strictly identical. Because of this symmetry, the intersection points are on the dotted 45-degree line and $\alpha_i^{**}(0)$ appears to be lower than α_i^*, $i = 1, 2$. According to Proposition 6, a decrease in θ will generate an increase in the willingnesses to pay α_i^{**}. As a result, Point Q will be moving to the North-East of Point P and the regions' willingnesses to pay insurance will be higher if the risks are pooled together (see Panel b). On the other hand, when the between-

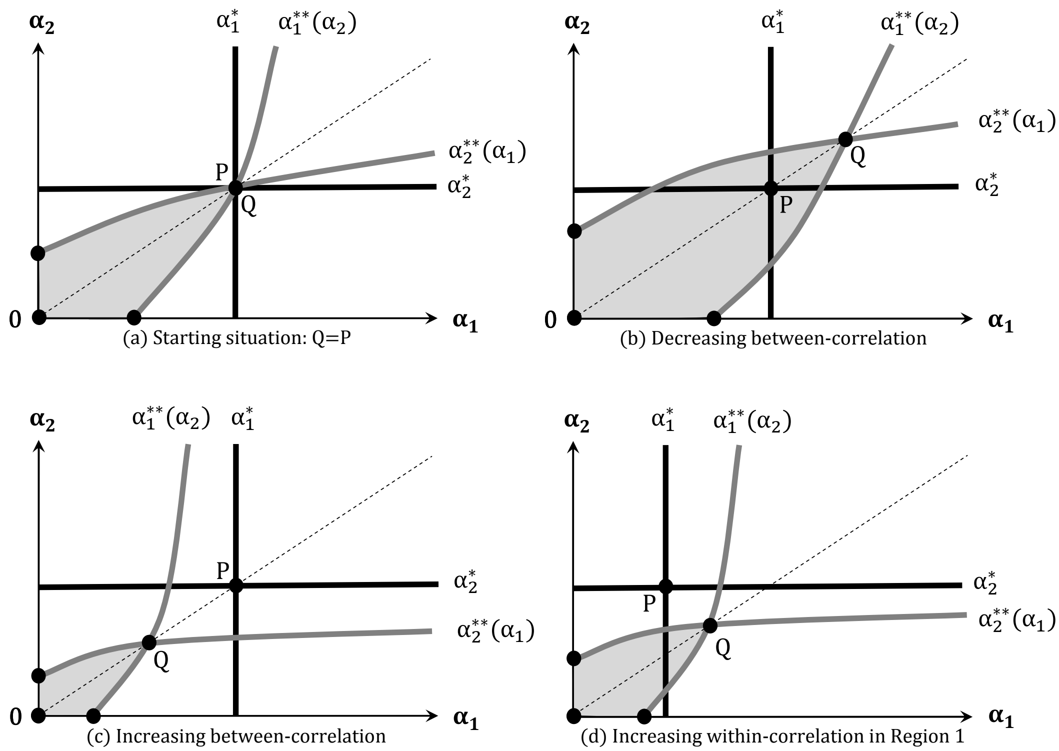

Figure 3. *Set of Nash Equilibria.*

correlation increases, pooling the risks will lead to a decrease in the willingnesses to pay: Point Q will move to the South-West of Point P (see Panel c).

Panel d displays an asymmetric case where only Region 1 faces an increase in the within-correlation, i.e., δ_1 increases. From Propositions 3, the intersection Point P will be moving to the West because we have $\partial\alpha^*/\partial\delta < 0$ in the one-region model. On the other hand, Point Q will be moving to the South-West because both Regions 1 and 2 will be affected by the increase in δ_1 (Propositions 6). When both regions are insured, the players have the same expected utility function. Their willingnesses to pay are necessarily equal, which implies that Point Q still will be on the 45-degree line. As a result, Region 1's willingness to pay insurance is higher when the risks are pooled together, which is not the case for Region 2.

The two-region model shows the important role the between-correlation can play in a federal program. In panel c for instance, the regions' willingnesses to pay are lower when the regions are pooled together. The reason is that the between-correlation is sufficiently high to increase the default risk and lower the benefit from insurance. In that case, to be attractive, a federal contract should propose a lower price than a regional contract. In the asymmetric case, a federal program can be more attractive to correlated regions than it is to non-correlated regions. For instance, in Panel d, pooling the risk from the starting Point P without changing the premium levels will have the following consequences. First, Region 1 will accept the pooling since α_1^{**} is higher than α_1^*. In that case, we are guaranteed that Region 1 will reach a utility level higher than the reversion utility level $pU(-l)$. On the other hand, Region 2 will not accept the pooling because α_2^{**} will be lower than α_2^*. In that case, the utility reached by Region 2 is necessarily lower than $pU(-l)$. Hence, Point P cannot be a situation where both regions choose to insure. The only possibility to reach the pooling is by decreasing the premium of Region 2. In contrast, with strictly identical regions (Panel *b*), pooling the risk from the starting Point P without changing the premium levels will lead to an increase in both expected utilities (α_1^{**} and α_2^{**} will be higher than α_1^* and α_2^*).

5. Robustness to dependency modeling

The present section tests the robustness of our theoretical framework with respect to dependency modeling. There are mainly two techniques that can be used to incorporate dependence, both implying *mixture models*, i.e., probabilistic models for density estimation using several distributions (see, e.g., Denuit et al., 2005). The first technique consists in using a discrete *common shock model*, with a dichotomous variable describing the occurrence of a catastrophe. The second focuses on a *frailty type model*, with a continuous variable describing the intensity of the catastrophe. We will be interested in the first approach, which allows to derive much more simple results. To our knowledge, common shock models have never been used in the context of natural catastrophes and, as such, the following section presents an innovative approach to investigate the models by simulations method.

5.1. Modeling within-correlations

In the one-region model, the distribution function of N (or equivalently X) can be defined as a *mixture model* that depends on three probabilities:

- $p^* = \mathbb{P}(\text{Cat}) \in [0;1]$ is the probability of a natural catastrophe, with $1 - p^* = \mathbb{P}(\text{No Cat})$.
- $p_N \in [0;1]$ is the probability for an individual to claim a loss in case of no natural catastrophe.
- $p_C \in [0;1]$ is the probability for an individual to claim a loss in case of natural catastrophe. We assume that $p_C \geq p_N$, i.e., a natural catastrophe increases risk occurrence.

The probability of loss for an individual is given by:

$$p = p_N(1 - p^*) + p_C p^*. \tag{11}$$

Conditional on the occurrence or not of a natural catastrophe, the risks between individuals are assumed to be independent. For example, if $p^* = p_N = p_C = 10\%$, the probability of loss is equal to $p = 10\%$, and the distribution of N is simply given by the binomial distribution $\mathscr{B}(n, 10\%)$. In contrast, if $p^* = 10\%$, $p_N = 0\%$ and $p_C = 100\%$, the probability of loss is also equal to $p = 10\%$, but the risks are highly correlated since the probability of claiming a loss is 1 in case of a catastrophe (i.e., we have a $p^* = 10\%$ chance that everybody claims a loss), and 0 otherwise (i.e., we have a $1 - p^* = 90\%$ chance of no loss).

More generally, given Equation 11, the distribution function of N (and X) can be defined by the following mixture of two binomial distributions ($\forall k = 0 \dots n$):

$$F(x) = \mathbb{P}(N \leq k) = \mathbb{P}(N \leq k|\text{No Cat}) \times \mathbb{P}(\text{No Cat}) + \mathbb{P}(N \leq k|\text{Cat}) \times \mathbb{P}(\text{Cat}) \quad (12)$$

$$= \sum_{j=0}^{k} \binom{n}{j} \left[(p_N)^j (1-p_N)^{n-j}(1-p^*) + (p_C)^j (1-p_C)^{n-j} p^*\right] \quad (13)$$

To simplify, we can set:

$$p_C = \frac{1}{1-\delta} p_N, \quad (14)$$

which allows us to write p_N and p_C as a function of p^*, p and δ (using Equations 11 and 14):

$$p_N = \frac{(1-\delta)p}{1-\delta+\delta p^*} \quad (15)$$

$$p_C = \frac{p}{1-\delta+\delta p^*} \quad (16)$$

The coefficient $\delta \in \left[0, \min\left\{1, \frac{1-p}{1-p^*}\right\}\right]$ is determinant in our analysis and can be seen as a proxy for the correlation between risks.

Proposition 7. *The coefficient δ is an increasing monotonic function of the correlation between the individual risks.*

Proof. See Appendix. □

Figure 4 provides an illustration of the cumulative distribution function F when $n = 500$, $p^* = 0.1$, $p = 0.3$, and $\delta = 0.4$. In that case, the probabilities p_N and p_C are equal to 0.28 and 0.47, respectively. It is possible to test the properties of F described in Equation 2: (i) On the bottom-panel of Figure 4, the individual probability p is also the expectation of X. (ii) When the loss probability p increases from 0.3 to 0.5 (see the top panel of Figure 5), the distribution function translates to the right, with $p_N = 0.47$ and $p_C = 0.78$. (iii and v) The probability of an extreme event, for instance $x = 0.8$, increases with p and δ (see the bottom panel of Figures 5 and 6, respectively). (iv) An increase in δ from 0.4 to 0.7 implies a reduction of p_N to 0.24 and an increase in p_C to 0.81, which means that when δ is high, either there is a small or an extreme event (see the top panel of Figure 6). (vi) In Figure 6, an increase in δ has no impact on p. Notice that our theoretical analysis was only interested in the right-hand side of the distribution function around $x^* = p_C$.

Figure 7 illustrates what happens with an infinite number of risks. With correlated risks (top-panel of Figure 7), the cumulative distribution function F still presents two bends but with a much sharper curve. Two possible states of nature can be observed depending on the occurrence or not of a natural disaster. The insurance company can set the premium so that $\bar{x}$ is between p_N and p_C. In that case, the company has a 10% likelihood to go bankrupt. The insurance company can also set the premium so that $\bar{x}$ is greater than p_C, which will guarantee a zero probability of ruin. With an infinite number of uncorrelated risks (bottom-panel of Figure 7), the function F presents only one bend and the Law of Large Numbers does hold. The insurer can set the premium so that $\bar{x}$ is greater than $p = 0.30$ which will guarantee a zero probability of ruin.

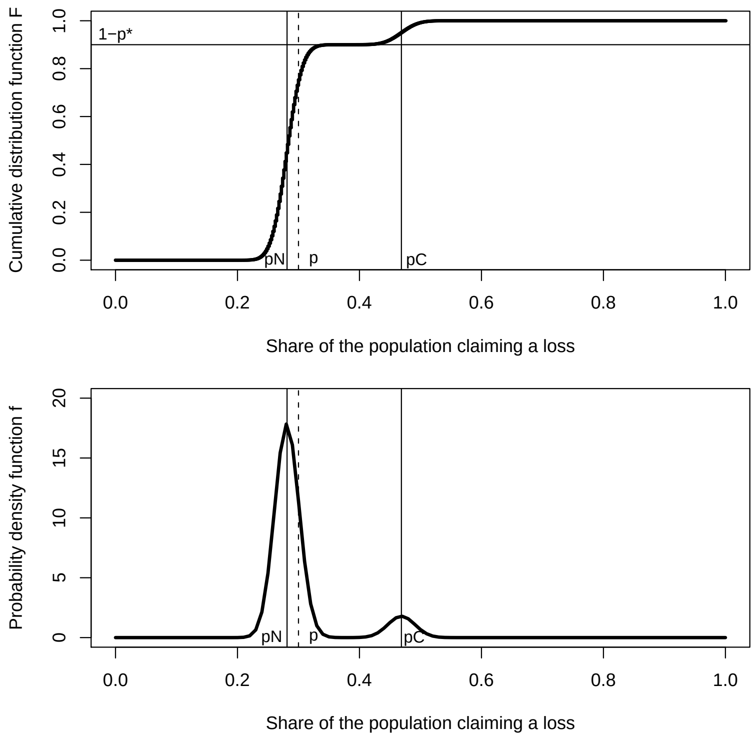

Figure 4. *Functions F and f ($n = 500$, p^* =0.1, p = 0.3, $\delta = 0.4$)*

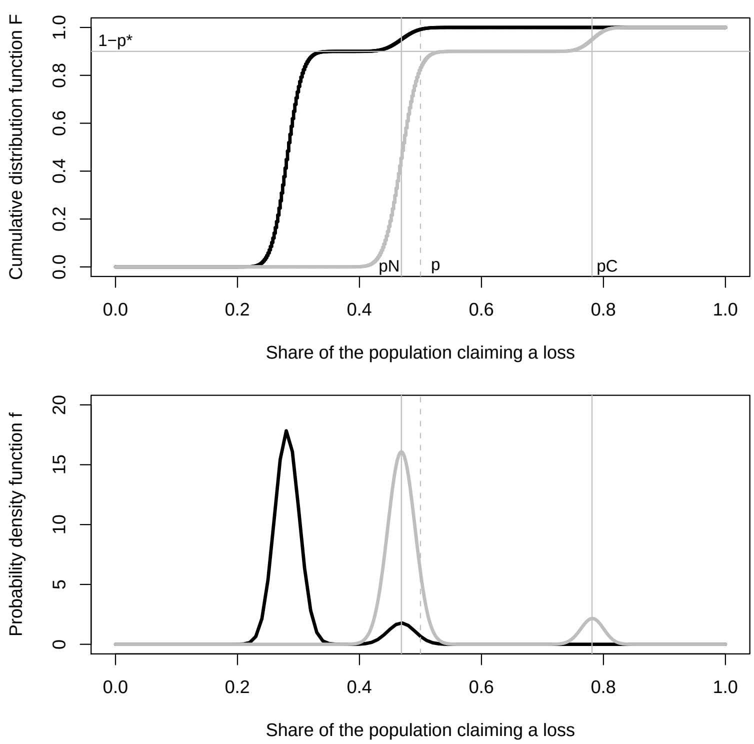

Figure 5. *Impact of an increase in p on F and f (p =0.3 then 0.5) .*

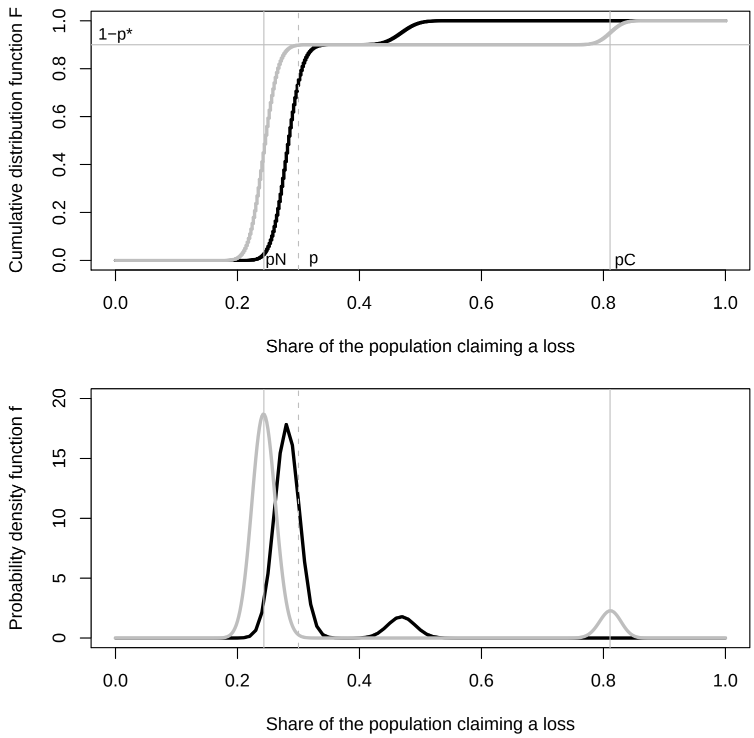

Figure 6. *Impact of an increase in δ on F and f (δ =0.4 then 0.7).*

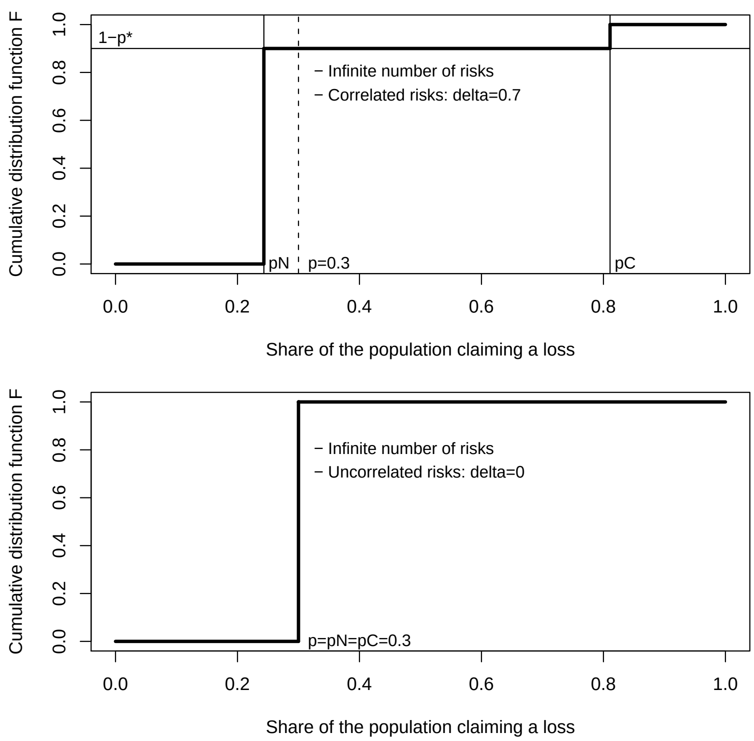

Figure 7. *Infinite number of risks: $n = +\infty$*

Table 4. *Catastrophe probabilities in the two-region economy.*

	Cat in Region 1		No Cat in Region 1	
	Cat in Region 2	No Cat in Region 2	Cat in Region 2	No Cat in Region 2
General case	θ	$p^* - \theta$	$p^* - \theta$	$l - 2p^* + \theta$
Perfectly independent	$(p*)^2$	$p^*(1-p^*)$	$p^*(1-p^*)$	$(1-p^*)(1-p^*)$
Positively dependent	p^*	0	0	$1-p^*$

5.2. *Modeling between-correlations*

Consider now two regions that have the same probability p^* of a catastrophe. The probabilities p_C and p_N can be different from one region to the other and will be denoted by P_C^1, P_N^1, P_C^2, P_N^2 for Region 1 and Region 2, respectively.

The distribution function F_0 of natural events over the whole economy can be derived from a mixture of F_1 and F_2 observed in each region. There are four possible cases depending on whether the regions are exposed to a catastrophe or not. The possible cases are described in Table 4. Let $p_{CC} = \theta$ denote the probability that Regions 1 and 2 are both victim of a catastrophe; $p_{CN} = 1 - p^*$ the probability that only Region 1 is hit by a catastrophe; $p_{NC} = 1 - p^*$ the probability that only Region 2 is hit, and $p_{NN} = 1 - 2p^* + \theta$ the probability that Regions 1 and 2 are not hit. The probability density function of X_0 can be defined as:

$$f_0(x) = p_{CC} f_{CC}(x) + p_{CN} f_{CN}(x) + p_{NC} f_{NC}(x) + p_{NN} f_{NN}(x), \tag{17}$$

with

$$f_{ab}(x) = \mathscr{B}(n_1, p_i^1) \star \mathscr{B}(n_2, p_i^2), \tag{18}$$

where $a, b \in \{C, N\}$ and $\star$ denotes the convolution operator. In this model, $p_{CC} = \theta$ represents the between-correlation.

Figure 8 displays the case where the risks between the regions are perfectly independent, i.e., $\theta = (p^*)^2$. In black are represented the cumulative distribution function F_1 (top panel of the figure) and the density f_1 of Region 1 (bottom panel) when $p = 0.3$, $n_1 = 500$, $\delta_1 = 0.5$. The grey curves display F_0 and f_0 when Region 1 is pooled with an identical region. In the vicinity of $x^* = p_C$, we can see that F_0 is below F_1, i.e., pooling Region 1 with another region reduces the probability of an extreme event. In contrast, when the risks are positively dependent, i.e., when $\theta = p^*$, F_0 is above F_1 in the vicinity of p_C (see Figure 9). In that case, pooling Region 1 with another region increases the probability of an extreme event. This corresponds to properties (v) and (vi) of the two-region model (see Equation 10).

In Figure 10, the risk between the regions are independent, i.e., $\theta = (p^*)^2$, but Region 1 has a higher within-correlation than Region 2 ($\delta_1 = 0.5$ and $\delta_2 = 0.7$). We can see that F_0 is below F_1 around p_C, which points out that pooling Region 1 with a more correlated area will be detrimental to Region 1. On the other hand, pooling Region 1 with a less correlated area will be beneficial to Region 1 (see Figure 11 where $\delta_1 = 0.5$ and $\delta_2 = 0.3$). This corresponds to properties (v) and (vi) of the two-region model (see Equation 10).

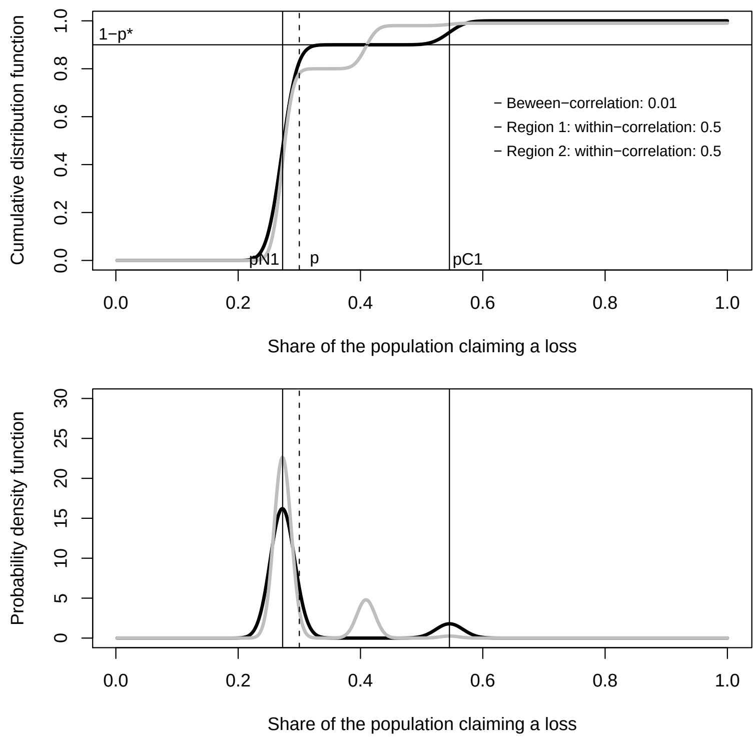

Figure 8. *F_1 and F_0 when the risks are perfectly independent.*

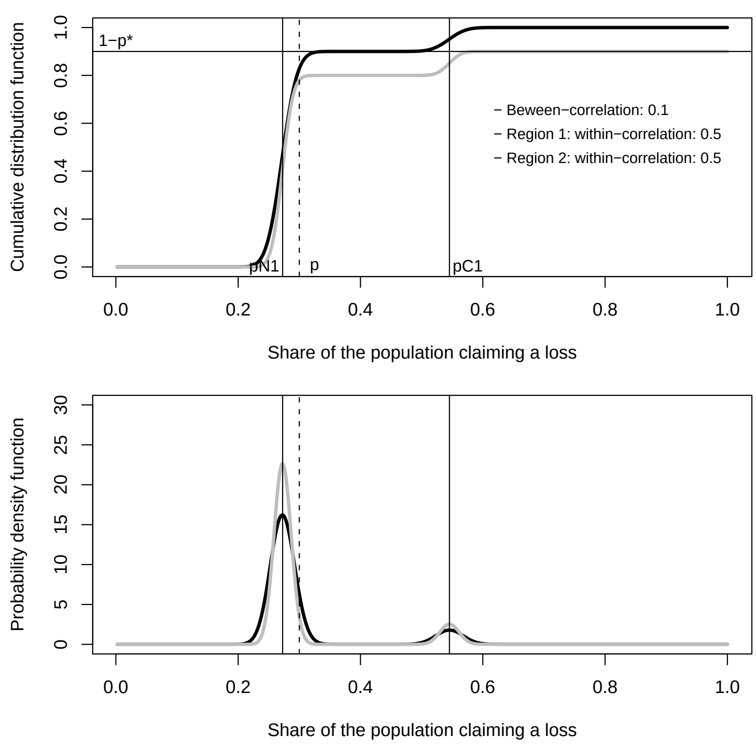

Figure 9. *F_1 and F_0 when the risks are positively dependent.*

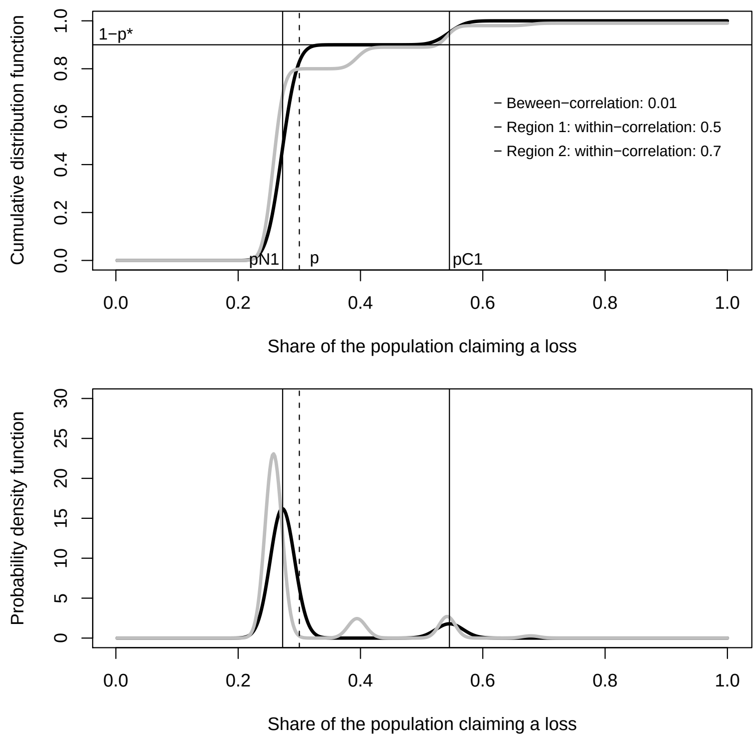

Figure 10. *F_1 and F_0 when Region 1 has a higher within-correlation.*

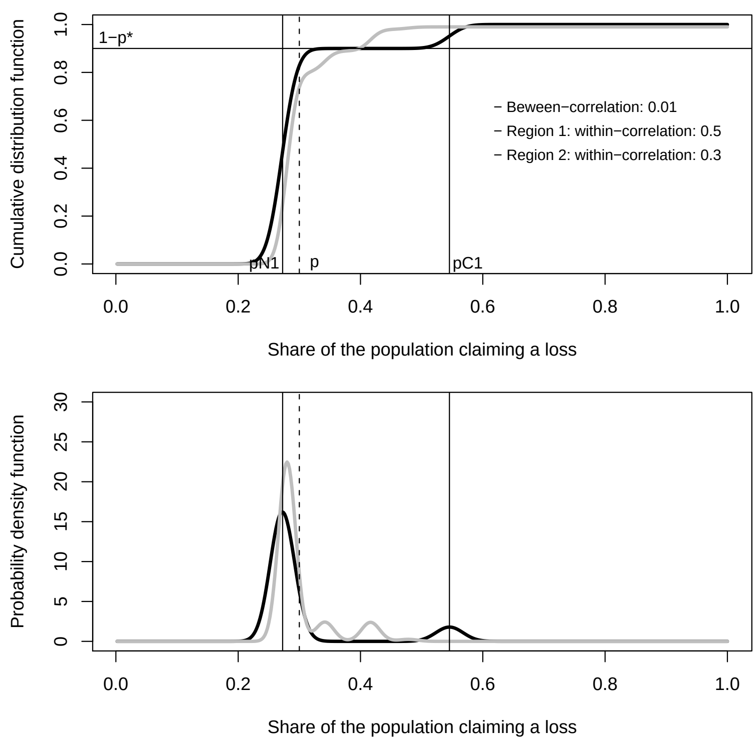

Figure 11. *F_1 and F_0 when Region 1 has a lower within-correlation.*

5.3. Application to the one-region model

This subsection investigates the one-region model by simulations using the mixture model of Subsection 5.1. Although we will not treat this issue, the simulations can be easily extended to the two-region model as well. For graphical convenience, the probability of a catastrophe p^* will be given and equal to 0.05, while the loss will be set to $l = 1$, n to 1000, and the utility function to $U(Y) = 100(1 - e^{-2Y})$. This function U is concave (i.e., policyholders are risk-averse) and belongs to the CARA class of utility function, i.e., leads to a Constant Absolute Risk Aversion according to the Arrow-Pratt measure of relative risk-aversion.

Figure 12 gives the expected utility V as a function of the premium α for $c = 0.05$, $\delta = 0.9$ and $p = 0.1$. The *grey curve* denotes the expected utility without government intervention. The *thin black curve* stands for the unlimited-guarantee scenario. The *large black curve* represents the expected utility when there is a zero default risk, i.e., $U(-\alpha)$. Confirming our expectations, the large black curve is above the other curves and the expected utility is higher with unlimited guarantee than without.

The two dots on the right hand side correspond to the willingness to pay (α^*). The values are obtained for the highest possible premiums such that inhabitants are indifferent between V and $pU(-l)$. These dots characterize the market equilibrium when the market is not perfectly competitive. The expected utility is equal to $pU(-l) = -63.9$ in both cases, with $\alpha^* = 0.206$ without government intervention, and $\alpha^* = 0.216$ with intervention. The two dots on the left hand side correspond to the lowest possible premiums such that the expected profit of the insurance company is non negative. This situation could be reached for instance with a competitive market or a regulated monopoly.

As demonstrated in Proposition 2, with a limited liability insurance, the expected utility initially increases with the premium and then decreases (see Figure 12 for instance). This result does not hold with government intervention. This may be easily explained. Recall that ruin occurs when $x > \left(\bar{x} = \frac{\alpha+c}{l}\right)$. If α is small, the probability of ruin is very high, almost close to 1, while the indemnity received by the policyholders is close to nc/N, i.e., very small. Without intervention, an increasing premium will generate an increase in the indemnity and will reduce the probability of ruin. When the probability of ruin becomes insignificant, the expected utility will decrease as usual with the premium. In contrast, with government intervention, full coverage will be always guaranteed. An increase in the premium will only serve the insurance company.

Figures 13 and 14 provide an illustration of Propositions 2 and 3. On the right hand side of Figure 13, a decrease in the correlation from 0.9 to 0.8 has a positive impact on the expected utility and the willingnesses to pay. The exact same situation could be reached with a decrease in p_C from 0.69 to 0.417, which highlights the importance of risk mitigation and prevention policies. Lastly, in Figure 14, an increase in the capital per head from $c = 0.05$ to 0.08 has a positive impact on the expected utilities. An increase in c leads to an increase in $\bar{x}$ and consequently reduces the probability of ruin $1 - F(\bar{x})$. As a result, we observe an upward shift in the expected utilities and a positive impact on the willingnesses to pay. The lowest possible premiums (the two dots on the left such that the expected profit is still positive) increase because the shareholders will be more exposed to industry failure (Proposition 1).

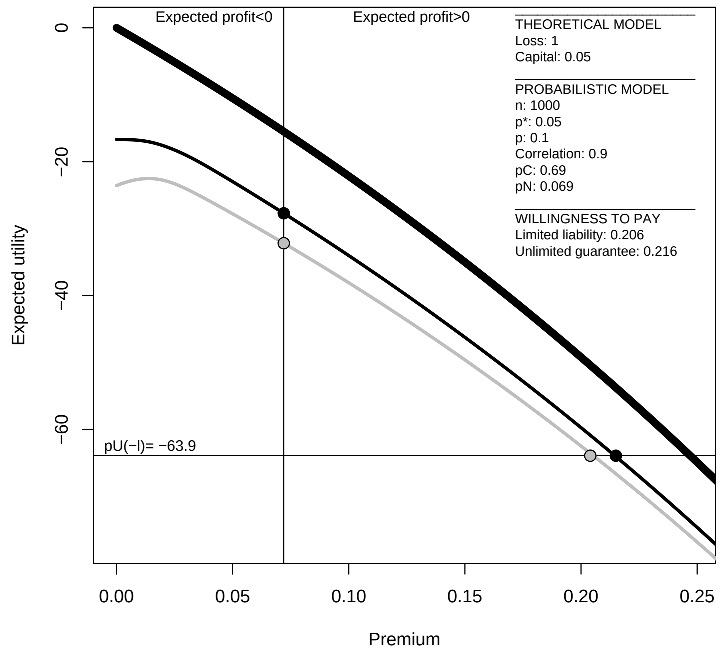

Figure 12. *Comparison of scenarios.*

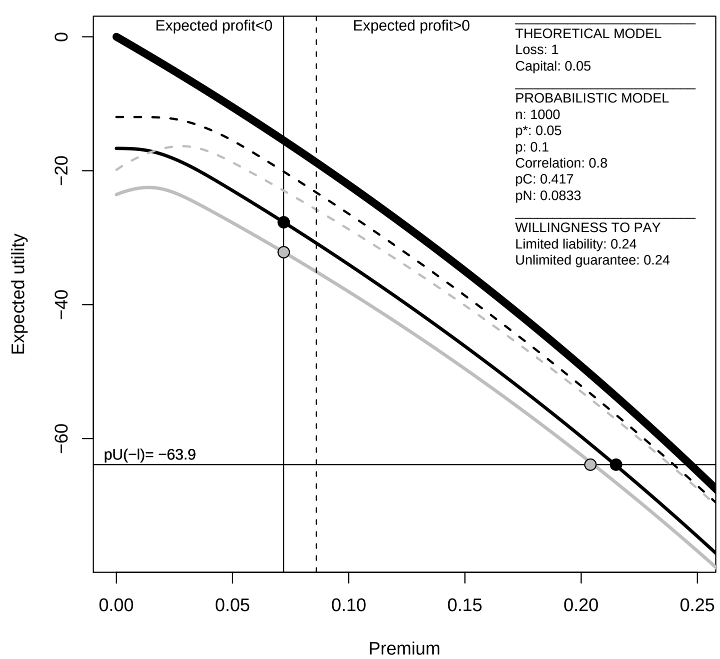

Figure 13. *Impact of a decrease in δ from 0.9 (plain line) to 0.8 (dotted line).*

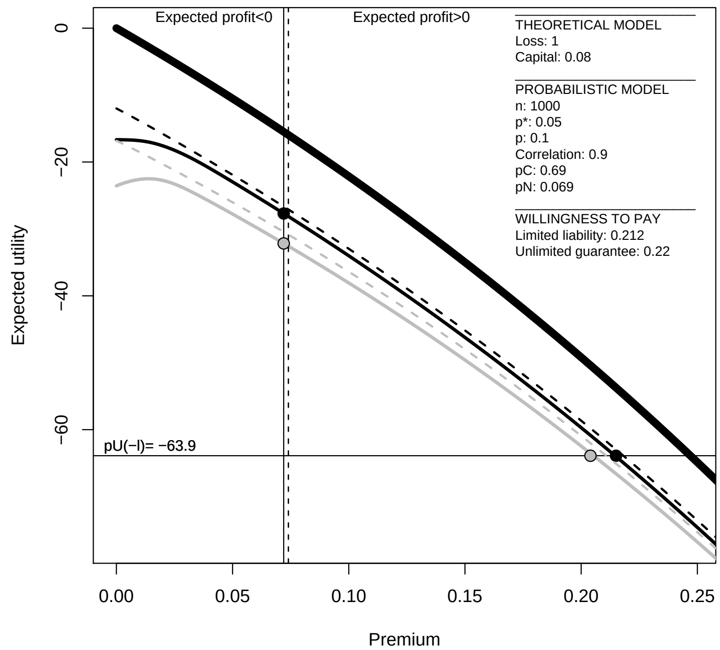

Figure 14. *Impact of an increase in c from* 0.05 *(plain line) to 0.08 (dotted line).*

Table 5. *Example of losses associated with catastrophic natural disasters.*[a]

	Region 1	Region 2	Region 3	Regions 1+2	Regions 1+3
1. Loss per inhabitant in Year 1	5	65	35	35	20
2. Loss per inhabitant in Year 2	95	35	65	65	80
3. Average of annual losses	50	50	50	50	50
4. Variance of annual losses	2025	225	225	225	900
5. Pearson correlation coefficient				-1	+1

[a] The number of inhabitants is the same in each region.

6. Policy implications of the models

The results we have obtained and discussed in the present paper have led to a better understanding of insurance markets when covers against natural catastrophes are in play. Three main elements were assumed to determinate the occurrence of natural events: first, the probability (p) for each individual to claim a loss, second, the within-correlation (δ) between the individual risks and, third, the between-correlation (θ), i.e., how the risks are correlated between two regions.

In practice, the model parameters can be easily approximated. Consider the example of Table 5. The average of the losses (Row 3) is an approximation for the level of risk in each jurisdiction, i.e., is related to p in our model. We can see that the level of risk is the same. In contrast, the time profile of the losses (Rows 1 and 2) is different, thus affecting the chances for the insurance industry to be solvent. The variance through time (Row 4) can actually be considered as a proxy for δ. On the one hand, the variance is extremely high in Region 1 because both a small and an extreme event are observed. On the other hand, the losses per capita in Regions 2 and 3 converge toward the average, pointing out a low within-correlation. The between-correlation θ can be approximated by the Pearson correlation coefficient (Row 5): the natural risks in Regions 1 are more correlated with those of Region 3 than they are with those of Region 2.

In our theoretical framework, the probability of ruin is mainly affected by δ. As a result, this parameter has an impact on the expected profit of the company as well as the policyholders' willingness to pay. An increase in δ will be detrimental to both the supply and demand sides. For instance, in Table 5, an insurer will prefer to insure Region 2 or Region 3 because these regions have a lower within-correlation. Similarly, the willingness to pay for an insurance contract will be higher in these regions, because the default risk is lower. In other words, insurance coverage is likely to be not provided in Region 1. How can we solve the problem of Region 1 in a Pareto-efficient manner? The present research highlights and allows to discuss several solutions within a unified microeconomic framework.

6.1. The role of capital resources

Policymakers have often chosen to intervene with capital requirements in the first place. According to our model, the willingness to pay for an insurance contract is a positive function of the company's capital. This relationship can also be observed empirically. In the US, several studies have found support for demand sensitivity to insurer's financial strength ratings (see for instance Sommer, 1996; Cummins and Danzon, 1997; Epermanis and Harrington, 2006). An increase in the capital requirements would benefit the potential purchasers, allowing the insurers to propose sufficiently high ratings for business. However, it should be stressed that an increase in the capital will also increase the exposition of the shareholders to industry failure, thus putting the insurance companies in a very uncomfortable position (see Subsection 3.3).

The decision to increase the capital requirements actually depends on the demand sensitivity to insurers' capital resources, a regulation that is not so easy to implement in practice. Another possibility would be to reduce the cost of access to capital itself, which will be beneficial for both the demand and supply sides. For instance, some authors have suggested the use of capital market instruments such as CAT bonds or CAT options to help insurance companies access capital in the financial markets. Another suggestion has been the creation of tax-deferred catastrophe reserves (Kousky, 2011).

6.2. A regulated premium

When faced with natural disasters, private insurers advocate high levels of premium for many reasons: (1) high premiums allows for protecting of people and businesses from financial ruin, (2) pre-funding of disasters is better than post-funding, (3) when assessing the risk of a disaster is difficult for a particular area, it is only rational for the industry to assume high disaster risk and charge high premiums. Simply put, premiums for catastrophic risks should be higher than for noncatastrophic risks. In our model we have shown that this assertion is true but only from the supply-side point of view.

A higher premium will allow to reduce the probability of ruin and lead to higher expected profits. However, from the demand-side point of view, we have shown that the willingness to pay for a catastrophe coverage is a negative function of δ, because correlated risks imply a higher default risk (see Subsection 3.2). For a same level of risk (p), a catastrophe coverage that is sold at a price higher than the price of a non-catastrophe coverage should be less attractive to potential purchasers.

Given the controversial impact of the premium, it seems difficult to think that a regulated price can be of any use, unless the idea is to solve the market inefficiencies due to imperfect competition and imperfect information (see, e.g., Epple and Schäfer, 1996; Jaffee and Russell, 1997). For instance, we have shown in Subsection 3.3 that a non perfectly competitive market could lead the industry to propose a price equal to the willingness to pay of policyholders, thus reducing to zero the benefit of an insurance coverage.

6.3. The importance of risk diversification

Our results confirm the idea that the natural catastrophe insurance industry is characterized by economies of scale, but only under certain circumstances. If the between-correlation is sufficiently low, a large company can reach a lower probability of ruin and sell insurance contracts at higher rates (see Subsection 3.3). In contrast, small companies might not offer catastrophe coverage because their potential weaknesses would prevent them to sell contracts at a price sufficiently high for business. From the two last columns of Table 5, we can see that an insurer would benefit from a pooling between Regions 1 and 2. However, this result holds only because the between-correlation is sufficiently low. This is not the case for instance with Region 1 and Region 3.

The growth of private reinsurance coverage throughout the world in recent decades has resulted in a much larger an efficient geographical pool of risk. However, despite the possibility of reinsurance coverage, many insurers still refuse to provide insurance coverage for too risky areas. This was the case in the US where insurers did not provide flood insurance coverage due to the hazard of flood typically being confined to a few areas. Other examples can be found in Germany, where some people living in Baden-Württemberg could not find any insurer to cover them against the periodically recurring floods (Epple and Schäfer, 1996). All these problems were solved by the creation of public programs.

6.4. Unlimited guarantee from the government

Because an unlimited guarantee insurance allow to spread the risks equally among the policyholders, the willingnesses to pay for insurance should be higher with government intervention. The consequence would be the possibility for the insurer to put forward higher premiums, which will reduce the insolvency probability, lead to higher expected profits, and could guarantee the existence of a catastrophe coverage (see Subsections 3.2 and 3.3).

Consider for instance a problem where $n = 100,000$ taxpayers face the following lotteries: assume that $x = 1\%$ of people , i.e., $N = 1,000$ victims, will be claiming a loss of $l = \$100,000$. The total loss will be \$100 million. Moreover, assume that the insurance industry is able to pay compensation for a maximum amount of \$80 million. In that case, \$20 million will not be paid by the insurance industry. If the government intervenes as an insurer of last resort, it does not matter whether you are claiming a loss or not because all the n taxpayers are supposed to participate to the insurance program to compensate the payment default. Everybody will pay $T = \frac{\$20 \text{ million}}{100,000} = \$200$ through additional taxes. On the other hand, without intervention, the 1,000 victims will receive only a reduced indemnity $I = \frac{\$80 \text{ million}}{1,000} = \$80,000$, i.e., will lose $I - l = \$20,000$. Since you have a $x = 1\%$ likelihood of being one of the victims, your expected loss is $0.01 \times 20,000 + 0.99 \times 0 = 200$ Euros. The limited liability scenario is consequently a mean preserving spread of the unlimited guarantee case. If people are risk-averse, they will prefer the unlimited guarantee scenario, even if it implies recurrent deficits. Of course, in practice, people do not know with certainty x and face a larger set of possible states of nature. However, this result has been generalized in Section 3 using the *Independence Axiom* of the von Neumann-Morgenstern Expected Utility Theory.

6.5. A federal program should be priced appropriately

If unlimited guarantee is an appropriate tool to spread the risks, it brings other questions to mind. To put our model in context it is useful to consider the US National Flood Insurance Program (NFIP). One of the frequently asked questions about NFIP is the following: '*Will policyholders in non-coastal states be paying more for flood insurance to support losses along the coast?*'. The answer to this question is that premiums are based on risk, not location. Two housing units with the same risk (p in our model) but located on terrains with different magnitude of damage (related to δ in our model) — for example, one in a shallow floodplain and the other in a steep and narrow mountain valley — will be charged the same rate (GAO, 2008). According to the two-region model developed in Section 4, such a pricing policy could lead to inefficiencies.

Depending on the value of the between-correlation, a federal program can be more attractive to correlated regions than it is to non-correlated regions. For instance, in Table 5, a federal program that comprises Region 1 and Region 3 cannot be politically viable in the long run. Inhabitants from Region 3 would not understand why they should pay the same price as Region 1 for a federal program that presents a higher default risk and higher probabilities of deficit than a regional program. Because of these negative pecuniary externalities, Region 3 should pay a lower fee.

The two-region model can be a useful normative tool for a regulation. The rates of a federal program should be computed based not only on the level of risks (p), i.e., on the expected losses (a basic actuarial principle), but also on how the risks are correlated within and between the regions (δ and θ), i.e., on the variance of the losses (which has never been applied to our knowledge). In particular, public officials must be prepared to announce rates lower than usual to attract low-correlation regions.

The importance of our result is not to be neglected. In January, 2011, a United States Representative from the state of Michigan, Candice Miller, has proposed legislation that aims at ending the government run flood insurance program (H.R.435, *National Flood Insurance Program Termination Act of 2010*). She points out that participants in some states, such as Michigan, have to pay for and cover the costs that some other states are incurring. This bill would close the program in the year 2013.

7. Concluding remarks

In conclusion, our paper tried to highlight some key mechanisms between the premium rate, the capital, the correlation of the risks and the decision to provide or buy insurance. Given our results, it is not surprising that so many industrialized nations have intervened in catastrophe insurance markets, even though the programs implemented have sometimes resulted in severe financial difficulties. Natural catastrophe insurance is a challenging issue to the public authorities since, by definition, catastrophic risks are uninsurable, i.e., may involve extremely large losses. However, compared to a purely private market, the chance of failure should be reduced: risk-averse policyholders will accept to pay higher rates for an unlimited guarantee insurance, thus reducing the probability of ruin. To limit the protests of the less correlated areas, we have shown that these rates should be computed based on how the risks are correlated within and between the jurisdictions involved.

We suspect that the present research could have implications in other situations as well. For instance, insurance against terrorism, public provision of fire protection, public health programs, unemployment insurance, could be subject to an analysis. Moreover, there are several problems of related interest which were not examined in the present paper such as the influence of risk mitigation, and the role of bounded rationality in insurance decisions. To some extent, our results also bring a new perspective to the theory of market failures: government intervention seems justifiable each time general interdependencies are in play in a limited-liability market, i.e., if an extreme event can potentially bankrupt or bring down the entire industry. These and other questions provide a formidable agenda for future research.

Appendix

Proof of Proposition 1

Recall that the profit of the insurance company is:

$$\Pi(\alpha, p, \delta, c) = \int_0^{\bar{x}} [n\alpha - xnl] f(x)dx - [1 - F(\bar{x})]cn \quad \text{with} \quad \bar{x} = [\alpha + c]/l.$$

From Leibniz rule, the partial derivative with respect to c is:

$$\frac{\partial \Pi}{\partial c} = \frac{\partial \bar{x}(c)}{\partial c}[n\alpha - \bar{x}(c)nl] f(\bar{x}(c)) - [1 - F(\bar{x}(c))]\, n + \frac{\partial F(\bar{x}(c))}{\partial c} cn.$$

Thus,

$$\frac{\partial \Pi}{\partial c} = \frac{1}{l}[n\alpha - \bar{x}nl] f(\bar{x}) - [1 - F(\bar{x})]\, n + \frac{1}{l} f(\bar{x})cn = -[1 - F(\bar{x})]\, n < 0.$$

From Leibniz rule, the partial derivative with respect to α is:

$$\frac{\partial \Pi}{\partial \alpha} = \frac{\partial \bar{x}(\alpha)}{\partial \alpha}[n\alpha - \bar{x}(\alpha)nl] f(\bar{x}(\alpha)) + \int_0^{\bar{x}(\alpha)} nf(x)dx + \frac{\partial F(\bar{x}(\alpha))}{\partial \alpha} cn.$$

Thus,

$$\frac{\partial \Pi}{\partial \alpha} = \frac{1}{l}[n\alpha - \bar{x}nl] f(\bar{x}) + nF(\bar{x}) + \frac{1}{l} f(\bar{x})cn = nF(\bar{x}) > 0.$$

Proof of Proposition 2

Scenario with limited liability. The expected utility V can be rewritten as:

$$V(\alpha, p, \delta, c) = U(-\alpha) - \int_{\bar{x}}^1 [U(-\alpha) - U(-\alpha - l + I(x))]\, xf(x)dx \quad \text{with} \quad I(x) = \frac{c + \alpha}{x}.$$

From Leibniz rule, the partial derivative with respect to α is:

$$\frac{\partial V}{\partial \alpha} = -U'(-\alpha) + \frac{1}{l}[U(-\alpha) - U(-\alpha - l + I(\bar{x}))]\, \bar{x} f(\bar{x}) - \int_{\bar{x}}^1 \left[-U'(-\alpha) + (1 - \frac{1}{x})U'(-\alpha - l + I(x))\right] xf(x)dx.$$

Since $\bar{x} = [\alpha + c]/l$ and $I(\bar{x}) = \frac{c+\alpha}{x} = l$ the middle term can be set to 0, which gives:

$$\frac{\partial V}{\partial \alpha} = -U'(-\alpha) - \int_{\bar{x}}^1 \left[-U'(-\alpha) + (1 - \frac{1}{x})U'(-\alpha - l + I(x))\right] xf(x)dx.$$

Consider now a first order development of U', for all $x > \bar{x}$,

$$U'(-\alpha - l + I(x)) \approx U'(-\alpha) + [-l + I(x)]U''(-\alpha).$$

Then

$$(1 - x)U'(-\alpha - l + I(x)) + xU'(\alpha) \approx U'(\alpha) + (1 - x)[-l + I(x)]U''(-\alpha).$$

Since $[-l + I(x)] = [I(x) - I(\bar{x})]$, we have

$$\frac{\partial V}{\partial \alpha} \approx -U'(-\alpha) + \int_{\bar{x}}^1 U'(\alpha) + [(1 - x)[I(x) - I(\bar{x})]U''(-\alpha)] f(x)dx,$$

with

$$-U'(-\alpha)+\int_{\bar{x}}^{1}U'(\alpha)f(x)dx=-U'(\alpha)\int_{0}^{\bar{x}}f(x)dx=-U'(\alpha)\times\mathbb{P}(X\leq\bar{x}),$$

where $\mathbb{P}(X>\bar{x})$ is the probability of ruin. Moreover,

$$U''(-\alpha)\times\int_{\bar{x}}^{1}(1-x)[I(x)-I(\bar{x})]f(x)dx=-U''(-\alpha)\times H(\bar{x}),$$

where $H(\bar{x})=\int_{\bar{x}}^{1}(1-x)[I(\bar{x})-I(x)]f(x)dx$ is a positive function decreasing with $\bar{x}$. Hence $\frac{\partial V}{\partial\alpha}\leq 0$ implies:

$$-U''(-\alpha)\times H(\bar{x})\leq U'(\alpha)\times\mathbb{P}(X>\bar{x})\Leftrightarrow\frac{\mathbb{P}(X\leq\bar{x})}{H(\bar{x})}\geq-\frac{U''(-\alpha)}{U'(\alpha)},$$

where $\dfrac{\mathbb{P}(X\leq\bar{x})}{H(\bar{x})}$ is a positive increasing function in $\bar{x}$ and $-\frac{U''(-\alpha)}{U'(\alpha)}$ stands for the Arrow-Pratt measure of relative risk-aversion. If U is convex, this measure is negative, which implies that $\partial V/\partial\alpha$ is always negative. When U is concave, an inflexion point can exist: (1) if $\bar{x}$ is large (close to 1), then $\partial V/\partial\alpha$ is necessarily negative; (2) if $\bar{x}$ is small, then $\partial V/\partial\alpha$ can be positive.

From Leibniz rule, the partial derivative with respect to c is:

$$\frac{\partial V}{\partial c}=\frac{1}{l}\left[U(-\alpha)-U(-\alpha-l+I(\bar{x}))\right]f(\bar{x})+\int_{\bar{x}}^{1}U'(-\alpha-l+I(\bar{x}))f(x)dx,$$

i.e., since $I(\bar{x})=l$,

$$\frac{\partial V}{\partial c}=\int_{\bar{x}}^{1}U'(-\alpha-l+I(\bar{x}))f(x)dx>0.$$

The partial derivative with respect to p is:

$$\frac{\partial V}{\partial p}=-\int_{\bar{x}}^{1}\left[U(-\alpha)-U(-\alpha-l+I(x))\right]x\frac{\partial f(x)}{\partial p}dx.$$

which is negative since $\frac{\partial f(x)}{\partial p}>0$ for all $\bar{x}>x^{*}$. Similarly, the partial derivative with respect to δ is:

$$\frac{\partial V}{\partial\delta}=-\int_{\bar{x}}^{1}\left[U(-\alpha)-U(-\alpha-l+I(x))\right]x\frac{\partial f(x)}{\partial\delta}dx,$$

which is negative since $\frac{\partial f(x)}{\partial\delta}>0$ for all $\bar{x}>x^{*}$.

Scenario with unlimited guarantee. The expected utility V can be written as:

$$V(\alpha,p,\delta,c)=\int_{0}^{\bar{x}}U(-\alpha)f(x)dx+\int_{\bar{x}}^{1}U(-\alpha-T(x))f(x)dx\quad\text{with}\quad T(x)=xl-\alpha-c.$$

From Leibniz rule, the partial derivative with respect to α is:

$$\frac{\partial V}{\partial\alpha}=\frac{1}{l}U(-\alpha)f(\bar{x})-F(\bar{x})U'(-\alpha)-\frac{1}{l}U(c-\bar{x}l)f(\bar{x})=-F(\bar{x})U'(-\alpha)<0.$$

The expected utility V can also be rewritten as:

$$V(\alpha,p,\delta,c)=U(-\alpha)-\int_{\bar{x}}^{1}\left[U(-\alpha)-U(-\alpha-T(x))\right]f(x)dx\quad\text{with}\quad T(x)=xl-\alpha-c.$$

From Leibniz rule, the partial derivative with respect to c is:

$$\frac{\partial V}{\partial c} = \frac{1}{l}\left[U(-\alpha) - U(-\alpha - T(\bar{x}))\right] f(\bar{x})dx + \int_{\bar{x}}^{1} U'(-\alpha - T(x)) f(x)dx,$$

i.e., since $T(\bar{x}) = 0$,

$$\frac{\partial V}{\partial c} = \int_{\bar{x}}^{1} U'(-\alpha - T(x)) f(x)dx > 0.$$

The partial derivative with respect to p is:

$$\frac{\partial V}{\partial p} = -\int_{\bar{x}}^{1} \left[U(-\alpha) - U(-\alpha - T(x))\right] \frac{\partial f(x)}{\partial p} dx,$$

which is negative since $\frac{\partial f(x)}{\partial p} > 0$ for all $\bar{x} > x^*$. The partial derivative with respect to δ is:

$$\frac{\partial V}{\partial \delta} = -\int_{\bar{x}}^{1} \left[U(-\alpha) - U(-\alpha - T(x))\right] \frac{\partial f(x)}{\partial \delta} dx,$$

which is negative since $\frac{\partial f(x)}{\partial \delta} > 0$ for all $\bar{x} > x^*$.

Proof of Proposition 3

Let $V_1 = V_1(\delta_1, \alpha)$ and $V_2 = V_2(\delta_2, \alpha)$ with $\delta_1 \leq \delta_2$. From Proposition 2 we know that $\frac{\partial \Pi}{\partial \alpha} > 0$. Let V_1^{-1} denote the inverse function of V_1, with $V_1^{-1} = \sup\{\alpha | V_1(\alpha) \geq v\}$. It is possible to prove that V_1^{-1} is decreasing. Indeed, if $v' \leq v''$ then we have:

$$\{\alpha | V_1(\alpha) \geq v''\} \subset \{\alpha | V_1(\alpha) \geq v'\} \quad \Rightarrow \quad \sup\{\alpha | V_1(\alpha) \geq v''\} \leq \{\alpha | V_1(\alpha) \geq v'\} \quad \Rightarrow \quad V_1^{-1}(v'') \leq V_1^{-1}(v').$$

From Proposition 2 we know that $V_1(\alpha) \geq V_2(\alpha)$. Since V_1^{-1} is a decreasing function, this is equivalent to:

$$V_1^{-1}(V_1(\alpha)) \leq V_1^{-1}(V_2(\alpha)) \quad \Leftrightarrow \quad \alpha \leq V_1^{-1}(V_2(\alpha)).$$

The willingness to pay is the premium such that people choose to buy insurance, i.e., $V_1(\alpha_1^*) = pU(-l)$ and $V_2(\alpha_2^*) = pU(-l)$. Consequently, for $\alpha = \alpha_2^*$, we have:

$$\alpha_2^* \leq V_1^{-1}(V_2(\alpha_2^*)) \quad \Leftrightarrow \quad \alpha_2^* \leq V_1^{-1}(pU(-l)) \quad \Leftrightarrow \quad \alpha_2^* \leq \alpha_1^*,$$

which implies that $\frac{\partial \alpha^*}{\partial \delta} \leq 0$. Similarly, since $\frac{\partial V}{\partial c} \geq 0$, we can prove that $\frac{\partial \alpha^*}{\partial c} \geq 0$.

Proof of Proposition 4

Denote α_{lim}^* and α_{unlim}^* the willingness to pay with limited liability and unlimited guarantee, respectively. Let V_{lim} and V_{unlim} denote the expected utilities. Since V_{unlim} is continuous and decreasing, there necessarily exists a decreasing inverse function V_{unlim}^{-1}. The result that V_{unlim} is greater than V_{lim} for any value of α implies:

$$\forall \alpha, \quad V_{unlim}(\alpha) \geq V_{lim}(\alpha) \quad \Leftrightarrow \quad V_{unlim}^{-1}(V_{unlim}(\alpha)) \leq V_{unlim}^{-1}(V_{lim}(\alpha)) \quad \Leftrightarrow \quad \alpha \leq V_{unlim}^{-1}(V_{lim}(\alpha)).$$

The willingness to pay is the premium such that people choose to buy insurance, i.e., $V_{unlim}(\alpha_{ulim}^*) = pu(-l)$ and $V_{lim}(\alpha_{unlim}^*) = pu(-l)$. Consequently, for $\alpha = \alpha_{lim}^*$, we have:

$$\alpha_{lim}^* \leq V_{unlim}^{-1}(V_{lim}(\alpha_{lim}^*)) \quad \Leftrightarrow \quad \alpha_{lim}^* \leq V_{unlim}^{-1}(pU(-l)) \quad \Leftrightarrow \quad \alpha_{unlim}^* \leq \alpha_{lim}^*.$$

Proof of Proposition 5

The proofs are similar to the proof that $\partial V/\partial\delta < 0$ and $\partial\alpha^*/\partial\delta < 0$ in Propositions 2 and 3.

Proof of Proposition 6

For Region i, an increase in α_j is similar to an increase in the economic capital. The proof that $\frac{\partial V_i}{\partial\alpha_j} > 0$ is consequently similar to the proof that $\frac{\partial V}{\partial c} > 0$ in Proposition 2. As a result, the proof that $\frac{\partial\alpha_i^{**}}{\partial\alpha_j} > 0$ is equivalent to the proof that $\frac{\partial\alpha^*}{\partial c} > 0$.

Proof of Proposition 7

Let Z^k be the dichotomous variable equal to 1 when individual k, $k = 1..n$, has a claim. The correlation between risks is given by:

$$\begin{aligned} corr(Z^k,Z^l) &= \frac{cov(Z^k,Z^l)}{p(1-p)}, \\ &= \frac{\mathbb{P}(Z^k=1,Z^l=1)-p^2}{p(1-p)}, \\ &= \frac{(p_C)^2p^*+(p_N)^2(1-p^*)-p^2}{p(1-p)}. \end{aligned}$$

Replacing p_N and p_C by Equations 15 and 16 gives the correlation as a function of δ:

$$\begin{aligned} corr(Z^k,Z^l) &= \frac{p}{1-p}\left[\frac{p^*+(1-p^*)(1-\delta)}{(1-\delta-\delta p^*)^2}-1\right], \\ &= \frac{p}{1-p}\left[g(\delta)-1\right]. \end{aligned}$$

The derivative of g with respect to δ is given by:

$$\frac{dg}{d\delta} = 2p^* \times \frac{2-\delta+\delta p^*}{1-\delta-\delta p^*}.$$

The derivative $\frac{dg}{d\delta}$ is positive for $\delta \in [0;1]$, which implies that $corr(X^k,X^l)$ is a positive function of δ. Moreover, if $\delta = 0$, then $p_N = p_C = p$ in Equations 15 and 16. In that case, we have $\mathbb{P}(Z^k = 1, Z^l = 1) = p^2$ and $corr(Z^k,Z^l) = 0$, which corresponds to the independent case. On the other hand, if $\delta = (1-p)/(1-p^*)$, then $p_N = (p-p^*)/(1-p^*)$ and $p_C = 1$. In that case, we have $\mathbb{P}(Z^k = 1, Z^l = 1) = (p^*(1-p)^2)/(1-p*)+p^2$ and $corr(Z^k,Z^l) = [p^*(1-p)]/[p(1-p^*)]$, which corresponds to the perfectly dependent case when $p = p^*$, i.e., when $\mathbb{P}(Z^k = 1, Z^l = 1) = p$ and $corr(Z^k,Z^l) = 1$. In other words, the lower δ, the closer p_N and p_C and the more independent the risks.

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