Legislative Bargaining and Public Good Provision: An Equilibrium Characterization

Working Paper

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November 28, 2011

Abstract

This paper characterizes a no-delay equilibrium in stationary strategies for a legislative bargaining game in which legislators can allocate available revenue to each legislator's district in addition to a public good. The model closely follows that of Banks and Duggan (2000) who establish the existence of such an equilibrium. The main result is that equilibrium proposals randomize amongst all legislators only if the game is symmetric. Equilibrium is fully characterized for the symmetric case.

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1 Introduction

Bargaining has long been a topic of interest to economists. One of the first extensive theoretical treatments of bargaining was given by Nash (1950) and it has received extensive attention ever since. The Nash bargaining game essentially asks the question of how two people come to an agreement over the division of one dollar between each other. The bargaining game was further elaborated on by Ståhl (1972) and later by Rubinstein (1982). The progression of bargaining theory is synthesized nicely in Binmore and Dasgupta (1987). This early work focused almost exclusively on 2-person bargaining games.

Political scientists, however, are often interested in situations in which bargaining involves more than just two individuals. In fact, legislative decisions about how to appropriate money to projects preferred by different legislatures can be thought of as the result of a bargaining process. Baron and Ferejohn (1989) extend the Nash-bargaining game to include a large number, n, of players in what is now referred to as a legislative bargaining game. Their model involves dividing a dollar amongst the legislators through a series of proposed allocations and majority voting. In each time period, one legislator is randomly recognized to propose a division of the dollar and then the proposal in enacted if it garners majority support. If the proposal fails to gain majority support the game moves to the next time period where another legislator is randomly selected to make a proposal. The game ends when a proposal gains majority support and the dollar is split accordingly with each player receiving payoffs which are discounted according to the amount of time is required to pass a proposal. The main results in legislative bargaining theory are reviewed in Austen-Smith and Banks (2005).

Kalandrakis (2004) characterizes a markov-perfect equilibrium to a 3-player divide the dollar legislative bargaining game in which policy (a division of the dollar) evolves through time over a sequence of bargaining rounds. In each round a proposal is pitted against a status quo in a majority rule vote. The status quo for the following round is the division of the dollar enacted in the previous round. In a small number of rounds the equilibrium converges so that the proposer in each round is able to capture the entire dollar for herself leaving nothing for the other legislators. This result rests critically on the assumption of weakly dominant voting strategies. Kalandrakis is able to characterize the equilibrium to the symmetric game by describing the equilibrium as a solution to a system of equations.

The Baron and Ferejohn (1989) paper marked spurred the beginning of what is now an evolving literature on legislative bargaining. While Baron and Ferejohn focus on the divide the dollar game, which is entirely distributional, many subsequent papers analyze legislative bargaining problems over spatial issue spaces. These include the papers of Banks and Duggan (Banks and Duggan (2000) and Banks and Duggan (2006)). Baron (1996) analyzes a dynamic legislative bargaining model over a uni-dimensional collective goods program. The dynamics of the status quo are the same as in Kalandrakis (2004) and generate a "generalized" median voter theorem. Cho and Duggan (2003) establish the uniqueness of stationary equilibrium in legislative bargaining models with a one dimensional spatial issue space and quadratic utilities. Jackson and Moselle (2002) consider the problem of dividing a dollar and simultaneously choosing a point in a one-dimensional policy space. Many of the legislative bargaining models that exist which analyze non-divide the dollar issues consider only policy spaces which are purely ideological. This is true for both Cho and Duggan (2003) and Jackson and Moselle (2002). Those that do allow for a policy space which could be linked to distributional issues through budgetary concerns (Banks and Duggan (2000) and Banks and Duggan (2006)) limit their analysis to general existence and comparative statics results and do not give a characterization of equilibrium in such a case. One exception is Jackson (2011) which analyzes a legislative bargaining model in which three legislators divide revenue between themselves and a public good. The focus of this model is on the institutional incentives of ear-marking rather than on bargaining. The bargaining model this paper employs is rather simplistic allowing for only one legislative round due to the technical difficulties imposed by earmarking.

Banks and Duggan (2000) develop a general legislative bargaining model that extends the Baron and Ferejohn framework to an abstract policy space and an abstract voting rule. This framework allows bargaining to take place over policy dimensions that could be abstract issues on a left-right continuum, divide-the-dollar issues, and could also involve the allocation of funds to public goods. Banks and Duggan (2006) further generalizes the model by allowing a general status-quo policy as opposed to the no allocation status quo which is implicit in Baron and Ferejohn (1989) and Banks and Duggan (2000). While Banks and Duggan establish the existence of equilibrium in a legislative bargaining game with majority rule voting and a policy space that includes both payoffs to legislators based on allocations to their home district and public goods, no characterization of such an equilibrium exists in the literature.

The present paper does not model the policy making process as dynamic across legislative sessions as in Kalandrakis (2004). Instead, the model is limited to a dynamic model of one legislative session in the spirit of Baron and Ferejohn (1989) and Banks and Duggan (2000). However, the method of solution closely mimics that of Kalandrakis. By limiting the model to one legislative session two generalizations are possible. Firstly, the present model considers a more general bargaining game that includes not only a divide the dollar distribution game but also includes expenditure on a public good. Secondly, the game need not be symmetric. Each player is explicitly allowed to have a distinct utility function. This paper explicitly characterizes equilibrium proposals and continuation values in a legislative bargaining game where revenue can be allocated to individual legislator's districts (as in a divide the dollar game) or to a public good. The distributive dimension is explicitly linked to the public good dimension through the budget.

2 Model

Three legislators, often indexed by labeled i, j, or k must allocate a given revenue, R, to a public good and a distribution to each legislator's district. Let x_i denote the revenue allocated to legislator i's district and let x_y denote revenue allocated to the public good. Legislator preferences depend on the allocation of revenue, $x = (x_1, x_2, x_3, x_y)$, and the number of time periods elapse before the allocation is received, t, as given by the following utility function where $0 < a < 1$.[1]

$$u_j(x,t) = \delta^t \left(x_j + b_j \left(x_y \right)^a \right)$$

The distribution of R across its possible uses is determined through a legislative bargaining game in the spirit of Baron and Ferejohn (1989) and Banks and Duggan (2000). In the first time period one of the legislators is randomly recognized to propose a distribution of revenue. Each legislator is selected to make a proposal with probability $\frac{1}{3}$. Legislators then vote to either pass the proposal or not. Let i be the recognized legislator and j be generic. If the proposal garners majority support then the game ends with each legislator receiving utility according to $u_j(x,0) = x^i_j + b_j \left(x^i_y \right)^a$. If the proposal is not passed then the game progresses to the next time period with another legislator being recognized (possibly the same as in the previous time period) to make a proposal and that proposal being voted on by majority rule. If the proposal is passed then each legislator gets utility according to $u_j(x,1)$ and if not then the game progresses to the next round of proposals and votes. The game goes on in this manner until a proposal is made which receives majority support.

3 Results

For any particular subgame perfect equilibrium let $v(t,g)$ be the vector of individual values that results from equilibrium play. The continuation value, $\delta v_i(t,g)$, is the value to the game for player i if the legislature rejects the current proposal and moves to subgame g. Ex ante values are given by v_i, $i = 1, \ldots, 3$. In a stationary equilibrium, $v_i(t,g) = v_i$ for all t. By theorem (1) in Banks and Duggan (2000) we know that a stationary no-delay

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[1] $a \in (0,1)$ guarantees that $u_i(x,t)$ is quasiconcave in x.

equilibrium exists and when $\delta < 1$ all stationary equilibria are no-delay. Therefore, no existence proof will be presented in this paper as this result has already been established in the literature. I now proceed to characterize the no-delay equilibrium in stationary strategies for the legislative bargaining game with a public good. I also make comparison's to the core of the bargaining game. From the Samuleson rule we see that in the bargaining core the level of public good spending must be

$$x_y^{**} = \min\left\{R, \left(\frac{1}{a(b_1+b_2+b_3)}\right)^{\frac{1}{a-1}}\right\}.$$

Let x_k^{ij} be the proposed distribution to legislator k in $i's$ proposal when legislator j is selected by i to be a coalition member (i,j, and k are generic, it is possible that $j = k$). The proposed spending on public good provision by legislator i when legislator j is chosen as coalition member is given by x_y^{ij}. The probability that i chooses j to be a coalition member is p_j^i and p_k^i is the probability i chooses k. Therefore a stationary proposal strategy for player i contains a proposed spending plan $x^i = (x_i^{ij}, x_j^{ij}, x_k^{ij}, x_y^{ij}, x_i^{ik}, x_j^{ik}, x_k^{ik}, x_y^{ik})$ and a probability distribution over choice of coalition member p_j^i and p_k^i.

Proposition 1. *Suppose that an SP equilibrium in stationary strategies exists in which all players use strictly mixed proposal strategies, then the game must be entirely symmetric with* $b_i = b$ *and* $v_i = v$ *for all* i.

Proof. Let $v = (v_1, v_2, v_k)$ be the continuation values given stationary proposal strategies (x^1, x^2, x^3).

For each player,i , the proposal strategy must solve the constrained optimization problem given below.

$$\max_{p_j^i, p_k^i, x_i^{ij}, x_j^{ij}, x_k^{ij}, x_y^{ij}, x_i^{ik}, x_j^{ik}, x_k^{ik}, x_y^{ik}} p_j^i\left(x_i^{ij} + b_i\left(x_y^{ij}\right)^a\right) + p_k^i\left(x_i^{ik} + b_i\left(x_y^{ik}\right)^a\right) \tag{1}$$

subject to

$$p_j^i \left(x_i^{ij} + b_i \left(x_y^{ij}\right)^a\right) + p_k^i \left(x_i^{ik} + b_i \left(x_y^{ik}\right)^a\right) \geq \delta v_i \quad 1a$$

$$x_j^{ij} + b_j \left(x_y^{ij}\right)^a \geq \delta v_j \quad 1b$$

$$x_k^{ik} + b_k \left(x_y^{ik}\right)^a \geq \delta v_k \quad 1c$$

$$x_i^{ij} + x_j^{ij} + x_k^{ij} + x_y^{ij} \leq R \quad 2a$$

$$x_i^{ik} + x_j^{ik} + x_k^{ik} + x_y^{ik} \leq R \quad 2b$$

$$p_j^i + p_k^i = 1 \quad 3a$$

$$p_j^i \geq 0 \quad 3b$$

$$p_k^i \geq 0 \quad 3c$$

$$x^i \geq 0 \quad 4$$

To solve for x^{i*} we must consider which constraints will bind. 1a will tend to be unbinding while 1b and 1c will bind with equality as the proposer will always choose to payoff its coalition members in a minimal way. This also directly implies that $x_k^{ij} = x_j^{ik} = 0$. 2a and 2b both bind with equality as it will always be optimal to propose a spending plan which uses all revenue. 3a must bind but 3b and 3c will not. It is now possible to use substitution to rewrite the optimization problem as follows.

$$\max_{p_j^i, p_k^i, x_y^{ij}, x_y^{ik}} p_j^i \left(R - x_y^{ij} - \delta v_j + b_j \left(x_y^{ij}\right)^a + b_i \left(x_y^{ij}\right)^a\right) + p_k^i \left(R - x_y^{ik} - \delta v_k + b_k \left(x_y^{ik}\right)^a + b_i \left(x_y^{ik}\right)^a\right) \tag{2}$$

subject to

$$p_j^i\left(R - x_y^{ij} - \delta v_j + b_j\left(x_y^{ij}\right)^a + b_i\left(x_y^{ij}\right)^a\right) + p_k^i\left(R - x_y^{ik} - \delta v_k + b_k\left(x_y^{ik}\right)^a + b_i\left(x_y^{ik}\right)^a\right) \geq \delta v_i \quad 1$$

$$p_j^i + p_k^i = 1 \quad 2a$$

$$p_j^i \geq 0 \quad 2b$$

$$p_k^i \geq 0 \quad 2c$$

$$x_y^{ij} \geq 0 \quad 3a$$

$$x_y^{ik} \geq 0 \quad 3b$$

Taking the derivative of the objective function wrt x_y^{ij} and setting it equal to zero yields

$$p_j^i\left(a(b_j + b_i)\left(x_y^{ij}\right)^{a-1} - 1\right) = 0.$$

In a strictly mixed strategy equilibrium it must be the case that $p_j^i > 0$ which directly implies that

$$x_y^{ij*} = \left(\frac{1}{a(b_j + b_i)}\right)^{\frac{1}{a-1}}.$$

Similarly the FOC wrt x_y^{ik} along with $p_k^i > 0$ yields

$$x_y^{ik*} = \left(\frac{1}{a(b_k + b_i)}\right)^{\frac{1}{a-1}}.$$

Taking the derivative of the objective function wrt p_j^i and setting it equal to zero yields

$$R - x_y^{ij} - \delta v_j + b_j\left(x_y^{ij}\right)^a + b_i\left(x_y^{ij}\right)^a = 0.$$

Substituting $x_y^{ij} = x_y^{ij*}$ implies that

$$\delta v_j = R - \left(\frac{1}{a(b_j + b_i)}\right)^{\frac{1}{a-1}} + (b_j + b_i)\left(\frac{1}{a(b_j + b_i)}\right)^{\frac{a}{a-1}}. \tag{3}$$

Similarly the FOC wrt p_k^i and substituting $x_y^{ik} = x_y^{ik*}$ implies that

$$\delta v_k = R - \left(\frac{1}{a(b_k + b_i)}\right)^{\frac{1}{a-1}} + (b_k + b_i)\left(\frac{1}{a(b_k + b_i)}\right)^{\frac{a}{a-1}}. \tag{4}$$

In equilibrium the following equations must then hold because conditions 3 and 4 must hold for each legislator.

$$\delta v_2 = R - \left(\frac{1}{a(b_2 + b_1)}\right)^{\frac{1}{a-1}} + (b_2 + b_1)\left(\frac{1}{a(b_2 + b_1)}\right)^{\frac{a}{a-1}} \tag{5}$$

$$\delta v_3 = R - \left(\frac{1}{a(b_3 + b_1)}\right)^{\frac{1}{a-1}} + (b_3 + b_1)\left(\frac{1}{a(b_3 + b_1)}\right)^{\frac{a}{a-1}} \tag{6}$$

$$\delta v_1 = R - \left(\frac{1}{a(b_1 + b_2)}\right)^{\frac{1}{a-1}} + (b_1 + b_2)\left(\frac{1}{a(b_1 + b_2)}\right)^{\frac{a}{a-1}} \tag{7}$$

$$\delta v_3 = R - \left(\frac{1}{a(b_3 + b_2)}\right)^{\frac{1}{a-1}} + (b_3 + b_2)\left(\frac{1}{a(b_3 + b_2)}\right)^{\frac{a}{a-1}} \tag{8}$$

$$\delta v_1 = R - \left(\frac{1}{a(b_1 + b_3)}\right)^{\frac{1}{a-1}} + (b_1 + b_3)\left(\frac{1}{a(b_1 + b_3)}\right)^{\frac{a}{a-1}} \tag{9}$$

$$\delta v_2 = R - \left(\frac{1}{a(b_2 + b_3)}\right)^{\frac{1}{a-1}} + (b_2 + b_3)\left(\frac{1}{a(b_2 + b_3)}\right)^{\frac{a}{a-1}} \tag{10}$$

Equations 5 and 10 can only both be true if $b_3 = b_1$. Equations 6 and 8 can only both be true if $b_1 = b_2$. Equations 7 and 9 can only both be true if $b_2 = b_3$. Together this implies $b_1 = b_2 = b_3$ which then implies that $v_1 = v_2 = v_3$. □

The next proposition now gives a characterization of no-delay equilibrium in stationary strategies when the game is fully symmetric.

Proposition 2. *Suppose that* $b_1 = b_2 = b_3$ *then a no-delay equilibrium in stationary strategies is characterized by the following*

1. *If* $R < \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}$ *then*

$$\begin{aligned} x_y^{i*} &= R \\ x_1^{i*} = x_2^{i*} = x_3^{i*} &= 0 \\ v &= bR^a \end{aligned}$$

2. *If* $R \geq \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}$ *and* $R < \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} + \left(\frac{3b}{\delta} - 2b\right)\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}}$ *then*

$$\begin{aligned} x_y^{i*} &= \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} \\ x_i^{i*} &= R - \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} \\ x_j^{i*} = x_k^{i*} &= 0 \\ v &= \left(\frac{1}{3}\right)\left(R - \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}\right) + b\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}} \end{aligned}$$

3. *If* $R \geq \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}$ *and* $R \geq \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} + \left(\frac{3b}{\delta} - 2b\right)\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}}$ *then*

$$\begin{aligned}
x_y^{i*} &= \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} \\
x_i^{i*} &= R - \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} - \left(\frac{\delta}{3}\right)\left(R - \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}\right) - b\left(\frac{2\delta}{3} - 1\right)\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}} \\
x_j^{i*} &= \left(\frac{\delta}{3}\right)\left(R - \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}\right) + b\left(\frac{2\delta}{3} - 1\right)\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}} \\
x_k^{i*} &= 0 \\
v &= \left(\frac{1}{3}\right)\left(R + 2b\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}} - \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}\right)
\end{aligned}$$

4.

$$p_j^i + p_k^i = 1$$

$$p_i^j + p_i^k = 1$$

Proof. Each item is addressed in turn.

1. Suppose $R < \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}$
2. Suppose $R \geq \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}$ and $R < \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} + \left(\frac{3b}{\delta} - 2b\right)\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}}$
3. Suppose $R \geq \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}}$ and $R \geq \left(\frac{1}{2ab}\right)^{\frac{1}{a-1}} + \left(\frac{3b}{\delta} - 2b\right)\left(\frac{1}{2ab}\right)^{\frac{a}{a-1}}$
4.

□

Corollary 1. *Suppose that* $b_1 = b_2 = b_3$*, then*

$$x_y^{i*} \leq x_y^{**}.$$

Proposition 1 is a powerful result. It establishes that strictly mixed proposal strategies will only be an equilibrium if the bargaining game is entirely symmetric. Therefore, when the game is not fully symmetric we do not need to search for equilibria with strictly mixed proposal strategies. If the game is not symmetric at least one player must be playing a pure proposal strategy. The following proposition characterizes an no-delay equilibrium in stationary strategies when $b_1 \leq b_2 < b_3$ where players 1 and 2 play pure proposal strategies but player 3 plays a mixed strategy in which she randomizes over which legislator (1 or 2) to include in a winning coalition.

Proposition 3. *Suppose that* $b_1 \leq b_2 < b_3$ *and* $R > \left(\frac{1}{a(b_1+b_2+b_3)}\right)^{\frac{1}{a-1}}$*, then a no delay equilibrium in stationary strategies is characterized by the following*

$$x_y^{13*} = x_y^{31*} = \left(\frac{1}{a(b_1+b_3)}\right)^{\frac{1}{a-1}}$$

$$x_y^{23*} = x_y^{32*} = \left(\frac{1}{a(b_2+b_3)}\right)^{\frac{1}{a-1}}$$

$$x_1^{13*} = R - x_y^{13*} - \delta v_3 + b_3(x_y^{13*})^a$$

$$x_2^{13*} = x_1^{23*} = 0$$

$$x_2^{23*} = R - x_y^{23*} - \delta v_3 + b_3(x_y^{23*})^a$$

$$x_3^{31*} = R - x_y^{31*} - \delta v_1 + b_1(x_y^{31*})^a$$

$$x_3^{32*} = R - x_y^{32*} - \delta v_2 + b_2(x_y^{32*})^a$$

Define A, B, C, and D as follows:

$$A \equiv R - x_y^{13*} + (b_1+b_3)x_y^{13*^{a}} + b_1 x_y^{23*^{a}}$$

$$B \equiv R - x_y^{23*} + (b_2+b_3)x_y^{23*^{a}} + b_2 x_y^{13*^{a}}$$

$$C \equiv R - x_y^{31*} + (b_1+b_3)x_y^{31*^{a}}$$

$$D \equiv R - x_y^{32*} + (b_2 + b_3)x_y^{32*^a}$$

$$v_1 = \frac{\frac{\delta^2 P(B-A)}{(3-\delta P)(3-\delta(1-P))(3-2\delta)} + \frac{\delta(PC+(1-P)D)-B}{3-\delta(1-P)}}{\frac{\delta^2 P(3-\delta(1-P))+\delta^2(1-P)(3-\delta P)}{(3-\delta P)(3-\delta(1-P))(3-2\delta)}}$$

$$v_2 = \frac{\frac{\delta^2 (1-P)(A-B)}{(3-\delta P)(3-\delta(1-P))(3-2\delta)} + \frac{\delta(PC+(1-P)D)-A}{3-\delta P}}{\frac{\delta^2 P(3-\delta(1-P))+\delta^2(1-P)(3-\delta P)}{(3-\delta P)(3-\delta(1-P))(3-2\delta)}}$$

$$v_3 = \frac{PC + (1-P)D + \left(\frac{\delta}{3-2\delta}\right)\left(\frac{AP}{3-\delta P} + \frac{B(1-P)}{3-\delta(1-P)}\right)}{\frac{\delta^2 P(3-\delta(1-P))+\delta^2(1-P)(3-\delta P)}{(3-\delta P)(3-\delta(1-P))(3-2\delta)}}$$

Where P solves the following equation

$$C - D = \delta(v_1 - v_2).$$

Proof. In the proposed equilibrium, players 1 and 2 play pure proposal strategies while player 3 plays a mixed proposal strategy. That is, players 1 and 2 always choose to include player 3 in a winning coalition while player 3 chooses to include player 1 with probability P and player 2 with probability $1 - P$.

Consider player 3's optimal proposal which solves the following constrained optimization problem.

$$\max_{P, x_y^{31}, x_y^{32}} P\left(R - x_y^{31} - \delta v_1 + (b_1 + b_3)x_y^{31^a}\right) + (1-P)\left(R - x_y^{32} - \delta v_2 + (b_2 + b_3)x_y^{32^a}\right)$$

s.t.

1.

$$0 \leq P \leq 1$$

2.

$$x_1^{31} = \delta v_1 - b_1 x_y^{31^a} \geq 0$$

3.

$$x_2^{32} = \delta v_2 - b_2 x_y^{32^a} \geq 0$$

4.

$$x_3^{31} = R - x_y^{31} - \delta v_1 + b_1 x_y^{31^a} \geq 0$$

5.

$$x_3^{32} = R - x_y^{32} - \delta v_2 + b_2 x_y^{32^a} \geq 0$$

6.

$$P\left(R - x_y^{31} - \delta v_1 + (b_1 + b_3) x_y^{31^a}\right) + (1 - P)\left(R - x_y^{32} - \delta v_2 + (b_2 + b_3) x_y^{32^a}\right) - \delta v_3 \geq 0$$

It is clear from this optimization problem that if $P \in (0,1)$ that the optimal public good spending proposal depends on which legislator 3 chooses to buy off with

$$x_y^{31*} = \left(\frac{1}{a(b_1 + b_3)}\right)^{\frac{1}{a-1}}$$

and

$$x_y^{32*} = \left(\frac{1}{a(b_2 + b_3)}\right)^{\frac{1}{a-1}}.$$

Next, expressions for v_1, v_2, and v_3 can be written

$$v_1 = \frac{1}{3}\left(R - x_y^{13*} - \delta v_3 + (b_1 + b_3) x_y^{13*^a}\right) + \frac{b_1 x_y^{23*}}{3} + \frac{P \delta v_1}{3}$$

$$v_2 = \frac{b_1 x_y^{13*}}{3} + \frac{1}{3}\left(R - x_y^{23*} - \delta v_3 + (b_2 + b_3) x_y^{23*^a}\right) + \frac{b_1 x_y^{23*}}{3} + \frac{(1 - P)\delta v_2}{3}$$

$$v_3 = \frac{2\delta v_3}{3} + \frac{1}{3}\left(P\left(R - x_y^{31*} - \delta v_1 + (b_1 + b_3)x_y^{31*^a}\right) + (1-P)\left(R - x_y^{32*} - \delta v_2 + (b_2 + b_3)x_y^{32*^a}\right)\right)$$

Using the definitions of A, B, C, and D as in listed in the proposition, expressions for v_1, v_2, and v_3 can be reduced to

$$v_1 = \frac{A}{3 - \delta P} - \frac{\delta v_3}{3 - \delta P} \tag{11}$$

$$v_2 = \frac{B}{3 - \delta(1-P)} - \frac{\delta v_3}{3 - \delta(1-P)} \tag{12}$$

$$v_3 = \frac{PC + (1-P)D}{3 - 2\delta} - \frac{\delta(Pv_1 + (1-P)v_2}{3 - 2\delta} \tag{13}$$

The first order condition on P directly results in the following equation

$$R - x_y^{31*} - \delta v_1 + (b_1 + b_3)x_y^{31*^a} = R - x_y^{32*} - \delta v_2 + (b_2 + b_3)x_y^{32*^a}.$$

Which reduces to

$$C - D = \delta(v_1 - v_2). \tag{14}$$

Equations 11-13 are linear in v_1, v_2, and v_3. It is therefore a straightforward exercise to setup the linear system of equations and solve for v_1, v_2, and v_3 by applying Kramer's rule. This procedure yields the expressions for v_1, v_2, and v_3 listed in the proposition. However, these equations all depend on the value of P. The equilibrium value of P must be found by solving equation 14.

□

Corollary 2. *Suppose that* $b_1 \leq b_2 < b_3$ *and* $R > \left(\frac{1}{a(b_1+b_2+b_3)}\right)^{\frac{1}{a-1}}$, *then*

$$x_y^{13*} = x_y^{31*} \leq x_y^{23*} = x_y^{32*} < x_y^{**}$$

(strict if $b_1 < b_2$*)*

Text to be added

4 Conclusion

Text to be added

References

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