Learning More by Doing Less

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Abstract

Self-interested agents (e.g., interest groups, researchers) produce verifiable evidence in an attempt to convince a principal (e.g., legislator, funding organization) to act on their behalf (e.g., introduce legislation, fund research). Agents provide less informative evidence than the principal prefers since doing so maximizes the probability the principal acts in their favor. If the principal faces budget or other constraints that limit the number of agents whose proposals she can support, then agents produce more-accurate evidence as they compete for priority. Under reasonable conditions, the principal is better off when her capacity to act is limited.

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1 Introduction

Imagine that a legislator must decide which earmark projects to bring before the appropriations subcommittee. Each project is supported by an interest group, who benefits only if its project is funded. In order to convince the legislator that its project deserves funding, an interest group may collect evidence about the project's impact on the local economy, or popularity among the legislator's constituents. Although it may be difficult for the interest group to falsify evidence outright, the evidence that it chooses to produce may not completely reveal all relevant information about the costs and benefits of the project. For example, an interest group decides how many constituents to poll about their support for a project. If it surveys few constituents, then the survey outcome (whatever it is) is not very informative about the project's popularity. If the interest group polls a large portion of constituents, then the resulting evidence is significantly more informative. In some cases, the legislator may be able to fund any project she believes has an expected benefit. However, empirical evidence exists to the contrary. According to (Frisch and Kelly 2010, 2011), the appropriations committee systematically limits the total amount of earmark funding that legislators can request for their home districts. A similar story could describe a legislator selecting which pieces of legislation to pursue; time and procedural constraints may prevent her from actively pursuing all reforms that she believes to be better than the status quo.

Alternatively, consider an organization offering grant money to researchers. Many grant applications require some description of preliminary findings, which may be difficult to falsify. However, these findings may vary in how informative they are about the project's overall promise. Highly speculative preliminary findings may not be extremely informative about the research project's ultimate chances of success. In many cases, one may conduct preliminary research that produces more-convincing evidence about a proposal's ultimate chance of success. Depending on the organization's budget, it may have the capability to approve all grant proposals if it believes it is appropriate to do so, or it may face binding budget constraints that potentially prevent it from funding all projects even if it believes all of them to be worthwhile.

Motivated by these examples and others, we develop a game theoretic model in which two independent, self-interested agents produce verifiable evidence in am effort to convince a principal to accept proposals of unknown quality. Each agent supports a different proposal. An agent benefits if the principal accepts his proposal, regardless of its quality. The principal, however, prefers to accept good proposals and reject bad proposals. We focus on situations in which neither principal nor agents know the qualities of the proposals (at the beginning of the interaction). For instance, both the interest groups and the legislator may be uncertain about constituent support for an earmark project or policy reform prior to polling. Before she makes decides which proposal(s) to accept, the principal observes evidence produced by the agents, which conveys information about proposal quality. This evidence is verifiable: agents cannot manipulate or misrepresent the evidence that they uncover (nor would they choose to keep it hidden). However, *before* uncovering evidence, agents determine how *informative* their evidence will be (e.g. the interest group chooses how many constituents to poll). After observing the evidence, the principal decides which, if any, proposals to accept. If the principal can accept all proposals, then her capacity is *unlimited*. However, as is more typical, the principal may be constrained in her ability to accept proposals. If she is unable to accept all proposals, then she has *limited* capacity.[1]

We begin by identifying a strong conflict of interest between the principal and agents that exists whenever capacity is unlimited.[2] The principal benefits from accepting good proposals and rejecting bad proposals. She therefore prefers that agents produce fully-informative evidence, ensuring that she always makes the right decision. Agents, on the other hand, want to maximize the probability that the principal accepts their proposals. Because of this, the agents produce less-than-fully-informative evidence. To understand why an agent keeps the principal less-informed, imagine first that the principal is initially optimistic about the proposal. If she would be willing to accept based on the prior alone, there is no reason for the agent to produce informative evidence. On the other hand, if the principal would reject the proposal based on the prior, then the agent must produce informative evidence in his favor for his proposal to be accepted. However, he will choose to supply evidence that is just informative enough for a good outcome to sway the principal in favor of the proposal. He will never make the principal fully informed. In fact, when capacity is unlimited, the evidence supplied by the agents is effectively worthless. The principal's payoff is the same as if she never observes the signal and only acts according to her prior.[3]

The primary goal of our analysis is to show that limited capacity can mitigate or eliminate this conflict of interest between principal and agents. First, we show that limited capacity increases the quality of evidence produced by the agents. Second, we show that these informational benefits can dominate the costs that come with limited capacity. The principal often expects be better off with limited capacity.

Under limited capacity, it is not enough for agents to convince the principal that their proposal is likely beneficial. They also must convince the principal their proposal is a better choice than the other proposal. In this sense, limited capacity creates a competition in which agents produce more-informative evidence as they vie for priority. In equilibrium, the principal is exposed to more informative evidence under limited capacity than under unlimited capacity. However, a more informed principal is not necessarily better off. Limited capacity imposes a capacity constraint that has the potential to make the principal worse off; she cannot accept both proposals even when she expects to benefit from doing so. Our second set of results demonstrate that the informational benefits of limited capacity can dominate the downside of being constrained. Indeed, as long as she is not extremely optimistic about both proposals, the principal strictly prefers limited capacity.[4]

The analysis proceeds in two parts. In Section 3, we begin with a relatively simple version

[1]These types of capacity constraints could arise for a number of different reasons; for example, the principal may have a limited budget (e.g. to fund projects or make purchases), or limited time (e.g. to introduce legislation).

[2]The discussion of the results in this paragraph focuses on the more-general model of Section 4. While the results also hold in the simpler binary model, some of the results in the simple case are less clear-cut.

[3]This is true even though there are no direct costs to producing informative evidence. Agents do not supply informative evidence in order to strategically manipulate the principal's beliefs, not to save on costs. Including an explicit cost of supplying evidence does not change the qualitative nature of the results.

[4]Again, this result is true in the model with general signal structures. In the simple game we initially present, the result is less clear cut.

of the game in which we make two simplifying assumptions. First, we assume that proposals are ex ante identical. Second, we model evidence in a simple fashion. We assume that evidence is the realization of a binary signal that correctly reflects the true quality of the proposal with some probability α, chosen by the agent. The simple signal structure leads to a straightforward interpretation: α represents the accuracy (informativeness) of the evidence presented by an agent. We rely on this version of the game to develop intuition for the results.

In Section 4, we relax both simplifying assumptions, developing a more-general version of the game. In this section we allow for heterogeneous proposals; the prior beliefs associated with each proposal need not be identical. We also do not constrain an agent's evidence production to be represented by a binary signal structure, nor do we require that signals belong to a particular parametric class. We allow agents to design the experiment (i.e. the signal) that they use to produce evidence in a general way. Despite the complexity of the game in this environment, we are able to fully solve the model and provide a complete characterization of the equilibrium under all parameter values. The primary results are similar under both the binary game and the general game: the principal is better informed and (sometimes) better off when her capacity is limited. In addition to showing that the main results hold in a general environment, the section provides a series of novel results concerning agent asymmetries that were not present in the more-simple game.

Section 5 discusses a number of applications, including the legislator and grant writing examples discussed above, as well as additional examples involving a consumer making purchase decisions, a college deciding which applicants receive admissions or scholarships, a firm executive deciding which products to bring to market or which divisions to expand, and the FDA choosing which drugs to approve. In each of these situations, limited capacity may be a reality of the decision making process. Whereas the costs of limited capacity are largely understood, we demonstrate that these limitations may also have informational benefits. These benefits can be so large that the principal prefers to have capacity limitations.

2 Related Literature

The insightful work of Austen-Smith and Wright (1992) (henceforth AW) presents a model of adversarial evidence production. As in our model, both the politician (principal) and lobbyists (agents) are uncertain about the qualities of competing alternatives, and the politician must rely on lobbyists to supply evidence. A number of key differences between AW and our model exist, however. First, our evidentiary structure is more complex. In AW, agents make a binary decision about whether or not to produce evidence; the accuracy of their evidence is not a choice variable. In our framework, agents choose not only whether to produce evidence,[5] they also choose the informativeness of the evidence that they do produce. The main focus of our analysis is on the informativeness of evidence chosen by the agents, a consideration that is not possible in AW.

[5] The decision not to produce evidence is equivalent to sending an uninformative signal.

Second, the agents in AW represent mutually exclusive policy alternatives (e.g., one interest group advocating in support of and the other against a smoking ban). In our framework, choosing an action that benefits one agent does not necessarily prevent the principal from simultaneously choosing an action that benefits the other agent; proposals are not, by nature, mutually exclusive alternatives. This does not mean that the principal is necessarily able to act in favor of both agents, just that her inability to do so is caused by factors such as limited budgets or time constraints and is not an inherent feature of the proposals under consideration. This allows our framework to describe a variety of situations which do not necessarily fit into the AW framework (e.g., college admissions, grant funding, earmarks). It also makes for a natural comparison between outcomes when the principal is able to act in favor of all agents (e.g., has a sufficiently large budget to fund all grant proposals if she sees fit) and outcomes when the principal is constrained (e.g., due to budget limitations) in her ability to support agents.[6]

In recent articles, Brocas et al. (in press) and Gul and Pesendorfer (in press) analyze dynamic models of adversarial evidence production, where—as in our analysis–the correct course of action is unknown to all parties, and agents supply evidence that is publicly observed. Like AW, however agents have opposing preferences over a single policy and agents do not control the quality of evidence directly.[7] Furthermore, in these models the decisions to acquire information are made in a sequentially rational manner. These works are therefore best suited to describing prolonged advertising or persuasion campaigns, rather than to the types of decisions we study.

Other articles relate to our underlying evidence production framework but do not consider either direct or indirect competition between agents. Brocas and Carrillo (2007) study a dynamic information acquisition game, in which a leader has the capability to generate information that influences the decision of a follower with different preferences.[8] Kamenica and Gentzkow (in press) consider the problem of a sender who tries to influence the action of a receiver by designing a signal whose realization will be observed by the receiver prior to choosing an action. Like Brocas and Carrillo, these authors characterize environments in which the sender benefits from persuasion. These results are closely related to our results for a decision making environment with unlimited capacity.

[6]Additional differences between AW and our paper include AW's assumptions that signal collection is costly, and that agents privately observe their signals before deciding whether to share them with the principal (although the search choice is observed). Instead, for reasons discussed in the analysis, we assume that signal collection is without cost and any signal is publicly observable (although these assumptions are not required for our main results to hold qualitatively). Although the AW assumptions provide a more realistic description of some settings, we believe they would likely distract from our results. Adding costs of signal accuracy, for example, will make agents even less likely to collect informative signals, and would distract from the more interesting finding that agents prefer to keep the principal less-than-fully informed, even when signals are costless to produce.

[7]Both articles are concerned with the effect of cost differences on equilibrium outcomes. In both analyses, a public draw from a given informative signal is revealed, as long as one party chooses to exert effort. Thus, taking a new draw can be regarded as a decision to reveal new information to the decision maker, affecting her beliefs about which policy is best. However, in these analyses no action directly affects the informativeness of the signal itself. (Note that if players could committ to acquire a certain number of signal realizations initially, the adversaries would be competing in a manner similar to our model.)

[8]Brocas and Carrillo's analysis is best suited to applications with a temporal component, for example, the problem of a committee chairman deciding when to suspend debate and call a vote on a proposal. The authors describe situations in which the leader benefits from controlling the flow of public information.

However, even there, our focus is quite different: we are primarily concerned with determining the extent to which limited capacity improves the incentives for agents to supply accurate signals. This comparison is impossible in their analysis, as they focus on the case of a single agent. Building on their earlier results, Gentzkow and Kamenica (2011) analyze a persuasion environment with multiple players. They show that (i) moving from collusive to competitive play, (ii) introducing additional senders, (iii) decreasing alignment of sender preferences increase the amount of information revealed in equilibrium. We look at an alternative to these remedies. In our environment a restricted action space for the decision maker improves information accuracy.

We focus on the agents' choice of evidence quality, and assume that evidence is perfectly observable by the principal. We could relax this assumption slightly, assuming that evidence becomes private information of the agent, who then decides whether to transmit this evidence to the principal. If the principal can observe that evidence has been generated, and communication and verification is costless, then the disclosure game unravels, so that evidence is effectively observed by the principal.[9] Henry (2009) considers the impact that mandatory research disclosure rules may have on an agent's decision to acquire evidence. Che and Kartik (2009) consider how differences of opinion between decision makers and agents affects the agents' incentives to acquire and transmit evidence.[10] Other articles assume that agents know their evidence ahead of time, and must choose whether to disclose it (e.g., Milgrom 1981, Milgrom and Roberts 1986, Bull and Watson 2004). Cotton (2009) presents a model in which agents must compete for access to disclose their evidence to a time-constrained decision maker. The preferences of agents in our paper are similar to advocates in Dewatripont and Tirole (1999).

3 A Model with Symmetric Proposals and Binary Evidence

A principal (she) and two agents (he) play a two-stage game. All parties are risk-neutral. The principal must decide whether to accept or reject each of two separate proposals. She prefers to accept a proposal only if the proposal is of sufficiently high quality. The agents are associated with separate proposals. Each agent wants the principal to accept his proposal regardless of quality. Initially, both the principal and agents are uncertain about the quality of each proposal. Before the principal chooses which proposals to accept, each agent can produce evidence about the quality of his proposal.[11]

To keep the analysis as straightforward as possible, we assume that each proposal is either "good" (i.e., $\tau_i = g$) or "bad" (i.e., $\tau_i = b$). If accepted, all good proposals provide the same net benefit and all bad proposals provide the same net loss to the principal. The true quality of each proposal is unknown to all players, but it is common knowledge that quality is an i.i.d. draw from

[9] See the analysis of Kamenica and Gentzkow (in press) for more information on this point.

[10] Austen-Smith (1998) considers similar questions in a model of political access.

[11] Although we present the model as one in which the principal decides whether to accept different proposals, nothing is changed if we adapt the terminology to refer to the principal awarding funding, admitting applicant, implementing policy, or making other decisions that directly affect agent payoffs.

a Bernoulli distribution:

$$\begin{aligned} \Pr(\tau_i = g) &= \gamma \\ \Pr(\tau_i = b) &= 1-\gamma. \end{aligned}$$

If the principal rejects proposal i, she earns proposal-specific payoff $w_i = 0$ independent of τ_i. If the principal implements proposal i, she earns proposal payoff $w_i = 1-\theta > 0$ when the proposal is good and $w_i = -\theta < 0$ when the proposal is bad. Preference parameter $\theta \in (0,1)$ represents the "stakes" inherent in the principal's decision: when θ is large, the downside of accepting bad proposals is high compared to the upside of accepting good proposals.[12] The principal's total payoff is the sum of her payoffs from each proposal: $w = w_1 + w_2$. Agents benefit only if their proposal is accepted, with i receiving $u_i = 1$ if proposal i is accepted and $u_i = 0$ if rejected.

In the first stage of the game, each agent simultaneously commissions independent research to produce verifiable information about the quality of its proposal. The research outcome is a publicly observable realization $S_i \in \{G, B\}$, of signal s_i with the following conditional distribution:[13]

$$\begin{aligned} \Pr(s_i = G \mid \tau_i = g) = \Pr(s_i = B \mid \tau_i = b) &= \alpha_i \\ \Pr(s_i = G \mid \tau_i = b) = \Pr(s_i = B \mid \tau_i = g) &= 1-\alpha_i. \end{aligned}$$

The signal realization (or outcome) reflects true quality of the proposal with probability $\alpha_i \in [\frac{1}{2}, 1]$. We therefore refer to α_i as the "accuracy" of s_i. Increasing accuracy improve the informativeness of the signal in the sense of Blackwell. A signal with the lowest accuracy, $\alpha_i = \frac{1}{2}$, is completely uninformative; a signal with the highest accuracy, $\alpha_i = 1$, is fully informative about quality.

Agent i controls the accuracy of signal s_i.[14] When an agent commissions research, he chooses the ability of the researcher. Higher ability researchers are more likely to produce accurate findings. Alternatively, the agent may influence the informativeness of the signal by choosing the research design directly. Once both agents choose their signal accuracy, these choices are observed by the principal. By implication, the principal is able to observe and correctly interpret the identity, credentials, and reputation of the researcher; alternatively, she is able to observe and effectively analyze the informativeness of the research design.[15] In a later section, we consider a significantly more general framework in which agents design general signals and demonstrate that our results

[12] This specification is without loss of generality. If the principal's payoff from accepting a good proposal is $v > 0$ while the payoff of accepting a bad proposal is $-c < 0$, dividing both payoffs by $v + c$ gives the specification defined in the text.

[13] Identical results would hold if the research outcome (signal) were private information for the agent, provided this research outcome was verifiable, i.e. could be witheld but not falsely reported. See Kamenica and Gentzkow (in press) for more information. Identical results would also hold if the signal realization were privately observed by the principal, i.e. if it is a subjective impression, rather than verifiable evidence.

[14] In our framework there is no exogenous cost for increasing signal accuracy. Therefore, if agents do not provide fully informative signals, it is because they prefer such signals for strategic reasons, not because more informative signals require too much effort or money to generate. Including an exogenous cost does not alter the qualitative nature of the results.

[15] In some fields, research protocols must be registered in a database before subjects are enrolled. Stiff penalties exist for violating the protocols.

continue to hold.

At the beginning of the second stage, the principal observes both signal realizations. On the basis of all available information (prior information, signal accuracies and signal realizations), the principal chooses which proposals, if any, to accept. Once the principal makes her decision, the true quality of any accepted proposal is then revealed, and all payoffs are realized. This is a game of imperfect but symmetric information, similar to the persuasion games analyzed in Kamenica and Gentzkow (in press) and Brocas et al. (in press).

The principal's ability to accept proposals may be either *limited* or *unlimited*. If capacity is unlimited, she can accept neither, either, or both proposals as she sees fit. In this case no link exists between proposals; the decision to accept each proposal is made independently. Alternatively, when capacity is limited, she can accept at most one proposal. This limitation may arise for a variety of reasons: she may be constrained by limited budgets or limited time, she may also be constrained by procedural or bureaucratic hurdles that require considerable effort to overcome. When capacity is limited, a decision to accept a proposal precludes the possibility of accepting the other proposal. Therefore, acceptance decisions cannot be made in isolation; the signals and accuracies for both proposals influence the principal's decision. The capacity of the decision making environment is common knowledge.

We solve for Perfect Bayesian Equilibria of this two stage game under the limited and unlimited capacity systems. In the first stage, agents simultaneously choose their respective signal accuracies, α_1 and α_2. Once both agents choose their accuracies, these become public. In the second stage, the signals are realized, and the principal decides which proposals to accept, subject to the constraints of the decision making environment.

3.1 Unlimited Capacity

We begin by analyzing a setting in which the principal is unconstrained in her ability to accept proposals. Therefore, the principal accepts any proposal for which the expected payoff from doing so is positive. This expected payoff depends on her posterior belief about the quality of the proposal after observing the realization of the signal. Let $\hat{\gamma}(S_i, \alpha_i)$ denote the principal's belief that proposal i is good after observing signal realization S_i, given accuracy α_i:

$$\hat{\gamma}(G, \alpha_i) = \frac{\gamma\alpha_i}{\gamma\alpha_i + (1-\gamma)(1-\alpha_i)}$$

$$\hat{\gamma}(B, \alpha_i) = \frac{\gamma(1-\alpha_i)}{\gamma(1-\alpha_i) + (1-\gamma)\alpha_i}$$

The principal accepts proposal i if

$$\hat{\gamma}_i(S_i, \alpha_i)(1-\theta) - (1-\hat{\gamma}_i(S_i, \alpha_i))\theta \geq 0 \iff \hat{\gamma}_i(S_i, \alpha_i) \geq \theta.$$

Thus the principal accepts if the updated probability of a good proposal is at least as great as the stakes, θ. Otherwise, she rejects the proposal. We assume that the principal accepts a proposal if she is indifferent between accepting and rejecting.

If $\gamma < \theta$, then the principal is predisposed against accepting: if her decision were based solely on her prior belief, she would reject each proposal. In this case, observing a bad signal realization reinforces the principal's beliefs that the proposal is bad, and she continues to favor rejection. Observing a good signal realization, however, improves her belief about the quality of the project. When the good realization is generated by a sufficiently accurate signal, it overturns her predisposition and causes her to accept the proposal. This is the case when

$$\hat{\gamma}(G, \alpha_i) \geq \theta \iff \alpha_i \geq \frac{\theta(1-\gamma)}{(1-\theta)\gamma + (1-\gamma)\theta}.$$

We define

$$r \equiv \frac{\theta(1-\gamma)}{(1-\theta)\gamma + (1-\gamma)\theta}.$$

Therefore, for a good signal realization to persuade the principal to accept a proposal that she is predisposed against, it must be that

$$\alpha_i \geq r.$$

If signal accuracy is sufficiently low, then a good signal outcome is not persuasive enough to overturn the principal's predisposition to reject.

If $\gamma \geq \theta$, then the principal is predisposed in favor of accepting; her prior is sufficiently optimistic that she would accept each proposal given her prior alone. In this case, observing a good signal realization only strengthens the principal's beliefs that the proposal is good, and she continues to favor accepting. Observing a bad signal realization, however, weakens her beliefs that the proposal is good. When the signal is sufficiently accurate, the bad realization overturns her predisposition and causes her to reject the proposal. This is the case when

$$\hat{\gamma}(B, \alpha_i) < \theta \iff \alpha_i > 1 - \frac{\theta(1-\gamma)}{(1-\theta)\gamma + (1-\gamma)\theta} = 1 - r.$$

If signal accuracy is sufficiently low, then a bad realization is not persuasive enough to overturn her predisposition in favor of accepting.

The above discussion is summarized in the following lemma.

Lemma 3.1 *When accepting capacity is unlimited, the principal's equilibrium strategy is:*

- *If predisposed in favor of accepting, $\gamma \geq \theta$, then the principal rejects proposal i if and only if $s_i = B$ and $\alpha_i \in (1-r, 1]$.*

- *If predisposed against accepting, $\gamma < \theta$, then the principal accepts proposal i if and only if $s_i = G$ and $\alpha_i \in [r, 1]$.*

A *persuasive* signal has the potential to overturn the principal's predisposition; no realization of a *weak* signal affects the principal's decision. When $\gamma \geq \theta$, a persuasive signal has accuracy $\alpha > 1 - r$. When $\gamma < \theta$, a persuasive signal has accuracy $\alpha \geq r$. Otherwise, the signal is weak.

Agents anticipate the principal's behavior when they choose signal accuracy. Suppose that the principal is predisposed in favor of accepting, i.e., $\gamma \geq \theta$. In this case, the principal implements the proposal even if the signal is weak. Choosing a persuasive signal introduces the possibility that the principal observes a bad realization and does not implement the reform. In this case, agents strictly prefer weak signals.

Alternatively, suppose that the principal is predisposed against accepting, i.e., $\gamma < \theta$. If an agent chooses a weak signal, then the proposal is rejected. If the agent chooses a persuasive signal, then the proposal is accepted if and only if $s_i = G$. Let $\sigma(\alpha)$ denote the probability of observing a good realization given accuracy α:

$$\sigma(\alpha) \equiv \alpha\gamma + (1-\alpha)(1-\gamma).$$

When the signal is persuasive, the proposal is accepted with probability $\sigma(\alpha)$, which is also the agent's expected payoff. An agent's optimal signal accuracy maximizes $\sigma(\alpha)$ subject to $\alpha \in [r, 1]$. If $\gamma < \frac{1}{2}$, then σ is strictly decreasing in α, and the agent prefers $\alpha = r$ to any higher value. He also prefers $\alpha = r$ to any weak signal, as supplying the marginally persuasive signal leads to a positive probability of the proposal being implemented. If $\theta < \frac{1}{2}$, then this is the only possible case. If, however, $\theta > \frac{1}{2}$, then we must also consider the possibility that γ is such that $\frac{1}{2} \leq \gamma < \theta$. In this case, the principal is predisposed against accepting proposals that are most likely good (and are more likely than not to generate good signals). In this case, $\sigma(\alpha)$ is strictly increasing in α and the agent prefers $\alpha = 1$ to any lower value.

These observations lead to the following description of the equilibrium outcome.

Lemma 3.2 *In equilibrium under unlimited capacity,*

- *If $\theta \leq \gamma$, then for each i the agent chooses a weak signal, the principal accepts the proposal, and*

$$E[w_i] = \gamma(1-\theta) - (1-\gamma)\theta = \gamma - \theta \quad \text{and} \quad E[u_i] = 1.$$

- *If $\gamma < q$ and $\gamma < \frac{1}{2}$, then for each i, $\alpha_i = r$, the principal accepts proposal i if and only if $s_i = G$, and*

$$E[w_i] = 0 \quad \text{and} \quad E[u_i] = \sigma(r) > \gamma.$$

- *If $\frac{1}{2} \leq \gamma < \theta$, then for each i $\alpha_i = 1$, the principal accepts proposal i if and only if $s_i = G$, and*

$$E[w_i] = \gamma(1-\theta) \quad \text{and} \quad E[u_i] = \gamma.$$

Although the principal finds a fully-informative signal optimal, agents choose $\alpha = 1$ in only one situation: when the principal is predisposed against accepting and proposals are most-likely good (i.e., when $\frac{1}{2} \leq \gamma < \theta$). In all other cases, the agent prefers to keep the principal less than fully-informed, and provides either a weak or marginally-persuasive signal. If the principal is predisposed in favor of accepting, then it is a dominant strategy for the agent to choose a weak signal. Doing so assures that the principal implements the proposal. If the principal is predisposed against accepting and proposals are most-likely bad ($\gamma < \frac{1}{2}$ and $\gamma < \theta$), then agents choose signal accuracies that are just strong enough to make the principal indifferent between accepting and rejecting their proposals if she observes a good signal realization. This maximizes the probability that the principal observes a favorable realization and implements the proposal.

In the unlimited capacity system, two critical factors determine the agents' incentives to supply accurate signals. The first critical factor is the monotonicity of $\sigma(\alpha)$. When the proposal is most-likely good ($\gamma > \frac{1}{2}$), increasing a persuasive signal's accuracy makes a good realization more likely, which increases the probability that the proposal will be accepted; accuracy is *success-enhancing.* However, when the signal is most-likely bad ($\gamma < \frac{1}{2}$), $\sigma(\alpha)$ is decreasing. In this case increasing the accuracy of a persuasive signal decreases the probability of generating a good realization and hurts the probability of proposal implementation; accuracy is therefore *success-diminishing.* Thus, the prior belief determines which signal accuracy is optimal among the persuasive signals. The second critical factor is the principal's predisposition toward the proposal, determined by the relationship between θ and γ. The principal's predisposition determines whether or not the agents prefer to produce persuasive or weak signals. If the principal is predisposed towards accepting, agents strictly prefer weak signals to persuasive ones; if the principal is predisposed against accepting, then agents strictly prefer persuasive signals to weak ones. The severity of the conflict of interest between the agents and the principal thus depends critically on the interaction between these two forces.

(The problem with) Commitment

The principal prefers $\alpha = 1$ to all lower values, as it allows for a fully-informed decision. The politician's sequentially rational strategy, however, implements any proposal for which $\hat{\gamma} \geq \theta$, even if $\alpha < 1$. The agents recognize this and prefer to produce a less than fully informative signal.

If the principal could credibly commit to a strategy at the onset of the game, she could guarantee her ideal outcome by committing to reject any proposal for which $\alpha_i < 1$. That is, she would commit to reject any proposal for which she is less-than-fully informed about its quality. The agents would react by always choosing $\alpha = 1$, guaranteeing a fully-informed policy decision. In some situations such commitment may be reasonable. If we imagine an infinitely repeated sequence of stage games between a long-lived principal and short-lived agents, the principal's commitment power could be derived from reputation (provided her discount factor is sufficiently high). However, in a static game, the principal would have to derive commitment power from writing some kind of contract, under which she commits to reject any proposal for which the signal is not fully revealing.

This type of contract is problematic for several reasons. First, these types of contracts may be prohibited for institutional or legal reasons. In politics, for example, legislators typically cannot contract with interest groups to choose favorable policy conditional upon some action by the agent.

Second, even if contracts are allowed, deriving commitment power from writing legally binding contracts, may prove difficult. Signal accuracy may not be verifiable in court, rendering any contract based on accuracy unenforceable. Furthermore, even if α were verifiable, both the principal and agent would prefer to disregard the contract and allow the proposal to be implemented if the agent produced a favorable, persuasive signal realization with $\alpha < 1$. In our analysis, we focus on a principal who cannot commit to accept only those proposals with perfectly informative signals.

3.2 Limited Capacity

In the previous section where the principal could implement as many proposals as she wanted, the principal's choice regarding one proposal was independent of her choice regarding the other proposal. In this section, the principal can implement at most one of the two proposals. Here, choosing to implement one proposal excludes the possibility of implementing the other proposal. Therefore, to get his proposal accepted, it is no longer enough for an agent to provide sufficient evidence to convince the principal that his proposal has a positive expected payoff. The agent must also convince the principal that his proposal is more-likely beneficial than the alternative option.

In this section, we derive the Perfect Bayesian Equilibria of the game under limited capacity. The characterization of equilibrium depends on (i) whether the principal is predisposed in favor or against proposals, and (ii) whether the proposal is most-likely good or bad. When $\theta = \frac{1}{2}$, the principal is predisposed in favor of proposals that are most-likely good and against proposals that are most likely bad. When $\theta < \frac{1}{2}$, the stakes are small enough that the principal may be predisposed in favor of proposals for which the probability of being good is less than $\frac{1}{2}$. Conversely, when $\theta > \frac{1}{2}$, the stakes are large enough that the principal may be predisposed against projects that are more likely to be good than bad. Given this, we must consider four mutually exclusive parameter cases.

1. When the principal is predisposed against proposals, and proposals are most-likely good.

2. When the principal is predisposed in favor of proposals, and proposals are most-likely bad.

3. When the principal is predisposed against proposals, and proposals are most-likely bad.

4. When the principal is predisposed in favor of proposals, and proposals are most-likely good.

In cases 1 and 2, limited capacity does not encourage agents to produce more-informative signals. In case 1, agents prefer to produce a fully-informative signal ($\alpha_1 = \alpha_2 = 1$) even when capacity is unlimited.[16] In case 2 agents prefer to keep the principal uninformed ($\alpha_1 = \alpha_2 = \frac{1}{2}$) even under limited capacity.[17] In these two cases, limited capacity does alter agent behavior.

In cases 3 and 4, limited capacity introduces competitive pressure between the agents, who must provide more-accurate evidence than the other agent to have priority when both agents produce

[16] Both critical factors line up in favor of signal accuracy: the principal is predisposed against, and, because proposals are most-likely good, accuracy is success-enhancing.

[17] In this case, both critical factors line up against signal accuracy: principal is predisposed in favor, and accuracy is success-diminishing.

favorable signal realizations. This competitive pressure results in the agents choosing more-accurate signals, and in a more-informed principal. The discussion below focuses on these two cases, first showing that limited capacity increases signal accuracy, then deriving conditions under which the principal prefers limited capacity.

3.3 Principal predisposed against and proposals most-likely bad

The first case we consider is one in which the principal is predisposed against accepting (i.e., $\gamma < \theta$) and proposals are most likely bad (i.e., $\gamma < \frac{1}{2}$). We have already shown that if capacity is unlimited, then in this case both agents will supply marginally persuasive signals, $\alpha_1 = \alpha_2 = r$. That is, both agents provide just-accurate-enough signals to make the principal indifferent between implementing and not implementing their proposals in the event that their signal produces a favorable realization. When the principal can implement at most one proposal, this is no longer an equilibrium.

In the case of limited capacity, an agent prefers to provide a marginally-more accurate signal than the other agent. Producing a marginally more accurate signal than one's opponent results in the agent having priority in the event that both signal realizations are favorable, while having only a marginally lower probability of producing favorable evidence. This leaves open the possibility of two types of equilibria. The first possibility involves both agents producing fully informative signals; in which case, marginally increasing accuracy is not possible. The second possibility involves the agents playing mixed strategies over a continuum of signal accuracies with the only possible mass point on $\alpha_i = 1$. We provide a detailed characterization of the equilibria in the appendix, showing that when θ is low agents mix over a continuum with support between the marginally persuasive accuracy r and some upper bound $\bar{\alpha} \in (r, 1]$, where $\bar{\alpha}$ is strictly increasing in θ up to $\bar{\alpha} = 1$. For higher θ, the upper bound $\bar{\alpha}$ is then decreasing in θ, and the mixing distribution develops a mass point on $\alpha_i = 1$.

We formally characterize the equilibrium in the appendix. Lemma 3.3 summarizes the equilibrium results.

Lemma 3.3 *Under limited capacity with $\gamma < \theta$ and $\gamma < \frac{1}{2}$, there exists a threshold value of $\hat{\theta} \in [\gamma, 1]$ such that:*

- *If $\theta \geq \hat{\theta}$, then the unique equilibrium involves both agents producing fully-informative signals, $\alpha_1 = \alpha_2 = 1$.*
- *If $\theta < \hat{\theta}$, then the only equilibria involve both agents playing mixed strategies such that $\alpha_1, \alpha_2 > r$ with probability 1.*

The threshold value $\hat{\theta}$ is strictly decreasing in γ, starting at $\hat{\theta} = 1$ when $\gamma = 0$. If $\gamma < \frac{3-\sqrt{5}}{2}$, then $\hat{\theta} > \gamma$ and thus the type of equilibrium depends on the exact value of θ. If $\gamma \geq \frac{3-\sqrt{5}}{2}$, then $\hat{\theta} = \gamma$ and all $\theta > \gamma$ exceed $\hat{\theta}$. Therefore, when γ is not too low, the only equilibrium involves both agents producing fully-informative signals. Proposition 3.4, the main result for this case, holds as long as $\gamma < \theta$ and $\gamma < \frac{1}{2}$, regardless of whether the equilibrium involves pure or mixed strategies.

Proposition 3.4 *When $\gamma < \theta$ and $\gamma < \frac{1}{2}$,*

- *With probability one, each agent provides a more accurate signal under limited capacity than under unlimited capacity.*
- *The principal prefers limited capacity to unlimited capacity.*

Under unlimited capacity, both agents provide marginally pivotal signals, setting $\alpha_1 = \alpha_2 = r$. Under limited capacity, as determined by Lemma 3.3, the agents either provide fully-informative signals, or they play a mixed strategy that involves both agents producing evidence that almost certainly has accuracy greater than r. Regardless of whether the limited capacity game achieves the pure strategy or mixed strategy equilibrium, the agents produce more informative evidence than in the equilibrium of the unlimited capacity game.

That the principal tends to be more informed under limited capacity does not in itself imply that the principal prefers limited capacity to unlimited capacity. This is because limited capacity prevents the principal from implementing both proposals, even when both have positive expected value. When the principal is predisposed against proposals, this potential cost of limited capacity never negates the expected benefits that come from more-accurate signals. Recall, that with unlimited capacity, agents supply marginally persuasive signals, and the principal is, at best, indifferent between implementing and rejecting proposals. Her equilibrium expected payoff under the unlimited capacity system is always zero. Under limited capacity, the probability that she accepts a proposal which generates a positive expected surplus is non-zero in each type of equilibrium. Thus her expected payoff is positive under limited capacity. Intuitively, limited capacity improves information accuracy, but has no expected cost when the principal is predisposed against proposals. The loss of an option to accept a proposal does not impose a cost on the principal, as her expected payoff under unlimited capacity is the same as if she rejects both proposals.

3.4 Principal predisposed in favor and proposals most-likely good

We turn to the case in which the principal is predisposed in favor accepting (i.e., $\theta < \gamma$), and proposals are most likely good (i.e., $\gamma > \frac{1}{2}$). We have already shown that in equilibrium under unlimited capacity, both agents produce weak signals and the principal always implements both proposals. In this section, the principal continues to be predisposed in favor of both proposals, but she only has the capacity to implement one of them. This introduces competitive pressure between the agents who are now concerned about convincing the principal that their proposal is more likely than the other proposal to be beneficial. This pressure causes agents to provide persuasive evidence under limited capacity as they compete for their proposal to have priority over the other proposal.

Because the principal is predisposed towards accepting, she is willing to implement any proposal with a good signal realization. If only one proposal generates a favorable signal realization, then she implements that proposal regardless of signal accuracy. If both proposals generate good realizations, then the principal accepts the proposal with the more-accurate signal. For sufficiently large γ, the competitive incentives result in both agents producing fully informative signals in equilibrium with $\alpha_1 = \alpha_2 = 1$. When this is the case, the probability of generating favorable evidence is sufficiently high that the agents prefer a tie at $\alpha_i = 1$ rather than deviating to produce weak evidence with the hope that the other agent generates an unfavorable signal realization. For lower γ, the fully-informative equilibrium cannot be sustained and there exists both a pure strategy equilibrium in which only one of the agents produce a fully-informative signal, and a symmetric mixed strategy equilibrium with mixing between fully-informative and marginally-persuasive signals. We describe these equilibria in Lemma 3.5.

Lemma 3.5 *Under limited capacity with $\theta < \gamma$ and $\gamma > \frac{1}{2}$*

- *If $\gamma \geq 2-\sqrt{2}$ then the unique Nash equilibrium is for each agent to produce a fully informative signal, $\alpha_1 = \alpha_2 = 1$.*
- *If $\gamma < 2-\sqrt{2}$ then many asymmetric Nash equilibria exist. In each Nash equilibrium, one agent produces a fully revealing signal $\alpha_i = 1$, and the other agent produces a weak signal, $\alpha_j \in [\frac{1}{2}, 1-r]$.*
- *If $\gamma < 2-\sqrt{2}$ a symmetric mixed strategy Nash equilibrium exists. Each agent chooses $\alpha_i = 1-r$ with probability $p = \frac{\gamma^2-4\gamma+2}{\gamma^2-2\gamma+1}$, and $\alpha_i = 1$ with probability $1-p$.*

Under unlimited capacity, both agents provide weak signals. Here under limited capacity, there always exists a pure strategy equilibrium in which at least one agent produces fully-informative evidence, making the principal more informed than in the unlimited capacity environment. For low enough γ, there also exists a mixed strategy equilibrium in which the principal is fully-informed with positive probability. Thus, the principal always expects to be better informed under limited capacity than under unlimited capacity.

Showing that the principal prefers limited to unlimited capacity is less straightforward here compared to the previous section. Here, limited capacity constrains the principal to implement at most one proposal even though from an ex ante perspective, she would like to implement both. In order for the principal to prefer limited capacity, it must be that the expected benefits of more accurate evidence dominate the expected costs of rejecting one proposal with positive expected value. This is the case when θ is not too small relative to γ. The potential benefits are summarized in Proposition 3.6.

Proposition 3.6 *When $\theta < \gamma$ and $\gamma > \frac{1}{2}$,*

- *In every pure strategy Nash equilibrium under limited capacity, at least one agent supplies a fully revealing signal.*
- *In every mixed strategy Nash equilibrium under limited capacity, agents never supply less accurate signals than in the unlimited capacity equilibrium, and supply fully-revealing signals with non-zero probability.*

- *For each γ, there exists a value $\tilde{\theta} < \gamma$ such that the principal prefers limited capacity to unlimited capacity if and only if $\tilde{\theta} \leq \theta < \gamma$.*

The benefit of limited capacity is an increase in evidence quality (i.e. signal accuracy). However, because she is predisposed in favor of the proposals, limited capacity has a cost: the principal is constrained to implement fewer policies than she would like to implement if capacity were unlimited. If proposals have relatively little downside risk (small θ) and relatively high probability of being good (high γ), then the principal has a strong prior predisposition in favor of accepting proposals. In this case, the option to implement a reform is very valuable. In this case, the informational benefits associated with limited capacity will not be large enough to overcome the costs of not being able to support both proposals. When θ is sufficiently close to γ, however, the informational benefits dominate the downside associated with the capacity constraint. As θ approaches γ, the principal's predisposition in favor of proposals becomes weaker, and the ex ante expected benefit from being able to implement a given proposal decreases. Decreasing the ex ante benefit from implementation in turn decreases the relative downside of limited capacity, which eventually allows the informational benefits of more accurate signals to dominate the costs from being constrained on the number of proposals that can be accepted.

3.5 Summary of results

In the previous sections, we demonstrated the following main results.

1. In the majority of cases under unlimited capacity, agents produce less informative evidence than the principal prefers. They strategically keep the principal less than fully informed since doing so maximizes the probability that the principal implements their proposal.

2. When the principal has limited capacity to implement proposals, agents increase the quality of evidence they produce as they compete with each other for priority. If the principal is predisposed against proposals that are most likely bad, or predisposed in favor of proposals that are most likely good, then more accurate signals are supplied under limited capacity than under unlimited capacity. This leads to a more-informed principal.

3. For a significant range of parameter values, the principal prefers (ex ante) to operate under limited capacity, even if she has ex ante beliefs that both proposals are worthwhile. Here, the informational benefits of limited capacity outweigh the capacity costs.

Figure 1 illustrates the values of θ and γ under which the principal prefers limited capacity to unlimited capacity. When θ and γ are in the shaded regions, the informational benefits dominate the expected costs of limited capacity.[18]

[18] The plot assumes that agents play the symmetric mixed strategy when $\frac{1}{2} < \gamma < 2 - \sqrt{2}$ and $\gamma > \theta$. Alternatively, one may assume that they play the asymmetric pure strategy in this range. This choice makes little difference for the qualitative nature of the results.

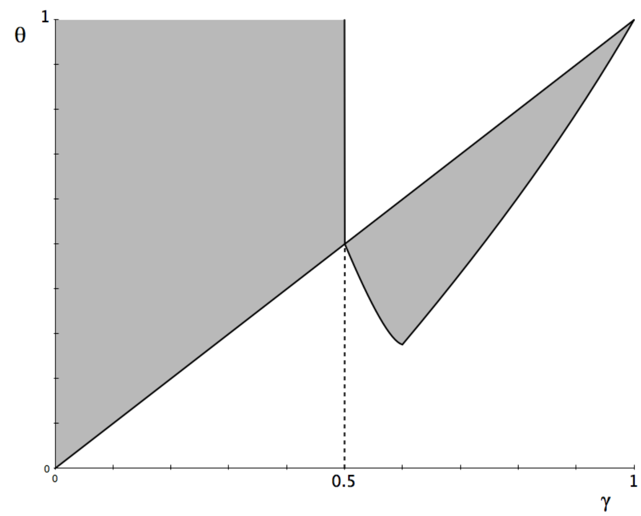

Figure 1: Range of parameters for which the principal prefers limited capacity

4 A Model with General Signals and Asymmetric Proposals

The previous section demonstrates that limited capacity often improves the informativeness of evidence supplied by agents, and that this benefit can outweigh the expected cost associated with capacity constraints. In demonstrating this result, we constrained agents to choose their signals from a simple parametric class and assumed that the prior beliefs about each proposal was identical. These simplifications allowed for a more-intuitive presentation of our results. In this section, we present our results in a significantly more general framework.

We expand the model of the last section in two ways. First, the prior beliefs associated with the proposals need not be identical. Let γ_i denote the ex ante probability that proposal i is good, where $\gamma_i \in (0,1)$ for $i = H, L$, and $\gamma_H \geq \gamma_L$. Second, we do not constrain the agents' choices of signals to any particular parametric class. Here, agents have considerably more freedom to design their signals.

Formally, signal (or *experiment*) Σ is a random variable, jointly distributed with proposal quality. We represent Σ as a pair of conditional random variables (Σ_g, Σ_b). If the true type of the proposal is g, then a realization of random variable Σ_g is observed; otherwise a realization of Σ_b is observed. We focus on random variables Σ_t, $(t = g, b)$ for which the cumulative distribution function has a countable number of discontinuities or mass points. Except at mass points, Σ_t admits a continuous density $f_t(x)$ which is strictly positive in some interval I_t. These are the only requirements on Σ_t, and any random variable with such a structure is *valid*. The set of mass points of random variable Σ_t is denoted M_t, a particular mass point in this set is denoted m_t^j, and the magnitude of the jump at m_t^j is μ_t^j. S_t is the set of possible realizations of Σ_t, and is thus $M_t \cup I_t$. Thus the cumulative distribution function of any valid random variable can be written

$$P_t(x) = F_t(x) + \sum_j H(x - m_t^j)\mu_t^j$$

where $H(x)$ is the Heaviside (or step) function. The associated density function $p_t(x)$ is defined using the Dirac delta function, δ.[19] Thus,

$$p_t(x) = f_t(x) + \delta(x - m_t^j)\mu_t^j$$

where $f_t(x)$ is continuous and equal to the derivative of $F_t(x)$.[20]

The public realization of signal Σ (also called *the outcome of the experiment*) conveys information about proposal quality. Because the true quality of each proposal is unknown, any signal realization $s \in S_g \cup S_b$ induces a posterior belief that the proposal is good, consistent with Bayes' Rule. This posterior belief depends on the prior, the signal realization, and on the signal structure itself. For any signal realization $s \in S_g \cup S_b$ the principal's posterior belief that the proposal is good given this signal realization is given by Bayes' rule:[21]

$$\hat{\gamma}(s) = \frac{\gamma p_g(s)}{\gamma p_g(s) + (1-\gamma)p_b(s)}$$

Once the experiment is chosen by the agent, but before the outcome of the experiment is realized, the value of the posterior belief $\hat{\gamma}(s)$ is a random variable $\Gamma = \widehat{\gamma}(\Sigma) = Pr(\tau = g|\Sigma)$. This random variable is thus the *ex ante* value of the principal's posterior belief. Observe that random variable Γ is valid, has support confined to the unit interval, and (according to the law of total expectation) has expected value equal to the prior belief, i.e. $E[\Gamma] = \gamma$. In the next Lemma we show that this is the *only* substantive restriction on random variable Γ.[22]

Lemma 4.1 *Consider any valid random variable* Γ *with support confined to the unit interval and expectation* γ*. If the prior belief is* γ *then there exists a signal* Σ *for which the ex ante posterior belief is* Γ.

This proposition considerably simplifies the analysis of this game. Instead of designing a signal directly, each agent i simultaneously chooses a valid random variable Γ_i, with support in the unit interval and expectation γ_i. This choice represents the ex ante value principal's posterior belief about proposal quality, and is equivalent to a choice of any experiment that generates the same distribution of posterior beliefs. We refer to this decision as a choice of a signal, although Γ technically represents an entire payoff-equivalent class of signals. Once both agents have chosen their Γ_i, these random variables are realized (i.e. the principal observes both the design and outcome of the experiment, and rationally updates her beliefs). On the basis of the realized posterior beliefs,

[19]Of course this density is not a proper function; any statement that defines a random variable by specifying such a density should be interpreted to mean that the cumulative distribution function of the variable is $P_t(x)$.

[20]Since we are dealing with random variables, all of the statements throughout the paper are true up to variations on sets of measure zero, a caveat which has no significant implication for our results.

[21]Recall that $\frac{a_1+b_1\delta(x)}{a_2+b_2\delta(x)}$ is equal to $\frac{a_1}{a_2}$ for $x \neq 0$ and is equal to $\frac{b_1}{b_2}$ for $x = 0$

[22]In recent papers, Kamenica and Gentzkow (in press) use a related representation of signals to study a general class of persuasion games, while Ganuza and Penalva (2010) use a related representation to study information disclosure in auctions.

the principal decides which proposals to accept, respecting the capacity constraints of the decision making environment.

4.1 Unlimited Capacity

To analyze the game with unlimited capacity, observe first that it is sequentially rational for the principal to accept all proposals for which the realized posterior belief is at least as large as θ. Anticipating the principal's sequentially rational behavior, in equilibrium each agent chooses random variable Γ_i to maximize $\Pr(\Gamma_i \geq \theta)$ subject to $E[\Gamma_i] = \gamma_i$ and $\Pr(0 \leq \Gamma_i \leq 1) = 1$.

Observe first that each agent always has the capability to choose a *fully revealing signal*:

$$Pr(\Gamma_i = 1) = \gamma_i \quad \text{and} \quad Pr(\Gamma_i = 0) = 1 - \gamma_i$$

This signal completely reveals the quality of the proposal with which it is associated. After observing the signal realization, the principal's beliefs are either one (she is sure the proposal is good) or zero (she is sure the proposal is bad). Consequently, she always chooses the right course of action. From the principal's perspective the fully revealing signal is optimal. If capacity is unlimited, however, agents never supply fully revealing signals. In fact, their signals are effectively worthless to the principal.

If the principal is predisposed in favor of implementing proposal i ($\gamma_i \geq \theta$), then agent i chooses a signal for which the entire support of Γ_i is above θ. The simplest way to do that is to concentrate all mass on γ_i so that $\Pr(\Gamma_i = \gamma_i) = 1$, but any signal for which all realizations are above θ achieves the same result: each proposal is always accepted. We refer to such signal as *weak*. Consistent with the results of the previous section, when the principal is predisposed in favor of accepting a proposal and capacity is unlimited, the agent associated with that proposal supplies a signal that never affects the principal's behavior.

If the principal is predisposed against proposal i, ($\gamma_i < \theta$) then agent i must choose a persuasive signal for a proposal to have a chance to be accepted. In this case, signals with non-zero probability mass on posterior beliefs strictly above θ are dominated.[23] Similarly, any posterior distributions which put non-zero probability mass on realizations between 0 and θ are dominated.[24] Thus, the optimal signal concentrates mass on only two posterior beliefs, 0 and θ. In order to satisfy the constraint on the expectation, the probability of generating the posterior belief θ must be equal to $\phi = \frac{\gamma_i}{\theta}$. Thus, the optimal signal requires only two realizations. One signal realization reveals that the proposal is bad for certain, while the good realization leaves the principal just indifferent between accepting and rejecting the proposal. This is qualitatively similar to the results of Section

[23] By concentrating all mass above θ in a mass point on θ, the agent generates a new random variable with the same probability of being greater than or equal to θ but with a smaller mean; the agent can then move additional probability mass from realizations below θ to the mass point on θ in order to satisfy the constraint. Doing so increases the agent's payoff.

[24] By moving the probability mass between 0 and θ into a mass point on 0, the agent can increase the probability mass that is weakly above θ without violating the constraint.

3.[25]

Lemma 4.2 *In equilibrium under unlimited capacity*

- *The principal accepts any proposal for which the realized posterior is greater than or equal to* θ.

- *If* $\theta \leq \gamma_i$, *then agent i chooses a weak signal. On the equilibrium path, the principal accepts the proposal.*

$$E[w_i] = \gamma_i - \theta \quad \text{and} \quad E[u_i] = 1.$$

- *If* $\gamma_i < \theta$ *then agent i chooses a signal such that* Γ_i *is equal to* θ *with probability* $\frac{\gamma_i}{\theta}$ *and zero with probability* $1 - \frac{\gamma_i}{\theta}$. *On the equilibrium path, the principal accepts if and only if* $\Gamma_i = \theta$.

$$E[w_i] = 0 \quad \text{and} \quad E[u_i] = \frac{\gamma_i}{\theta}.$$

This result demonstrates that the conflict of interest identified in the previous sections also exists in this framework: although agents have the capacity to fully reveal the quality of their proposals, in equilibrium they never choose to do so. Moreover, the evidence that they choose to provide is effectively worthless to the principal. She would obtain the same expected payoff if she did not observe the signal realization and simply acted according to her prior belief.

4.2 Limited Capacity

In this section we establish the two main results. First, when capacity is limited, agents supply more-informative equilibrium signals as they compete for priority. Second, for a wide range of parameters, the benefit of receiving more informative signals outweighs the expected cost of limited capacity; in these cases, the principal expects to do better when capacity is limited.

To analyze the game with limited capacity, observe first that the principal can accept at most one proposal, even though she expects a positive payoff from any proposal for which the realized posterior belief is at least as large as θ. She will therefore choose to accept the proposal that generates the highest posterior belief, provided this posterior belief exceeds θ.

In the first stage of the game, agents H and L ($\gamma_H \geq \gamma_L$), simultaneously design valid random variables Γ_H, Γ_L with support confined to the unit interval. The mean of each player's random variable is constrained:

$$E[\Gamma_H] = \gamma_H \quad \text{and} \quad E[\Gamma_L] = \gamma_L$$

These random variables represent the ex ante posterior beliefs induced by the agent's experiment. Once both agents have made their choices, each random variable is realized; that is, the outcome of the experiment is observed. The principal then implements the proposal that generates the highest

[25]The optimal signal in the parametric class of 3, however, is not optimal, because the principal's belief following a bad realization is not zero, as it is under the optimal signal.

realized posterior belief, provided this realization is greater than θ. If both random variables have the same realization above θ each proposal is equally likely to be implemented; if both random variables have the same realization below θ, both proposals are rejected.[26]

In this game, agents want to make their proposals appear as good as possible, that is, they want to concentrate as much mass as possible on high realizations of the posterior belief. However, because of the feasibility constraint, any probability mass on realizations higher than γ_i must be offset by probability mass on realizations below γ_i. Thus, reducing the prior belief associated with a proposal limits the ability of the agent to reveal good information about the proposal.[27]

The normal form representation of this game is closely related to the normal form of a full-information symmetric all-pay auction with the inclusion of mixed strategies, though important differences exist. In the standard symmetric full-information all-pay auction, each agent's strategy is a choice of non-negative bid. The agent who chooses the highest bid wins a prize, but all participants must pay their bids. The prize is equally valued by all players, and this value is common knowledge. A mixed strategy in this game is a choice of random variable, which represents a player's random bid. The player whose realized bid is highest wins the prize and, in expectation, pays a price equal to the mean of his random bid.[28] In our framework, agents also design random variables, and the agent whose random variable generates the highest realiation has his proposal accepted (provided a threshold is reached). Unlike an all-pay auction, the mean of each agent's strategy is constrained. In our framework, the agent's strategy represents the posterior belief associated with his signal. Bayesian rationality therefore requires that the expected value of an agent's strategy must be equal the prior belief (which can be different for each agent).[29] Despite this important distinction, our analysis brings to light a connection between persuasion games and all-pay auctions.[30]

4.2.1 Equilibrium

In this section we describe the equilibrium of the game for all possible parameter values.[31] Qualitatively, all of the equilibria of this game share a similar structure. The disadvantaged player, L,

[26] Conitzer and Wagman (2011) analyze a model related to our underlying framework. There are, however, a number of features of our environment that are absent in their analysis: we allow for different means and a minimum outcome simultaneously. Also, our random variables are confined to the unit interval. These differences have significant implications for the results.

[27] Also observe that in the game we consider, the inclusion of mixed strategies does not expand the available strategies for the players. Any mixed strategy in our setting is simply a mixture of valid random variables with the same mean, which is equivalent to a new valid random variable with the same mean, which is an admissible "pure" strategy.

[28] Thus, with two players, i's expected payoff from choosing mixed strategy B_i against mixed strategy B_j in the full information symmetric all pay auction is $vPr(B_i > B_j) + \frac{1}{2}vPr(B_i = B_j) - E[B_i]$

[29] Applied to the all-pay auction setting, this requirement forces each bidder to adhere to a (potentially different) budget constraint that holds *in expectation* only. While in the all pay auction agent i chooses best response B_i to maximize $vPr(B_i > B_j) + \frac{1}{2}vPr(B_i = B_j) - E[B_i]$, in our game player i's best response maximizes $Pr(\Gamma_i > \Gamma_j) + \frac{1}{2}Pr(\Gamma_i = \Gamma_j)$ *subject to* $E[\Gamma_i] = \gamma_i$

[30] Less substantial differences also exist. Unlike the standard treatment of the full information all-pay auction, in the game we consider there is both a maximum possible realization (equivalent to a bid cap) and a minimum realization required for the proposal to be allocated (equivalent to a reservation price).

[31] A detailed derivation of each equilibrium can be found in the appendix.

chooses a strategy that consists of some combination of the following: a mass point on zero, uniform mixing between θ and some value $\bar{\gamma}$ no greater than one, and a mass point on one. The advantaged player, H, chooses a strategy that consists of some combination of the following: a mass point on zero, a mass point on θ, uniform mixing between θ and the same $\bar{\gamma}$, and a mass point on one.

Lemma 4.3 *In every equilibrium, agents' strategies are of the following type:*

$$\Gamma_H = \begin{cases} 0 & \textit{with probability} \quad f_{H0} \\ \theta & \textit{with probability} \quad f_{H1} \\ U[\theta, \bar{\gamma}] & \textit{with probability} \quad f_{H2} \\ 1 & \textit{with probability} \quad f_{H3} \end{cases}$$

$$\Gamma_L = \begin{cases} 0 & \textit{with probability} \quad f_{L0} \\ U[\theta, \bar{\gamma}] & \textit{with probability} \quad f_{L1} \\ 1 & \textit{with probability} \quad f_{L2} \end{cases}$$

$$f_{H0} + f_{H1} + f_{H2} + f_{H3} = f_{L0} + f_{L1} + f_{L2} = 1$$

Within this structure, the nature of the equilibrium depends critically on the strength of competition between the agents. The severity of competition depends on the interplay of two related factors. The first factor is the optimism of the prior beliefs associated with each proposal; speaking roughly, as the prior beliefs for both proposals increase so that both proposals appear better ex ante, competition between the agents becomes more fierce. As a response, the agents supply more valuable signals to the principal. The severity of competition, is undermined, however, by the degree of asymmetry between the two prior beliefs γ_H and γ_L. When this asymmetry is high, the initial advantage of player H is high. This initial advantage undermines competition between the agents.

Consistent with this discussion, we expect that if both proposals are very likely to be good and the asymmetry between the proposals is low, then competition between agents is likely to be fierce. Fierce competition, in turn, forces agents to supply fully revealing signals. Indeed, this intuition is borne out by the following lemma.

Lemma 4.4 *If $\gamma_L \geq \frac{2-2\theta}{2-\theta}$ then it is a Nash Equilibrium for each player to choose a fully revealing strategy.*

On the other hand, if γ_H is high, and γ_L is relatively low, the fierce competition of the previous proposition is somewhat undone by the degree of asymmetry between the proposals. From the perspective of agent L, in order to have a hope of proposal acceptance, he must still supply a fully revealing signal, however, agent H need not supply a fully revealing signal. Because it is so likely that proposal L is rejected when its quality is discovered, agent H does not need to ever reveal that his proposal is bad to stay competitive. That is, the posterior associated with a bad signal realization does not need to be zero for the advantaged agent. We refer to a signal that concentrates probability mass on only two posteriors, 0 and θ as a *quasi-revealing* signal.[32] Whenever, γ_H is high, and γ_L is relatively low, the advantaged agent chooses a quasi-revealing signal in equilibrium, rather than a fully revealing signal.

Lemma 4.5 *If $\gamma_L \leq \frac{2-2\theta}{2-\theta}$ and $\gamma_H \geq \frac{2-2\theta+\theta^2}{2-\theta}$ then it is a Nash Equilibrium for player L to use a fully revealing signal, and for player H to use a quasi-revealing signal: $f_{H1} = \frac{1-\gamma_H}{1-\theta}$, $f_{H3} = \frac{\gamma_H-\theta}{1-\theta}$ and $f_{H0} = f_{H2} = 0$.*

Next, we consider the equilibria that exist for other values of the prior beliefs. In these cases, competition is somewhat muted compared to the fully revealing case; in addition to sometimes putting positive probability mass on signal realizations that fully reveal the proposal type, in equilibrium agents put positive probability mass on posterior beliefs in an interval $[\theta, \bar{\gamma}]$. We first consider the case of relatively small differences between the proposals.[33]

Lemma 4.6 *If $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\gamma_L \leq \frac{1}{2}(1-\theta^2)$ then the strategies of Lemma 4.3 constitute an equilibrium for*

$$\bar{\gamma} = \gamma_L + \sqrt{\gamma_L^2 + \theta^2}$$

$$f_{H0} = 1 - f_{H1} - f_{H2}, \quad f_{H1} = \frac{\gamma_H - \gamma_L}{\theta}, \quad f_{H2} = 1 - \frac{\sqrt{\gamma_L^2 + \theta^2} - \gamma_L}{\theta}, \quad f_{H3} = 0$$

$$f_{L0} = 1 - f_{L1}, \quad f_{L1} = f_{H2}, \quad f_{L2} = 0$$

Thus, we find that for small degrees of asymmetry between the proposals, and for relatively small values of both prior beliefs, both agents send signals that sometimes reveal that their proposal is bad. In addition, the posterior belief about each proposals is uniformly distributed between θ and some maximum realization $\bar{\gamma} \leq 1$ with equal probability for both proposals. The only difference between the agent's strategies, is that agent H's signal is less likely to reveal that his proposal is bad for certain. Next, we describe the equilibrium for an intermediate range of priors:

Lemma 4.7 *If $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\frac{1}{2}(1-\theta^2) \leq \gamma_L \leq \frac{2-2\theta}{2-\theta}$, then the strategies of Lemma 4.3 constitute an equilibrium for*

$$\bar{\gamma} = 2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2}$$

$$f_{H0} = 1 - f_{H1} - f_{H2} - f_{H3}, \quad f_{H1} = \frac{\gamma_H - \gamma_L}{\theta}, \quad f_{H2} = \frac{\bar{\gamma} - \theta}{2 - \bar{\gamma}}, \quad f_{H3} = \frac{2 - 2\bar{\gamma}}{2 - \bar{\gamma}}$$

$$f_{L0} = 1 - f_{L1} - f_{L2}, \quad f_{L1} = f_{H2}, \quad f_{L2} = f_{H3}$$

From the lemma, we see that the equilibrium is different in two ways from the case discussed previously. First, because both priors are higher than in the previous case, both agents are able to send signals that sometimes reveal that the proposals are good; that is, both ex ante posterior beliefs have mass points on realization 1. Second, the upper end of the support of the uniform

[32] In order to choose a quasi-revealing signal, the prior must exceed θ.

[33] That is, when $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$.

$\bar{\gamma}$ in this case is just two minus its counterpart in the previous proposition. This equilibrium has a similar property to the previous one: the only difference between agents strategies is agent H's mass point on θ.

We turn now to the case of large differences in prior beliefs about the two proposals. As in the previous case, we present two propositions, one for the case of relatively pessimistic priors, and another for the case of relatively optimistic priors. Whenever the difference in prior beliefs is large, the advantaged agent never chooses a signal that ever reveals that his proposal is bad for certain.

Lemma 4.8 *If $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\gamma_H \leq \frac{1}{2}(1+\theta^2)$, then the strategies of Lemma 4.3 constitute an equilibrium for*

$$\bar{\gamma} = \gamma_H + \sqrt{\gamma_H^2 - \theta^2}$$

$$f_{H0} = 0, \;\; f_{H1} = 1 - f_{H2}, \;\; f_{H2} = \frac{2(\gamma_H - \theta)}{\bar{\gamma} - \theta}, \;\; f_{H3} = 0$$

$$f_{L0} = 1 - f_{L1}, \;\; f_{L1} = \frac{2\gamma_L}{\bar{\gamma} + \theta}, \;\; f_{L2} = 0$$

This equilibrium is somewhat reminiscent of the high asymmetry, high γ_H equilibrium in which agent L supplies a fully revealing signal, and agent H supplies a quasi-revealing signal. Unlike that scenario, however, neither prior belief is high enough that the agent can actually put probability mass on posterior belief realization 1. In equilibrium it is not worth it for an agent to send a signal that would actually reveal a good proposal, as the probability that the proposal is good is low. Instead, of concentrating mass on one, the agents spread it uniformly in an interval from θ to some threshold $\bar{\gamma} \leq 1$. For intermediate priors and high asymmetry, agents begin to reveal good proposals, as demonstrated by the following lemma:

Lemma 4.9 *If $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\frac{1}{2}(1+\theta^2) \leq \gamma_H \leq \frac{2-2\theta+\theta^2}{2-\theta}$, then the following strategies constitute an equilibrium:*

$$\bar{\gamma} = 2 - \gamma_H - \sqrt{\gamma_H^2 - \theta^2}$$

$$f_{H0} = 0, \;\; f_{H1} = 1 - f_{H2} - f_{H3}, \;\; f_{H2} = \frac{\bar{\gamma} - \theta}{2 - \bar{\gamma}}, \;\; f_{H3} = \frac{2 - 2\bar{\gamma}}{2 - \bar{\gamma}},$$

$$f_{L0} = 1 - f_{L1} - f_{L2}, \;\; f_{L1} = \frac{2\gamma_L(\bar{\gamma} - \theta)}{(2 - \bar{\gamma})^2 - \theta^2}, \;\; f_{L2} = \frac{4\gamma_L(1 - \bar{\gamma})}{(2 - \bar{\gamma})^2 - \theta^2}$$

Qualitatively, the only difference between the intermediate prior case and the low prior case is the appearance of the mass points on 1. As in the case of small differences in prior beliefs, the mixing threshold $\bar{\gamma}$ is two minus the threshold for small priors.

We illustrate the six propositions that characterize the Nash equilibrium of the first stage game between agents in Figure 2.

We would like to highlight two key points about these equilibria. First, as agents compete, probability mass is spread: the probability of posterior realizations above θ is non-zero in all equilibria. Second, in all equilibria, the probability of either accepting proposal L or rejecting both

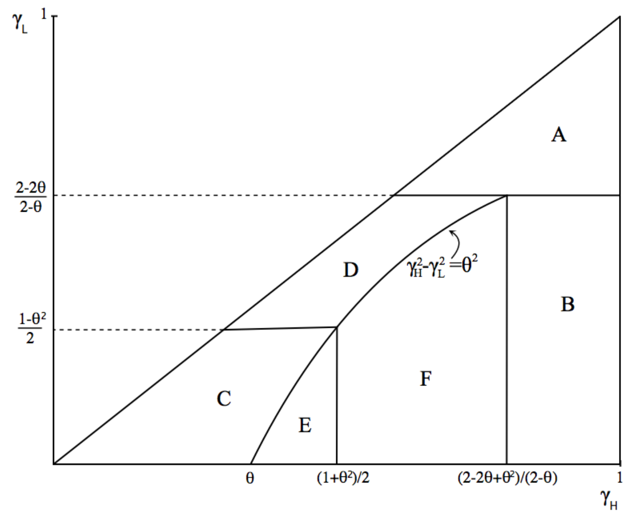

Figure 2: Equilibrium Cases. Labels follow propositions in alphabetical order.

proposals is non-zero. While agent H does possess an advantage, it is never the case that in any equilibrium the principal always prefers to accept proposal H.

4.3 The Information Benefit of Limited Capacity

We now show that under limited capacity agents supply more informative signals than under unlimited capacity. To establish this result we apply results from Ganuza and Penalva (2010). These authors define a *convex order* among random variables.

Definition 4.1 *Y is greater than Z in the convex order if for all convex real-valued function $\phi, E[\phi(Y)] \geq E[\phi(Z)]$ provided the expectation exists.*

These authors then use this definition to define a notion of signal precision that they call *integral precision*. We adapt their definition to our setting.

Definition 4.2 *Signal Σ_1 is more integral precise than signal Σ_2 if the ex ante posterior belief induced by Σ_1 is greater than the ex ante posterior belief induced by Σ_2 in the convex order.*

The authors then show that with a binary type space (such as ours), the notion of integral precision is equivalent to Blackwell informativeness. That is, for a given prior, Σ_1 is more integral precise than Σ_2, if and only if Σ_1 is Blackwell-sufficient for Σ_2 (i.e. Σ_1 is more informative than Σ_2).[34]

In order to demonstrate that each agent's equilibrium signal under limited capacity is more Blackwell informative than his equilibrium signal under unlimited capacity it is enough to show that the agent's equilibrium strategy under limited commitment dominates his strategy under unlimited commitment in the convex order. In our case, the agent's strategies have the same mean; therefore,

[34] Σ_1 is Blackwell sufficient for Σ_2 if Σ_2 is a garbling of Σ_1. In this case, any decision maker prefers to observe signal Σ_1 to signal Σ_2.

the convex order is equivalent to the reverse of second order stochastic dominance.[35] Thus, if each agent's equilibrium strategy under limited capacity is second order stochastic dominated by his equilibrium strategy under unlimited commitment, then both signals are more Blackwell informative under limited commitment than under unlimited commitment. In the next proposition, we establish that this is indeed the case.

Proposition 4.10 *Each agent's equilibrium strategy under limited capacity is more Blackwell informative than the agent's equilibrium strategy under unlimited capacity.*[36]

This proposition illustrates the informational benefit of limited capacity. Because agents compete (indirectly) under the limited capacity system, they choose to supply signals that are more informative about the true quality of their proposals. This result is stronger than the similar result in Section 3, where it held when the principal's predisposition was aligned with the priors.

Although agents supply more informative signals under limited capacity, the principal is not always better off because her actions are constrained: she may not be able to accept a proposal that she expects is beneficial. In the next section, we show that the benefits of limited capacity frequently outweigh the costs; the principal frequently does better under a limited capacity system.

4.4 Limited Capacity is Preferred

As demonstrated in the previous section, limited capacity comes with benefits, but it also comes with costs. If both proposals generate posterior beliefs that are greater than θ, under limited capacity the principal is only able to accept one, although she expects to benefit by accepting both. In this section, we show that the benefits of limited capacity often outweigh the costs; in these cases, the principal prefers to have with limited capacity.

If $\gamma_H \leq \theta$, then it is not difficult to see that the principal prefers limited capacity. In this case, under unlimited capacity, the principal's expected payoff is zero, equal to her payoff of rejecting both proposals. Recall that the principal's expected payoff of accepting a proposal which she believes is good with probability γ is $\gamma - \theta$. In the limited capacity equilibrium, the probability that the largest of the two realizations of the posterior belief is strictly greater than θ is non-zero (given that $\gamma_L > 0$). Thus, there is a positive probability that the principal accepts a proposal that brings her a strictly positive expected payoff. Her expected payoff in this equilibrium is therefore strictly positive.

Intuitively, if the principal is predisposed against both proposals and capacity is unlimited, the agents supply signals which leave the principal (at best) indifferent between accepting and rejecting. Thus there is no cost associated with limited capacity, but because limited capacity motivates the agents to supply signals that are more informative, it is strictly beneficial and always preferred.

[35] If E[Y]=E[Z] then Y second order stochastic dominates Z if and only if for every *concave* function ϕ $E[\phi(Y)] \geq E[\phi(Z)]$. As the negative of a concave function is convex, strategy Γ_1 is second order stochastic dominated by strategy Γ_2 if and only if Γ_1 is greater than Γ_2 in the convex order.

[36] Here we assume that if the principal is predisposed in favor of an agent's proposal, the agent supplies an uninformative signal.

A related argument demonstrates that the principal continues to prefer limited capacity when $\gamma_L \leq \theta \leq \gamma_H$. With these parameters under unlimited capacity, the principal always accepts proposal H and is at best indifferent between accepting and rejecting proposal L, resulting in expected payoff $\gamma_H - \theta$. If the principal observes the equilibrium signals from the limited capacity game, but is forced to accept H (and reject L) her payoff will be $\gamma_H - \theta$, identical to her payoff in the unlimited capacity equilibrium. In the limited capacity equilibrium, however, the principal is not constrained to always accept H and reject L; in fact, the probability that in the limited equilibrium the principal chooses to accept L or reject both proposals is non-zero. Her expected payoff in the limited capacity equilibrium therefore exceeds her payoff when she is constrained, which is $\gamma_H - \theta$.

We have therefore demonstrated that the principal strictly prefers the equilibrium with limited capacity to the equilibrium with unlimited capacity whenever $\gamma_L \leq \theta$, independent of γ_H. Because the equilibrium changes in a continuous way as the parameters change, there is some region in which $\gamma_L > \theta$ in which the principal also prefers the equilibrium with limited capacity. We therefore have the following proposition:

Proposition 4.11 *If $\gamma_L, \gamma_H \leq \theta$, then the principal's expected payoff always is strictly higher under limited capacity compared to unlimited capacity. If $\gamma_H > \theta$, then there exists a value $\tilde{\gamma}_L \in (\theta, \gamma_H)$ such that the principal's expected payoff is strictly higher under limited capacity when $\gamma_L < \tilde{\gamma}_L$.*

This region is illustrated in Figure 3.

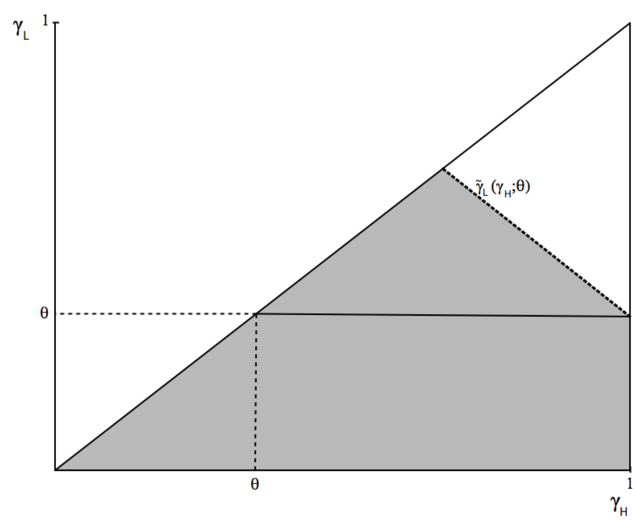

Figure 3: Limited capacity is preferred in the shaded region.

5 Applications

5.1 Legislative Decision Making

Limited capacity is a reality of the legislative process. An appropriation committee may not be willing to allot individual legislators funds to support all potential earmarks they believe beneficial. For example, Frisch and Kelly (2010, 2011) present evidence that during the 2006 budget cycle the Chairman of the US House Appropriations Subcommittee for Labor-Health and Human Services allowed each rank-and-file member of the U.S. House to request up to $400,000 in earmark funding from his subcommittee. The allotted amount increased systematically for subcommittee members, principals in at-risk districts and those in leadership positions. If a legislator requested a larger amount of funding from the subcommittee, the funding was rejected or cut down to the allotted amount. This process means that legislators must carefully decide for which earmark projects to request funding. Legislators often respond to their own limited capacity to request earmarks by setting up an application process through which potential beneficiaries may apply, and lobby for funding.

Similarly, time and staff constraints may prevent a politician from introducing legislation on each policy proposal she or her constituents support. In an attempt to convince the legislator that its policy proposal is worth the legislator's time and effort, an interest group can collect evidence confirming the merits of its proposal. Our story is consistent with Hall and Deardorff (2006)'s story of "lobbying as legislative subsidy," where special interest and lobby groups promise assistance (e.g., provide help conducting research or writing legislation) to time constrained politicians in an effort to convince the politicians to take up their cause. In our model, some of the assistance—conducting research and helping the politician better understand the implications of a policy—may come before the legislator decides which policies to pursue.

We have shown that limited legislative capacity can entice special interest groups to produce more informative evidence about the merits of their projects or policies. These constraints have the potential to improve politician (and constituent) wellbeing, even though they may sometimes prevent good policies from being implemented or benifical earmark projects from being funded.

5.2 Grant Writing

The second motivating example from the introduction involved grant writing. A funding organization must choose which research or community development proposals to accept. If the funding organization could back all projects that it believes worthy of funding, then applicants with ex ante promising projects have no incentive to produce additional evidence about the merits of its project, and applicants with projects the funding organization is predisposed not to accept will collect just enough evidence about the quality of their project to change the funding decision in their favor.

Funding organizations, however, rarely have the ability to back all projects. The organization often must decide which of the promising projects is most-promising. The Robert Wood Johnson Foundation makes this clear on their website:

> "Due to the volume of proposals we receive, many excellent projects that meet our criteria still do not receive funding."

One way that applicants make their proposals stand out is by producing evidence about the likely success or contributions of their project. For researchers, this means providing a more-detailed description of their qualifications, research methods and policy implications, or producing preliminary results. Before applying for funding for a large scale field experiment, for example, researchers often run smaller trials. The number of treatments and the number of subjects in their experiments affect the informativeness of the trials about the eventual success of the project. Even in theoretical work, the researchers decide whether to develop a formal model and preliminary propositions prior to submitting an application.

The analysis shows how competition for limited funding between researchers leads to the allocation of funding to projects that are on average more promising than those funded by an organization that is able to back all projects it believes beneficial. Although the budget limitation may result in fewer promising projects being funded, the informational benefits of limited capacity often outweigh the potential costs, and the funding organization may in fact prefer a limited budget. Increasing an organization's budget may make the organization worse off.

5.3 College Admissions

The process of admission to elite undergraduate colleges has become increasingly competitive. With the increased competition has come a greater emphasis on extracurricular activities. Our model suggests a reason for this. A college admissions officer at an elite school wants to admit students who are most-likely high ability. Applicants provide a partially informative signal about their ability through their high school grade point average. A high GPA, however, is not perfect evidence that the student is high ability if moderate-ability students have a chance of achieving a high GPA through extra work and tutoring. Students can devote time to extra curricular activities, which communicates to the admissions officer that they maintained their GPA while devoting time to other non-studious activities. Devoting time to the other activities makes maintaining a high GPA less likely for moderate ability students, and thus improves the informativeness of a good academic outcome.[37]

5.4 Firm Expansion and Product Launch

A firm executive may be looking to expand operations, and not know which divisions offer the most-promise for expansion. The capital available for expansion may be enough to fund expansion for only one division, even if the executive believes that multiple divisions are worth expanding. Prior to choosing which division to expand, the division managers may propose strategic plans

[37] One of the authors is reminded of his high school valedictorian, who got a perfect score on the SAT, twice. He took the exam a second time to demonstrate that the perfect score he earned on the first attempt was no accident, increasing the informativeness of his signal.

and compile verifiable evidence about the profitability of the expansion of their divisions. Our results suggest that limitations to the amount of funds available for investment may lead to better investment decisions regarding these funds. That is, limited resources causes competition between the division managers, incentivizing them to produce more informative information about the future plans and profitability of their divisions.

A similar story may be told in which a firm is limited in its capacity to introduce new products to the market, and must decide which successful R&D projects to mass produce. Limited capacity may improve the quality of evidence produced by project or division managers about the quality or marketability of their respective products. The analysis suggests that a firm manager who is constrained in the number of divisions she can expand, or the number of products she can put into production may be make better decisions and be better off than a manager with unconstrained capacity.

5.5 Product Information and Pre-purchase Sampling

Firms produce different, independent products. Both producers and consumers may be uncertain about the match quality between a given product and an individual consumer's needs. To enable consumers to learn about the potential enjoyment they will receive from a purchase, the sellers can allow consumers to interact with a good prior to making a purchase decision (e.g., hands on interaction with iPads at the Apple Store, test drives at car dealerships, free samples at a chocolate shop). The more interaction the firm allows prior to purchase, the more informative the consumer's impression is about the good. If consumers are without budget constraints, the sellers would need to provide consumers with just enough interaction with the products to convince them that it is most-likely in their best interest to make a purchase. When consumers are budget constrained, however, convincing them that the project is most-likely worthwhile is not enough. Producers must convince consumers that their product is likely a better purchase than the other products. Our model predicts that limited consumer budgets lead to increased pre purchase interactions with products, which makes for a better-informed purchase decision. Our analysis illustrates that consumers may be better off when they can afford to purchase fewer products. Budget constraints lead sellers to provide consumers with more pre-purchase interaction, increasing the probability that the consumers buy products they find worthwhile.

5.6 FDA Approval

The US Food and Drug Administration (FDA) regulates pharmaceuticals. Included in this is the control of whether drugs are approved for over-the-counter or prescription use. To gain FDA approval, drug companies conduct clinical trials to provide sufficient evidence that their produce is safe and effective for human use. There is currently no official limit to the number of new drugs that the FDA can approve every year. Our analysis shows that limiting the number of annual new drug approvals could theoretically increase consumer wellbeing. This would be because competition for a limited number of approvals could incentivize drug companies to conduct more-extensive reviews of their product before applying for FDA approval. Of course, we are ignoring the financial costs and potential health effects of increased time to market associated with additional trials, and the political and public relations problems that may arise from not approving drugs that the evidence suggests will most-likely save or improve lives.

6 Conclusion

We develop a model of persuasion in which a principal decides whether to implement each of two independent proposals (e.g., earmark requests, policy reforms, grant funding). Agents advocating on behalf of the proposals can produce evidence about the quality of their respective proposal, enabling the principal to make a more informed decision. The principal prefers the agents to produce the most-informative evidence possible. Agents, however, strategically choose evidence quality to maximize the probability that the principal implements their proposal (maximizing their own payoffs rather than the principal's payoff). When the principal can implement all proposals she believes are worthwhile (i.e., the case of unlimited capacity), the agents typically will not produce fully informative evidence, leaving the principal less than fully informed about proposal quality.

Within this framework, we show that the principal can often be both better informed and better off when she is constrained in the number of proposals she can implement. When the principal is unable to implement all proposals, the agents are concerned about their proposal being given priority over the alternatives in the event that the principal would like to implement more proposals than she has capacity. If the principal observes evidence in favor of two proposals, but can only implement one of them, then she will give priority to the one with the more-informative evidence—that is, the one she is more certain is high quality. The agents react to this by increasing the quality of the evidence they produce. Under limited capacity, the agents produce more informative evidence compared to the case in which the legislator had unlimited capacity to implement proposals. We then derive reasonable conditions under which this informational benefit dominates the expected cost, and the principal prefers to be limited in the number of proposals she can implement.

The model highlights an informational benefit of limited capacity and competitive advocacy that has not been focused on before in the literature. The framework may be applied to understand incentives in a variety of settings, lending insight into informational lobbying, consumer sampling, college admissions and preliminary research in grant applications, among others.

7 Appendix

7.1 Detailed analysis the symmetric agent, binary signal model under limited capacity

7.1.1 Predisposed against and proposals most-likely bad

In this case, $\gamma < \theta$ and $\gamma < \frac{1}{2}$. We first characterize principal behavior under limited capacity. Since the principal is predisposed against accepting, she certainly rejects any proposal for which the signal is weak or the realization is bad. If only one proposal generates a favorable signal from a persuasive distribution, then the principal implements that proposal. If both signals generate good realizations, she accepts the proposal she believes is more likely to be good; that is the one with the more-accurate signal quality. Formally:

- If $\alpha_i \in [\frac{1}{2}, r)$ and $\alpha_j \in [\frac{1}{2}, r)$ reject both proposals.
- If $\alpha_i \in [r, 1]$ and $\alpha_j \in [\frac{1}{2}, r)$ accept proposal i if and only if $s_i = G$. Always reject proposal j.
- If $\alpha_i \in [r, 1]$ and $\alpha_j \in [r, 1]$ and $s_i = B$ and $s_j = B$, reject both proposals.
- If $\alpha_i \in [r, 1]$ and $\alpha_j \in [r, 1]$ and $s_i = G$ and $s_j = B$, accept proposal i.
- If $\alpha_i \in [r, 1]$ and $\alpha_j \in [r, 1]$ and $s_i = G$ and $s_j = G$, accept proposal i if $\alpha_i > \alpha_j$ and accept each proposal with equal probability if $\alpha_i = \alpha_j$.

This illustrates the impact of limited capacity. By supplying a more accurate signal, an agent gives his issue priority: if both signals are persuasive, and both realizations are good, his proposal will be the one implemented. Because the proposals are most-likely bad, however, increasing accuracy reduces the probability of generating a good signal. In equilibrium, agents trade off the benefit of taking priority against the reduction in the probability of generating a good signal.

Given the principal's strategy, we construct each agent's first period expected payoff as a function of both signal accuracies. Suppose agents choose α_i, α_j:

$$\begin{array}{llll} \text{if} & \alpha_i \in [\frac{1}{2}, r) & \text{then} & u_i(\alpha_i, \alpha_j) = 0 \\ \text{if} & \alpha_i \in [r, 1] \text{ and } \alpha_i > \alpha_j & \text{then} & E[u_i(\alpha_i, \alpha_j)] = \sigma(\alpha_i) \\ \text{if} & \alpha_i \in [r, 1] \text{ and } \alpha_i < \alpha_j & \text{then} & E[u_i(\alpha_i, \alpha_j)] = \sigma(\alpha_i)(1 - \sigma(\alpha_j)) \\ \text{if} & \alpha_i \in [r, 1] \text{ and } \alpha_i = \alpha_j & \text{then} & E[u_i(\alpha_i, \alpha_j)] = \sigma(\alpha_i)\left(1 - \frac{\sigma(\alpha_j)}{2}\right). \end{array}$$

Each of these payoffs is straightforward to understand. If an agent produces a weak signal, his proposal is rejected. If an agent produces a persuasive signal with higher accuracy than the other agent, his issue is decided on its merits. It is implemented if and only if it generates a good signal. If agent i produces a persuasive signal that is less accurate than the other agent, his proposal is accepted only if it generates a good signal realization and the other proposal generates a bad realization. Finally, if both agents produce persuasive signals of identical accuracy, then proposal i is accepted if $s_i = G$ and $s_j = B$. If both proposals generate good signal realizations, then each proposal is accepted with equal probability. A weak signal is strictly dominated by producing a marginally persuasive signal and would never be part of an equilibrium strategy.

We start by characterizing the unique pure strategy equilibrium of this stage game, $\alpha_i = \alpha_j = 1$. Suppose that agent j chooses a fully informative signal, $\alpha_j = 1$. In this case, the probability that proposal j generates a good signal realization is $\sigma(1) = \gamma$. Therefore, by choosing $\alpha_i = 1$, agent i expects payoff

$$\gamma\left(1 - \frac{\gamma}{2}\right)$$

Any choice of $\alpha_i \in [r, 1)$ gives expected payoff $\sigma(\alpha_i)(1 - \gamma)$. Since $\gamma < \frac{1}{2}$, function $\sigma(\alpha)$ is strictly decreasing and an agent's optimal choice of $\alpha \in [r, 1)$ is $\alpha = r$, which gives expected payoff

$$\sigma(r)(1 - \gamma) = \frac{\gamma(1 - \gamma)^2}{(1 - \theta)\gamma + (1 - \gamma)\theta}$$

In order for $\alpha_i = \alpha_j = 1$ to be a Nash equilibrium, it must be that

$$\gamma\left(1-\frac{\gamma}{2}\right) \geq \frac{\gamma(1-\gamma)^2}{(1-\theta)\gamma+(1-\gamma)\theta} \iff \theta \geq \hat{\theta} \equiv \frac{3\gamma^2-6\gamma+2}{2\gamma^2-5\gamma+2}$$

The above definition of $\hat{\theta}$ describes the value of $\hat{\theta}$ from Lemma 3.3. No other symmetric pure strategy equilibrium exists, as a deviation to $\widehat{\alpha}_i = \alpha_j + \varepsilon$ gives higher expected payoff than $\alpha_i = \alpha_j$. No asymmetric pure strategy equilibrium exists, as the agent supplying the signal with higher accuracy could always benefit by slightly reducing his signal accuracy.

If both agents choose $\alpha = 1$, the most profitable deviation for an agent is to produce a marginally persuasive signal, ceding full priority to the other agent but maximizing the probability of generating a good signal realization. Such a deviation is profitable if the probability of generating a good realization when choosing the marginally persuasive signal is high; this in turn is the case when the marginally persuasive signal accuracy r is close to $\frac{1}{2}$. When the stakes θ increase, the cost of mistakenly implementing a bad proposal is high; thus the principal requires a more informative signal in order to overturn her predisposition towards rejection. In other words, increases in θ increase the value of r. Thus a sufficiently high θ ensures that both agents produce fully-informative signals in equilibrium.

Next, we provide a partial characterization of all mixed strategy equilibria of the first stage game. This partial characterization is based on several straightforward observations about the nature of the equilibrium. For our main result, the partial characterization is sufficient. Consider a mixed strategy Nash equilibrium in which each agent's signal accuracy is the realization of random variable A. We assume that this mixed strategy Nash equilibrium involves randomization, i.e. it is not the pure strategy equilibrium $\alpha_1 = \alpha_2 = 1$ already characterized.

Lemma 7.1 *Under the limited capacity legislative system with $\gamma < q$ and $\gamma < \frac{1}{2}$, in any symmetric mixed strategy Nash equilibrium of the stage game, the following four properties must hold:*

1. *Weak signals are outside of the support of A.*
2. *The only possible mass point is $\alpha = 1$.*
3. *The smallest signal accuracy inside the support of A is r.*
4. *If disjoint intervals $[x_1, x_2]$ and $[x_3, x_4]$ are in the support of A, then the entire interval $[x_1, x_4]$ is inside the support of A.*

Proof.

(1) Uninformative signals are strictly dominated.

(2) If the mixed strategy equilibrium strategy has a mass point at a, then it can be described as follows: with probability ϕ each agent selects $\alpha = a$. With probability $1-\phi$ each agent draws a signal accuracy from CDF $F(x)$ (which is possibly discontinuous itself, i.e. has mass points). Consider the following deviation for agent i. With probability ϕ agent selects $\alpha = a + \varepsilon$. With probability $1-\phi$ agent i draws signal accuracy from CDF $F(x)$ With probability $(1-\phi)^2$ both choose to draw from $F(x)$, in this case the payoff is unchanged. With probability $(1-\phi)\phi$ the other agent plays his mass point, while the deviating agent plays from $F(x)$. In this case the deviator's payoff is unchanged. With probability $(1-\phi)\phi$ the other agent plays from $F(x)$ while the deviator plays the mass point. Because the mass point under the deviation is arbitrarily close to the original mass point the payoff is also arbitrarily close to the original payoff. With probability ϕ^2 both players play the mass point.

In this case, the deviation gives expected payoff $\sigma(a+\varepsilon)$ instead of $\sigma(a)\left(1-\frac{\sigma(a)}{2}\right)$. Thus for small values of ε this deviation causes a discrete increase in the agent's expected payoff. When ε is very small this increase is approximately $\frac{\sigma(a)^2}{2}>0$. This deviation is profitable. The only value of a for which no such deviation exists is $a=1$.

(3) Imagine that $\underline{\alpha}$, the lowest element of the support of A, were strictly greater than r. Compare $\underline{\alpha}$ to r. As there is no mass point at $\underline{\alpha}$, with probability 1 the other agent's signal accuracy is strictly higher than both $\underline{\alpha}$ and r. Therefore, the expected payoff from choosing $\underline{\alpha}$ for certain is $\sigma(\underline{\alpha})(1-E[\sigma(\alpha_j)])$, while the payoff to choosing r is $\sigma(r)(1-E[\sigma(\alpha_j)])$. Because $\gamma<\frac{1}{2}$, $\sigma(x)$ is decreasing; hence,

$$\sigma(r)(1-E[\sigma(\alpha_j)])>\sigma(\underline{\alpha})(1-E[\sigma(\alpha_j)])$$

Therefore, if the smallest element of the support is larger than r, then playing the mixed strategy is dominated by choosing r.

(4) Imagine that disjoint intervals $[x_1,x_2]$ and $[x_3,x_4]$ are in the support of A, but interval (x_2,x_3) is outside the support of A. Compare playing x_2 to x_3. Both of these signal accuracies have the same probability of being larger than the other player's accuracy, and both accuracies have the same probability of being less than the other player's signal accuracy. However, in both of these cases, the payoff associated with pure strategy x_2 is higher than the payoff associated with x_3. This contradicts the indifference condition. ■

This Lemma is not difficult to understand. As weak signals are dominated they are outside of the support of the mixed strategy equilibrium. No interior mass point can exist in a symmetric mixed strategy equilibrium; if both strategies have the same mass point, then agents choose the same accuracy with positive probability. If both proposals generate good realizations and the accuracies are the same (which happens with nonzero probability), a tie occurs, and each proposal is accepted with probability $\frac{1}{2}$. By just slightly increasing the accuracy associated with the mass point, an agent can assure that all ties break in his favor, which increases his payoff by a discrete amount, and is therefore profitable. The only possible mass point is therefore at $\alpha=1$. If the smallest signal accuracy in the support of the mixed strategy $\underline{\alpha}$ were strictly greater than r, then whether agent i chose $\alpha_i=r$ or chose $\alpha_i=\underline{\alpha}$, his accuracy is always less than the accuracy of the other agent, which always gives the other agent decision making priority. Thus whether $\alpha_i=r$ or $\alpha_i=\underline{\alpha}$, proposal i is implemented if $s_j=B$ and $s_i=G$. However, because increased accuracy reduces the probability of generating a good signal (and does not affect the probability that $s_j=B$), the signal with $\alpha_i=r$ has a higher probability that $s_i=G$. Therefore if $\underline{\alpha}>r$ then choosing $\alpha_i=r$ dominates the mixed strategy. The intuition for the last part of the Lemma is similar: under the assumptions, pure strategies x_2 and x_3 give an agent the same priority in the decision making process, but accuracy x_2 is more likely to generate a good signal realization.

Only two types of mixed strategies are consistent with Lemma 7.1. The first type of mixed strategy calls for agents to choose their signal strengths from a continuous CDF with support on $[r,\bar{\alpha}]$, where $\bar{\alpha}\le 1$. The second type of mixed strategy equilibrium calls for agents choosing signal strengths from a CDF with continuous support on $[r,\bar{\alpha}]$ and mass point on 1. Under both feasible mixed strategies, both agents choose signal qualities greater than r with probability one, since the mixed strategy CDFs do not allow for a mass point on r and r is the smallest accuracy in the support. Compared to the case of unlimited capacity, here limited capacity almost certainly results in a more informed principal. Because the principal has an expected payoff of zero in this case under unlimited capacity, and her expected payoffs are strictly higher here under limited capacity, she strictly prefers limited to unlimited capacity.

Although it is not essential for the main results, we believe that a complete characterization of the mixed strategy equilibria lead to a better understanding of the effects limited capacity has on agent incentives. Below, we complete the characterization of the mixed strategy equilibrium for $\gamma < \theta, \gamma < \frac{1}{2}$ in the cases in which no pure strategy equilibrium exists. In an interest of space, and because the characterization is not required for the results in the body of the paper, we do not walk through the derivation. Define $\bar{\alpha}_1$ and $\bar{\alpha}_2$ as follows,

$$\bar{\alpha}_1 = \frac{1}{(1-2\gamma)^2} \frac{3\gamma - 2\gamma^2 - 1}{(\theta + \gamma - 2\theta\gamma)^2} \Big(\left(4\gamma - 4\gamma^2 - 1\right)\theta^2 + \left(4\gamma^2 - 2\gamma\right)\theta + \left(\gamma^3 - 2\gamma^2\right) + \gamma\sqrt{\left(4\gamma^2 - 4\gamma + 1\right)\theta^2 + \left(2\gamma - 4\gamma^2\right)\theta + \left(\gamma^4 - 2\gamma^3 + 2\gamma^2\right)} \Big)$$

$$\begin{aligned} \bar{\alpha}_2 &= \frac{1}{(2\gamma-1)^2} \frac{1-3\gamma+2\gamma^2}{(4\gamma^2-4\gamma+1)\theta^2+(2\gamma-4\gamma^2)\theta+(4\gamma^3-11\gamma^2+12\gamma-4)} \\ &\quad \times \Big(\left(4\gamma^2 - 4\gamma + 1\right)\theta^2 + \left(2\gamma - 4\gamma^2\right)\theta + \left(5\gamma^3 - 14\gamma^2 + 14\gamma\right) \\ &\quad + \gamma\sqrt{\left(4\gamma^2 - 4\gamma + 1\right)\theta^2 + \left(2\gamma - 4\gamma^2\right)\theta + \left(\gamma^4 - 2\gamma^3 + 2\gamma^2\right)} - 4 \Big). \end{aligned}$$

- Under limited capacity with $\gamma < \theta$, $\gamma < \frac{1}{2}$ and

  $$\theta \leq \frac{1}{(1-2\gamma)^2}\left((1-\gamma)\sqrt{(1-2\gamma)^3} - \left(\gamma - 2\gamma^2\right)\right)$$

  the unique mixed strategy Nash equilibrium of the first stage game is characterized as follows. Each agent chooses his signal accuracy α_i equal to the realization of random variable A with support on $[r, \bar{\alpha}_1]$. The density of A is given by

  $$f(x) = \frac{\sigma(\bar{\alpha}_1)(1-2\gamma)}{\sigma(x)^3}.$$

  Each agent's equilibrium payoff is given by $Eu_i = \sigma(\bar{\alpha}_1)$.

- Under limited capacity with $\gamma < \theta$, $\gamma < \frac{1}{2}$ and

  $$\frac{1}{(1-2\gamma)^2}\left((1-\gamma)\sqrt{(1-2\gamma)^3} - \left(\gamma - 2\gamma^2\right)\right) < \theta < \frac{3\gamma^2 - 6\gamma + 2}{2\gamma^2 - 5\gamma + 2}$$

  the unique mixed strategy Nash equilibrium of the first stage game is characterized as follows. With probability $\phi = \frac{2\gamma - 2\sigma(A)}{\gamma^2 - 2\sigma(A)\gamma}$ an agent chooses a fully informative signal $\alpha = 1$. . With probability $1 - \phi$, α is equal to the realization of random variable A with support on $[r, \bar{\alpha}_2]$ The density of A is given by

  $$f(x) = \sigma(\bar{\alpha}_2)\gamma^2 \frac{1-2\gamma}{2\gamma - \gamma^2 - 2\sigma(\bar{\alpha}_2)(1-\gamma)} \frac{1}{\sigma(x)^3}.$$

  Each agent's equilibrium payoff is given by $\sigma(\bar{\alpha}_2)(1 - \gamma\phi)$.

- For larger θ the Nash equilibrium is the pure strategy equilibrium derived previously.

7.1.2 Predisposed in favor and proposals most-likely good

Here we derive equilibria for the case when $\theta < \gamma$ and $\gamma > \frac{1}{2}$. The following summarizes the principal's equilibrium strategies, as discussed in the body of the paper:

- If $s_i = G$ and $s_j = B$, the principal accepts proposal i.

- If $s_1 = s_2 = G$. the principal accepts proposal i if $\alpha_i > \alpha_j$. If $\alpha_i = \alpha_j$ accept each proposal with probability $\frac{1}{2}$.
- If $s_1 = s_2 = B$, the principal accepts proposal i if and only if $\alpha_i < \alpha_j$ and $\alpha_i \leq 1-r$. If $\alpha_i = \alpha_j \leq 1-r$ accept each proposal with probability $\frac{1}{2}$.

Given the principal's strategy, it is straightforward to construct each agent's expected payoff as a function of both signal accuracies. Suppose agents choose α_i, α_j,

$$\begin{array}{llll}
\text{if} & \alpha_i > \alpha_j & \text{then} & E\left[u_i\left(\alpha_i, \alpha_j\right)\right] = \sigma\left(\alpha_i\right) \\
\text{if} & \alpha_i \in (1-r, 1] \text{ and } \alpha_i < \alpha_j & \text{then} & E\left[u_i\left(\alpha_i, \alpha_j\right)\right] = \sigma\left(\alpha_i\right)\left(1-\sigma\left(\alpha_j\right)\right) \\
\text{if} & \alpha_i \in [\frac{1}{2}, 1-r] \text{ and } \alpha_i < \alpha_j & \text{then} & E\left[u_i\left(\alpha_i, \alpha_j\right)\right] = 1-\sigma\left(\alpha_j\right) \\
\text{if} & \alpha_i \in (1-r, 1] \text{ and } \alpha_i = \alpha_j & \text{then} & E\left[u_i\left(\alpha_i, \alpha_j\right)\right] = \sigma\left(\alpha_i\right)\left(1-\frac{\sigma(\alpha_j)}{2}\right) \\
\text{if} & \alpha_i \in [\frac{1}{2}, 1-r] \text{ and } \alpha_i = \alpha_j & \text{then} & E\left[u_i\left(\alpha_i, \alpha_j\right)\right] = \frac{1}{2}.
\end{array}$$

If agent i produces a more accurate signal than agent j, proposal i is implemented if and only if proposal i generates a good signal. If agent i produces a less informative signal than agent j and this signal is persuasive, then proposal i is implemented if and only if issue j generates a bad signal and i generates a good signal. On the other hand if i's signal is less informative than j and is also weak, then proposal i is implemented if and only if j generates a bad signal. If both agents produce a signal with the same persuasive accuracy, then proposal i is implemented whenever $s_i = G$ and $s_j = B$; furthermore, if both proposals generate good realizations, each is accepted with equal probability. Finally, if both agents produce the same weak signal, then i is implemented if it generates a good realization and j does not; otherwise each proposal is implemented with probability $\frac{1}{2}$. In this case, the probability of i being implemented is just

$$\sigma\left(\alpha\right)\left(1-\sigma\left(\alpha\right)\right)+\frac{1}{2}\left(\sigma\left(\alpha\right)^2+\left(1-\sigma\left(\alpha\right)\right)^2\right)=\frac{1}{2}$$

As in the previous case, an agent who supplies a more informative signal receives priority in the decision making process. Furthermore, because accuracy is success enhancing, it is easy to see that supplying a fully informative signal $\alpha_i = 1$ is a best reply to any less than fully informative signal $\alpha_j < 1$.[38] Thus, in any pure strategy Nash equilibrium, at least one agent supplies a fully informative signal. If the best reply to a fully informative signal is also a fully informative signal, then $\alpha_1 = \alpha_2 = 1$ is the unique Nash equilibrium of the first stage game. This is not always the case, however. If agent i replies to a fully informative signal by also issuing a fully informative signal, his payoff is

$$\gamma\left(1-\frac{\gamma}{2}\right)$$

When both agents produce fully-informative signals, both proposals have equal priority, and a good signal realization is required for a proposal to be accepted. Rather than supply a fully informative signal as a response to a fully informative signal, agent i may prefer to produce a weak signal. In doing so, agent i cedes priority to agent j; however, if proposal j generates a bad signal realization, proposal i will be implemented, regardless of i's signal realization. If a good signal realization is not sufficiently likely when $\alpha = 1$ (i.e. γ is not large enough), an agent may prefer to sacrifice priority in order to remove the requirement that his

[38]Suppose $\alpha_j < 1$. Because $\sigma(\alpha)$ is increasing, $\alpha_i = 1$ is clearly better than any other $\alpha_i > \alpha_j$. Among accuracies less than α_j, weak or marginally persuasive accuracies are preferred to strictly persuasive values. Furthermore, because $\sigma\left(\frac{1}{2}\right) = \frac{1}{2}$ and $\sigma(\alpha)$ is increasing, $\sigma(\alpha) > \frac{1}{2}$. Hence $1-\sigma(\alpha)$, the payoff to any weak signal, is less than $\frac{1}{2}$, which is itself less than the payoff to a fully revealing signal.

proposal generate a good realization in order to be implemented.

A fully revealing signal from each agent $\alpha_1 = \alpha_2 = 1$ is the unique Nash equilibrium of the first stage game if and only if

$$\gamma\left(1 - \frac{\gamma}{2}\right) \geq 1 - \gamma \iff \gamma \geq 2 - \sqrt{2}.$$

If, on the other hand, $\gamma < 2 - \sqrt{2}$, then the best response to a fully-informative signal is a weak signal $\alpha \in \left[\frac{1}{2}, 1 - r\right]$. Because agent j prefers to produce a fully-informative signal whenever agent i produces a weak signal, a multitude of asymmetric equilibria exist. In each of these equilibria one of the agents supplies a fully revealing signal and the other agent supplies a weak signal.

There also exists the possibility of mixed strategy equilibria. Consider the following mixed strategy: with probability p an agent chooses $\alpha_i = 1 - r$, and with probability $1 - p$ an agent chooses a fully revealing signal $\alpha_i = 1$. First we show that for an appropriate choice of p, an agent receives the same expected payoff from choosing pure strategies $\alpha_i = 1 - r$ and $\alpha_i = 1$ against this mixed strategy. We then verify that this expected payoff is not less than the expected payoff of choosing any other pure strategy $\alpha \in [\frac{1}{2}, 1)$ against this mixed strategy.

If agent i anticipates that agent j uses this mixed strategy, i's payoff from choosing $\alpha_i = 1 - r$ is $\frac{1}{2}p + (1 - p)(1 - \gamma)$. His expected payoff of choosing $\alpha_i = 1$ is $p\gamma + (1 - p)\left(\gamma - \frac{1}{2}\gamma^2\right)$.The value of p for which the agent is indifferent between these pure strategies is given by equation:

$$\frac{1}{2}p + (1 - p)(1 - \gamma) = p\gamma + (1 - p)\left(\gamma - \frac{1}{2}\gamma^2\right)$$

$$p = \frac{\gamma^2 - 4\gamma + 2}{\gamma^2 - 2\gamma + 1}$$

$$\gamma \in (\frac{1}{2}, 2 - \sqrt{2}] \rightarrow p \in (0, 1].$$

Each agent's payoff from either pure strategy is $u = \frac{2\gamma - 3\gamma^2}{2(1-\gamma)^2}$

$$\begin{aligned} \gamma &= \frac{1}{2} \rightarrow u = \frac{1}{2} \\ \gamma &= 2 - \sqrt{2} \rightarrow u = \sqrt{2} - 1. \end{aligned}$$

To complete the derivation, it must be shown that no pure strategy in the interior of the interval $\alpha \in [\frac{1}{2}, 1)$, if played for certain, gives a higher payoff than u against this mixed strategy. Consider the payoff to pure strategy $\widetilde{\alpha}_i < 1 - r$; to the payoff of playing $\alpha_i = 1 - r$ for certain. If the other agent plays $\alpha_j = 1$, then the two strategies give the same payoff $1 - \gamma$. However, if the other agent plays $\alpha_j = 1 - r$, then by also playing $\alpha_i = 1 - r$ agent i receives payoff $\frac{1}{2}$. By playing $\widetilde{\alpha}_i$ agent i receives payoff $1 - \sigma(1 - r)$. Because $1 - r > \frac{1}{2}$ and success is effort enhancing, $\sigma(1 - r) > \sigma\left(\frac{1}{2}\right) = \frac{1}{2}$. Thus $1 - \sigma(1 - r) < \frac{1}{2}$. Any pure strategy $\widetilde{\alpha}_i < 1 - r$ has a smaller expected payoff than $\alpha_i = 1 - r$ against this mixed strategy. Next we compare the payoff of playing pure strategy $1 - r < \widetilde{\alpha}_i < 1$ to playing pure strategy $\alpha_i = 1$ against this mixed strategy. If the other agent chooses $\alpha_j = 1 - r$ then playing $\widetilde{\alpha}_i < 1$ is dominated by playing $\alpha_i = 1$. If the other agent plays $\alpha_j = 1$, then the payoff to playing $\alpha_i = 1$ is $\gamma - \frac{1}{2}\gamma^2$. The payoff to playing $\widetilde{\alpha}_i < 1$ is $\sigma(\widetilde{\alpha}_i)(1 - \gamma)$. As

$\sigma(\widetilde{\alpha}_i) < \sigma(1) = \gamma$ it follows that

$$\gamma - \frac{1}{2}\gamma^2 > \gamma(1-\gamma) > \sigma(\alpha)(1-\gamma)$$

Therefore playing any pure strategy $1-r < \widetilde{\alpha}_i < 1$ is worse than playing $\alpha_i = 1$ against this mixed strategy. Hence, $\alpha_i = 1$ and $\alpha_i = 1-r$, give the same payoff when played against this mixed strategy, and all other possible pure strategies give a worse payoff when played against this mixed strategy. This completes the derivation of the symmetric mixed strategy equilibrium.

To determine whether the principal prefers limited or unlimited capacity, recall that under unlimited capacity agents supply weak signals, and the principal approves both proposals, $E[w] = 2(\gamma-\theta)$. Unlike the case in which the principal was predisposed against the proposals, if the principal is predisposed in favor of the proposals, the option to accept a proposal has (ex ante) value in equilibrium. Limited capacity therefore imposes an expected cost on the principal, as she does not have the option to implement one of the proposals that she would choose to implement if capacity were unlimited. For the limited capacity system to be preferred, it must result in a sufficiently large increase in signal accuracy; accepting at most one proposal with better information about its quality must dominate accepting both proposals with no additional information about their quality. In the case where $\gamma \geq 2-\sqrt{2}$, limited capacity induces an equilibrium in which both signals are fully revealing. If at least one of the proposals is good, then a good proposal is implemented. Thus when $\gamma \geq 2-\sqrt{2}$ the principal prefers the limited capacity system (ex ante) if and only if

$$(1-\theta)\left(2\gamma-\gamma^2\right) \geq 2(\gamma-\theta) \iff \theta \geq \frac{\gamma^2}{\gamma^2-2\gamma+2}$$

Next consider the case when $\gamma < 2-\sqrt{2}$. If limited capacity induces a fully-informative signal by only one of the agents (as always happens in the asymmetric pure strategy equilibrium and sometimes happens in the symmetric mixed strategy equilibrium) the principal implements the proposal associated with the fully informative signal if and only if it is a good proposal. If the realization of the fully informative signal is bad, she implements the proposal about which she is uninformed. If she anticipates that agents play the asymmetric pure strategy equilibrium in the case $\gamma < 2-\sqrt{2}$, then for such γ, the principal prefers limited capacity if and only if

$$\gamma(1-\theta) + (1-\gamma)(\gamma-\theta) \geq 2(\gamma-\theta) \iff \theta \geq \gamma^2$$

If the principal anticipates that agents play the symmetric mixed strategy equilibrium in the case $\gamma < 2-\sqrt{2}$, then for such γ the principal prefers limited capacity if and only if

$$(1-p)^2(1-\theta)\left(2\gamma-\gamma^2\right) + 2p(1-p)\left(\gamma(1-\theta)+(1-\gamma)(\gamma-\theta)\right) + p^2(\gamma-\theta) \geq 2(\gamma-\theta)$$
$$\iff$$
$$\theta \geq \frac{5\gamma^4-17\gamma^3+15\gamma^2-4\gamma}{5\gamma^3-11\gamma^2+8\gamma-2}$$

The threshold value $\tilde{\theta}$ from Proposition 3.6 follows immediately from the above analysis. If $\gamma \geq 2-\sqrt{2}$ then $\tilde{\theta} = \frac{\gamma^2}{\gamma^2-2\gamma+2}$. If $\gamma < 2-\sqrt{2}$ and the principal anticipates that agents play one of the asymmetric pure strategy Nash equilibria of the first stage, then $\tilde{\theta} = \gamma^2$. If $\gamma < 2-\sqrt{2}$ and the principal anticipates that agents play the mixed strategy Nash equilibrium in the first stage, then $\tilde{\theta} = \frac{5\gamma^4-17\gamma^3+15\gamma^2-4\gamma}{5\gamma^3-11\gamma^2+8\gamma-2}$. Whenever $\theta \in [\tilde{\theta}, \gamma)$ it is better for the principal to implement a single proposal with access to better information about proposal quality, than to decide to implement both proposals based solely on prior information.

7.1.3 When limited capacity does not increase signal accuracy

In the body of the paper, we only briefly discuss the parameter cases under which limited capacity does not increase signal accuracy. This includes when the principal is predisposed against policies that are most-likely good, or predisposed in favor of policies that are most-likely bad. When she is predisposed against proposals that are most likely good, the agents provide fully-informative signals under both unlimited and limited capacity and therefore the principal can be no more informed than she was under unlimited capacity. The slightly less-straightforward case is when the principal is predisposed in favor of proposals which are most-likely bad. In this case, the agents supply weak signals when capacity is unlimited. Here, we establish that that the unique Nash equilibrium under limited capacity involves the agents choosing completely uninformative / weak signals, $\alpha_1 = \alpha_2 = \frac{1}{2}$.

We first show that the best response of player i to any signal accuracy $\alpha_j > \frac{1}{2}$ is a weak signal, that is strictly less than α_j. Observe that $\sigma\left(\frac{1}{2}\right) = \frac{1}{2} > \sigma(\alpha)$ for any $\alpha > \frac{1}{2}$. Suppose first that the other agent chooses $\alpha_j > \frac{1}{2}$. By choosing $\alpha_i \in [\frac{1}{2}, r]$ and $\alpha_i < \alpha_j$, agent i assures himself payoff $1 - \sigma(\alpha_j)$ .Notice that

$$
\begin{aligned}
1 - \sigma(\alpha_j) &> \frac{1}{2} > \sigma(\alpha_i) \\
&\text{and} \\
\sigma(\alpha_i) &> \sigma(\alpha_i)(1 - \sigma(\alpha_j)) \\
&\text{and} \\
\sigma(\alpha_i) &> \sigma(\alpha_i)\left(1 - \frac{\sigma(\alpha_j)}{2}\right).
\end{aligned}
$$

Thus the payoff of choosing any $\alpha_i \in [\frac{1}{2}, r]$ and $\alpha_i < \alpha_j$ against $\alpha_j > \frac{1}{2}$ is greater than the payoff of choosing any other value of α_i. If $\alpha_j = \frac{1}{2}$, then agent i's payoff from choosing $\alpha_i = \frac{1}{2}$ is just $\frac{1}{2}$ which is greater than his payoff of choosing any $\alpha_i > \frac{1}{2}$. Thus $\alpha_1 = \alpha_2 = \frac{1}{2}$ is a Nash equilibrium, and no other Nash equilibrium exists.

7.2 Details involving the general model

Detailed involving the general model are provided in the online appendix.

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8 ONLINE APPENDIX

The remainder of the document is intended for publication as an online appendix. Here, we walk through the analysis of the general game in detail. There does not exist a one-to-one relationship between the claims in this document and the Lemmas and Propositions in the paper. Although each of the results in the paper follow immediately from one or more of the claims proven here. Lemma 4.1 is restated by Claim 1. Lemma 4.2 follows the analysis in the body of the paper, as well as Claim 2. Lemma 4.3 follows from the characterization of all possible equilibria (Claims 3 through 8). Lemma 4.4 follows from Claim 3, Lemma 4.5 from Claim 8, Lemma 4.6 from Claim 5, Lemma 4.7 from Claim 6, Lemma 4.8 from Claim 4, and Lemma 4.9 from Claim 7. Propositions 4.10 and 4.11 follow from the Lemmas and the analysis in the body of the paper.

Claim 1: Consider any valid random variable Γ with support confined to the unit interval and expectation γ. If the prior belief is γ then there exists a signal for which the ex ante posterior belief is Γ.

Proof. As Γ is valid, it has a countable set of mass points M. Let m_j represent an element of M and μ_j represent the jump in the CDF at m_j. The density of Γ is given by

$$p(x) = g(x) + \sum_j \delta(x - m_j)$$

for x in the support of Γ, a subset of the unit interval. Define two new random variables, Γ_b, Γ_g by their densities as follows:

$$p_g(x) = \frac{x}{\gamma} p(x) \quad \text{and} \quad p_b(x) = \frac{1-x}{1-\gamma} p(x).$$

Observe that the supports of Γ_g, Γ_b coincide exactly with the support of Γ. Observe also that if $E[\Gamma] = \gamma$ then these densities do indeed integrate to one. Consider the signal given by the pair (Γ_g, Γ_b). For this signal, the posterior belief associated with a draw of s is

$$\frac{\gamma p_g(s)}{\gamma p_g(s) + (1-\gamma) p_b(s)} = \frac{\gamma(\frac{s}{\gamma}) p(s)}{\gamma(\frac{s}{\gamma}) p(s) + (1-\gamma)(\frac{1-s}{1-\gamma}) p(s)} = s$$

Thus, for this signal, the posterior belief associated with a draw of s from this signal structure is simply s itself. The density of the posterior belief is therefore equal to the density of a draw from this signal structure:

$$\gamma p_g(x) + (1-\gamma) p_b(x) = p(x)$$

Thus, we have constructed a signal structure for which the ex ante posterior belief is Γ.[39] ■

Claim 2: Any random variable Γ for which $Pr(0 < \Gamma < \theta) > 0$ is never a best response.

Proof. Suppose that in response to the action of player j player i were to choose random variable Γ_i which puts strictly positive probability mass on realizations in the interval $(0, \theta)$. Consider a new random variable $\hat{\Gamma}_i$ constructed in the following way: 1) move all mass in the interval $(0, \theta)$ to a mass point on 0. This reduces the expected value, but does not affect the agent's payoff. Next distribute some mass from the mass point on 0 to a new mass point on 1 in such a way that the expected value of the variable $\hat{\Gamma}_i$ is γ_i. This new random variable dominates the original, because the probability mass on 1 leads to a strict increase in the probability of winning. ■

[39] This construction is not unique.

Claim 3: If $\gamma_L \geq \frac{2-2\theta}{2-\theta}$ then it is a Nash Equilibrium for each player to choose a fully revealing strategy.
Proof. An agent uses a *fully-revealing strategy* when he chooses a Bernoulli random variable. That is, player i's fully revealing strategy is

$$Pr(\Gamma_i = 1) = \gamma_i \quad \text{and} \quad Pr(\Gamma_i = 0) = 1 - \gamma_i.$$

A player can always use a fully revealing strategy.

Suppose that player j uses a fully informative strategy. Because random variables with realizations in $(0, \theta)$ cannot be best responses, player i's best response G_i has the following structure

$$Pr(G_i = 0) = \phi_L, \quad Pr(\theta \leq G_i < 1) = \phi_M, \quad Pr(G_i = 1) = \phi_H.$$

Let $g_i = E[G_i|\theta \leq G_i < 1]$. If there is nonzero probability mass strictly between θ and 1, then $g_i > \theta$. Thus, either player's best response to a fully revealing strategy can be described by the following maximization problem:

$$\begin{array}{cc} max_{\phi_M, \phi_H, g} & \phi_M(1-\gamma_j) + \phi_H(1-\frac{\gamma_j}{2}) \\ \text{s.t.} & \phi_M g_i + \phi_H = \gamma_i, \;\; \phi_M + \phi_H \leq 1, \\ & \phi_M \geq 0, \;\; \phi_H \geq 0, \;\; g_i \geq \theta \end{array}$$

Thus for any best response it must be that $\phi_M g_i + \phi_H = \gamma_i$ and $\phi_M + \phi_H = x \leq 1$. Solving for ϕ_M and ϕ_H gives

$$\phi_H = \frac{\gamma_i - g_i x}{1 - g_i}, \quad \phi_M = \frac{x - \gamma_i}{1 - g_i}$$

In addition,

$$\phi_H \geq 0 \text{ and } \phi_M \geq 0 \iff \gamma_i \leq x \leq \frac{\gamma_i}{g_i}$$

The payoff of playing this strategy is simply

$$u(x, g_i) = \frac{x - \gamma_i}{1 - g_i}(1 - \gamma_j) + \frac{\gamma_i - g_i x}{1 - g_i}(1 - \frac{\gamma_j}{2})$$

Observe that if some combination of (x, g_i) is feasible, then any combination with the same value of x but smaller value of g_i is also feasible. Because $\frac{du}{dg_i} = -\frac{1}{2}\gamma_i \frac{x-\gamma_i}{(1-g_i)^2} < 0$ a best response must have $g_i = \theta$. Thus,

$$\begin{array}{rl} u(x, \theta) = & \frac{x-\gamma_i}{1-\theta}(1-\gamma_j) + \frac{\gamma_i - \theta x}{1-\theta}(1 - \frac{\gamma_j}{2}) \\ = & \frac{2(1-\gamma_j)-(2-\gamma_j)\theta}{2(1-\theta)} x + \frac{\gamma_i \gamma_j}{2(1-\theta)}. \end{array}$$

Therefore, provided the coefficient on x is negative, the best response will be to set x to its smallest feasible value, i.e. γ_i; if this is so, the best reply is $\phi_M = 0$ and $\phi_H = \gamma_i$, the fully revealing strategy. Thus, provided

$$\frac{(2-\gamma_j)\theta - 2(1-\gamma_j)}{2(1-\theta)} \leq 0 \iff \gamma_j \geq \frac{2-2\theta}{2-\theta}$$

is satisfied for both values, γ_H, γ_L each player's best response to a fully revealing signal is also a fully revealing signal. If the inequality holds for γ_L it also holds for γ_H. ∎

Claim 4: If $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\gamma_H \leq \frac{1}{2}(1+\theta^2)$ then the following strategies constitute an equilibrium:

$$\Gamma_H = \begin{cases} \theta & \text{with probability} \quad 1 - f_H \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad f_H \end{cases}$$

$$\Gamma_L = \begin{cases} 0 & \text{with probability} \quad 1 - f_L \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad f_L \end{cases}$$

$$\bar{\gamma} = \gamma_H + \sqrt{\gamma_H^2 - \theta^2}$$

$$f_H = \frac{2(\gamma_H - \theta)}{\bar{\gamma} - \theta} = 1 - \frac{\gamma_H - \sqrt{\gamma_H^2 - \theta^2}}{\theta}$$

$$f_L = \frac{2\gamma_L}{\bar{\gamma} + \theta} = \frac{\gamma_L}{\theta}\left(1 - \frac{\sqrt{\gamma_H^2 - \theta^2}}{\theta + \gamma_H}\right)$$

$$E[u_p] = \frac{1}{2}(\bar{\gamma} - \theta)(f_H(1 - f_L) + f_L(1 - f_H)) + f_H f_L\left(\frac{2}{3}\bar{\gamma} - \frac{1}{3}\theta\right)$$

Proof. First we establish that the proposed strategies are admissible.
By assumption $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2} \leq \gamma_H$, therefore $\bar{\gamma}$ is a real number. These same conditions imply that $\bar{\gamma} \geq \theta$. Furthermore, this condition implies that $f_H \geq f_L$ (this condition is not necessary for admissibility but it will play a role later). As $\theta \leq \gamma_H, f_H \geq 0$, f_L is obviously positive. Finally, substituting and simplifying gives $f_H - 1 = -\frac{\gamma_H - \sqrt{\gamma_H^2 - \theta^2}}{\theta} < 0$; $f_L \leq f_H \leq 1$. If $\gamma_H \leq \frac{1}{2}(1+\theta^2)$ then $\bar{\gamma} \leq 1$. Finally, we check that both random variables have the required expectations.

$$E[\Gamma_L] = \frac{2\gamma_L}{\bar{\gamma} + \theta}\frac{\bar{\gamma} + \theta}{2} = \gamma_L$$

$$E[\Gamma_H] = \left(1 - \frac{2(\gamma_H - \theta)}{\bar{\gamma} - \theta}\right)\theta + \frac{2(\gamma_H - \theta)}{\bar{\gamma} - \theta}\frac{\bar{\gamma} + \theta}{2} = \gamma_H$$

We now show that the proposed strategies are mutual best responses. According to Lemma 1, any best reply to Γ_H has the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_M - \phi_H \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta + \epsilon$ leads to all ties at θ breaking in favor of player L.

In order for $\hat{\Gamma}_L$ to be admissible, it must be that $\phi_M \bar{g}_M + \phi_H \bar{g}_H = \gamma_L$, which implies

$$\bar{g}_M = \frac{\gamma_L - \phi_H \bar{g}_H}{\phi_M}$$

Consider the expected payoff of playing $\hat{\Gamma}_L$ against Γ_H:

$$\begin{aligned}
&\phi_M(1 - f_H + f_H \int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\ g_M(x)\ dx) + \phi_H \\
=\ &\phi_M(1 - f_H + f_H \frac{\bar{g}_M - \theta}{\bar{\gamma}-\theta}) + \phi_H \\
=\ &\phi_M(1 - f_H + f_H \frac{\frac{\gamma_L - \phi_H \bar{g}_H}{\phi_M} - \theta}{\bar{\gamma}-\theta}) + \phi_H \\
=\ &\phi_H \frac{\bar{\gamma} - \theta - f_H \bar{g}_H}{\bar{\gamma}-\theta} + \phi_M \frac{\bar{\gamma} - \theta - f_H \bar{\gamma}}{\bar{\gamma}-\theta} + \frac{\gamma_L f_H}{\bar{\gamma}-\theta}.
\end{aligned}$$

Observe that the coefficient on ϕ_M is equal to 0:

$$\begin{aligned}
\bar{\gamma} - \theta - f_H \bar{\gamma} \ &= \ \bar{\gamma} - \theta - \frac{2(\gamma_H - \theta)}{\bar{\gamma}-\theta}\bar{\gamma} \\
= \ \frac{\gamma_H^2 - \theta^2 - (\bar{\gamma} - \gamma_H)^2}{\bar{\gamma}-\theta} \ &= \ \frac{\gamma_H^2 - \theta^2 - (\sqrt{\gamma_H^2 - \theta^2})^2}{\bar{\gamma}-\theta} \\
= \ 0. &
\end{aligned}$$

Thus, the payoff of any admissible best response $\hat{\Gamma}_L$ does not depend on the value of ϕ_M or n the random variable G_M. Moreover, as no mass point exists in G_H at the left endpoint, $\bar{g}_H > \bar{\gamma}$. Therefore $\bar{\gamma} - \theta - f_H \bar{\gamma} = 0 \rightarrow \bar{\gamma} - \theta - f_H \bar{g}_H < 0$. Hence, in any best response, it must be that $\phi_H = 0$. Therefore, a random variable is a best response to Γ_H if and only if it has the structure of Γ_L, with $\phi_H = 0$. As the strategy Γ_L proposed in the proposition, satisfies these criteria, Γ_L is a best reply to Γ_H.

As we have already shown, any best reply to Γ_L has the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_L - \phi_M - \phi_H \\ \theta & \text{with probability} \quad \phi_L \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Also, G_M has no mass point at θ.

In order for $\hat{\Gamma}_H$ to be admissible, it must be that $\phi_L \theta + \phi_M \bar{g}_M + \phi_H \bar{g}_H = \gamma_H$, which implies

$$\bar{g}_M = \frac{\gamma_H - \phi_H \bar{g}_H - \phi_L \theta}{\phi_M}.$$

Consider the expected payoff of playing $\hat{\Gamma}_H$ against Γ_L:

$$\begin{aligned}
& \phi_L(1-f_L)+\phi_M(1-f_L+f_L\int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\; g_M(x)\; dx)+\phi_H \\
= & \ \phi_L(1-f_L)+\phi_M(1-f_L+f_L\frac{\bar{g}_M-\theta}{\bar{\gamma}-\theta})+\phi_H \\
= & \ \phi_L(1-f_L)+\phi_M(1-f_L+f_L\frac{\frac{\gamma_L-\phi_H\bar{g}_H-\phi_L\theta}{\phi_M}-\theta}{\bar{\gamma}-\theta})+\phi_H \\
= & \ \phi_H\frac{\bar{\gamma}-\theta-f_L\bar{g}_H}{\bar{\gamma}-\theta}+(\phi_L+\phi_M)\frac{\bar{\gamma}-\theta-f_L\bar{\gamma}}{\bar{\gamma}-\theta}+\frac{\gamma_H f_L}{\bar{\gamma}-\theta}.
\end{aligned}$$

Observe that the coefficient on $(\phi_L+\phi_M)$ is positive, because, as demonstrated previously, $\theta \le \sqrt{\gamma_H^2-\gamma_L^2} \rightarrow f_H \ge f_L$. Moreover, because $\bar{g}_H > \bar{\gamma}$the coefficient on ϕ_H is strictly less than the one on $(\phi_L+\phi_M)$. Therefore, in the best response, $\phi_H = 0$ and $\phi_L+\phi_M = 1$. As the strategy in the proposition satisfies these criteria, it is a best response. ■

Claim 5: If $\sqrt{\gamma_H^2-\gamma_L^2} \le \theta$ and $\gamma_L \le \frac{1}{2}(1-\theta^2)$ then the following strategies constitute an equilibrium:

$$\Gamma_H = \begin{cases} 0 & \text{with probability} \quad 1-f_{H1}-f_{H2} \\ \theta & \text{with probability} \quad f_{H1} \\ U[\theta,\bar{\gamma}] & \text{with probability} \quad f_{H2} \end{cases}$$

$$\Gamma_L = \begin{cases} 0 & \text{with probability} \quad 1-f_L \\ U[\theta,\bar{\gamma}] & \text{with probability} \quad f_L \end{cases}$$

$$\bar{\gamma} = \gamma_L+\sqrt{\gamma_L^2+\theta^2}$$

$$f_{H1} = \frac{\gamma_H-\gamma_L}{\theta}$$

$$f_{H2} = f_L = 1-\frac{\sqrt{\gamma_L^2+\theta^2}-\gamma_L}{\theta}$$

$$E[u_p] = \frac{1}{2}(\bar{\gamma}-\theta)(f_{H2}(1-f_L)+f_L(1-f_{H2}))+f_{H2}f_L(\frac{2}{3}\bar{\gamma}-\frac{1}{3}\theta)$$

Proof. First, we establish that the proposed strategies are admissible. Obviously, $\bar{\gamma} \ge \theta$, $f_{H1} \ge 0$. A simple calculation shows that $\sqrt{\gamma_L^2+\theta^2}-\gamma_L \le \theta$, and therefore, $f_{H2} = f_L \ge 0$. Furthermore, $f_{H1}+f_{H2}-1 = \frac{\gamma_H-\sqrt{\gamma_L^2+\theta^2}}{\theta}$. As $\theta \ge \sqrt{\gamma_H^2-\gamma_L^2}$ this difference is negative. Thus, all probabilities are valid. Observe that $\gamma_L \le \frac{1}{2}(1-\theta^2) \rightarrow \bar{\gamma} \le 1$. Next we demonstrate that the expected values are correct:

$$E[\Gamma_L] = f_L(\frac{\theta+\bar{\gamma}}{2}) = (1-\frac{\sqrt{\gamma_L^2+\theta^2}-\gamma_L}{\theta})(\frac{\theta+\gamma_L+\sqrt{\gamma_L^2+\theta^2}}{2}) = \gamma_L$$

$$E[\Gamma_H] = f_{H1}\theta+f_{H2}(\frac{\theta+\bar{\gamma}}{2}) = \theta\frac{\gamma_H-\gamma_L}{\theta}+\gamma_L = \gamma_H$$

Next, we demonstrate that the strategies are mutual best replies. According to Lemma1, any admissible best reply to Γ_H, denoted $\hat{\Gamma}_L$ must have the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1-\phi_M-\phi_H \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta+\epsilon$ leads to all ties at θ breaking in favor of player L. In order for this strategy to be admissible, it must be that

$$\phi_M \bar{g}_M + \phi_H \bar{g}_H = \gamma_L \iff \bar{g}_M = \frac{\gamma_L - \phi_H \bar{g}_H}{\phi_M}$$

The expected payoff of using such a strategy against Γ_H is given by:

$$\begin{aligned} & \phi_M(1-f_{H2}+f_{H2}\int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\ g_M(x)\ dx) + \phi_H \\ = & \ \phi_M(1-f_{H2}+f_{H2}\frac{\bar{g}_M-\theta}{\bar{\gamma}-\theta}) + \phi_H \\ = & \ \phi_M(1-f_{H2}+f_{H2}\frac{\frac{\gamma_L-\phi_H\bar{g}_H}{\phi_M}-\theta}{\bar{\gamma}-\theta}) + \phi_H \\ = & \ \phi_H \frac{\bar{\gamma}-\theta-f_{H2}\bar{g}_H}{\bar{\gamma}-\theta} + \phi_M \frac{\bar{\gamma}-\theta-f_{H2}\bar{\gamma}}{\bar{\gamma}-\theta} + \frac{\gamma_L f_{H2}}{\bar{\gamma}-\theta}. \end{aligned}$$

Observe that the coefficient on ϕ_H is always less than the coefficient on ϕ_M, hence, for a best response it must be that $\phi_H = 0$. Furthermore, observe that the coefficient on $\phi_M = 0$. To see this, note

$$\bar{\gamma}(1-f_{H2}) = (\gamma_L + \sqrt{\gamma_L^2+\theta^2})\frac{(-\gamma_L+\sqrt{\gamma_L^2+\theta^2}}{\theta} = \frac{\gamma_L^2-\gamma_L^2+\theta^2}{\theta} = \theta.$$

Thus, the payoff of any strategy of the type $\hat{\Gamma}_L$ is independent of ϕ_M and G_M. Therefore, any admissible random variable of structure $\hat{\Gamma}_L$is a best response, provided $\phi_H = 0$. As the strategy Γ_L is consistent with these requirements, it is a best response.

We now consider the best response to Γ_L. According to Lemma 1, any best reply to Γ_L has the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1-\phi_L-\phi_M-\phi_H \\ \theta & \text{with probability} \quad \phi_L \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Also, G_M has no mass point at θ.

In order for $\hat{\Gamma}_H$ to be admissible, it must be that $\phi_L \theta + \phi_M \bar{g}_M + \phi_H \bar{g}_H = \gamma_H$, which implies

$$\bar{g}_M = \frac{\gamma_H - \phi_H \bar{g}_H - \phi_L \theta}{\phi_M}.$$

Consider the expected payoff of playing $\hat{\Gamma}_H$ against Γ_L:

$$\begin{aligned}
& \phi_L(1-f_L) + \phi_M(1-f_L+f_L \int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\, g_M(x)\; dx) + \phi_H \\
= \;& \phi_L(1-f_L) + \phi_M(1-f_L+f_L \frac{\bar{g}_M-\theta}{\bar{\gamma}-\theta}) + \phi_H \\
= \;& \phi_L(1-f_L) + \phi_M(1-f_L+f_L \frac{\frac{\gamma_H - \phi_H \bar{g}_H - \phi_L\theta}{\phi_M}-\theta}{\bar{\gamma}-\theta}) + \phi_H \\
= \;& \phi_H \frac{\bar{\gamma}-\theta-f_L\bar{g}_H}{\bar{\gamma}-\theta} + (\phi_L+\phi_M)\frac{\bar{\gamma}-\theta-f_L\bar{\gamma}}{\bar{\gamma}-\theta} + \frac{\gamma_H f_L}{\bar{\gamma}-\theta}.
\end{aligned}$$

Observe that the coefficient on $(\phi_L + \phi_M) = 0$ because, $f_L = f_{H2}$ and, as demonstrated previously $\bar{\gamma}(1-f_{H2}) - \theta = 0$. Thus, the payoff to any admissible $\hat{\Gamma}_L$ is independent of ϕ_L, ϕ_M, G_M. However, because $\bar{g}_H > \bar{\gamma}$ the coefficient on ϕ_H is negative. Thus, in a best response, it must be that $\phi_H = 0$. Hence, any random variable of the structure $\hat{\Gamma}_H$ is a best response, provided $\phi_H = 0$. As the strategy in the proposition satisfies these criteria, it is a best response. ■

Claim 6: If $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\frac{1}{2}(1-\theta^2) \leq \gamma_L \leq \frac{2-2\theta}{2-\theta}$ then the following strategies constitute an equilibrium:

$$\Gamma_H = \begin{cases} 0 & \text{with probability} \quad 1 - f_{H1} - f_{H2} - f_{H3} \\ \theta & \text{with probability} \quad f_{H1} \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad f_{H2} \\ 1 & \text{with probability} \quad f_{H3} \end{cases}$$

$$\Gamma_L = \begin{cases} 0 & \text{with probability} \quad 1 - f_{L1} - f_{L2} \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad f_{L1} \\ 1 & \text{with probability} \quad f_{L2} \end{cases}$$

$$\bar{\gamma} = 2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2}$$

$$f_{H1} = \frac{\gamma_H - \gamma_L}{\theta}$$

$$f_{H2} = f_{L1} = \frac{\bar{\gamma}-\theta}{2-\bar{\gamma}} = \frac{(\gamma_L - \sqrt{\gamma_L^2+\theta^2})(\gamma_L + \sqrt{\gamma_L^2+\theta^2} - (2-\theta))}{\theta^2}$$

$$f_{H3} = f_{L2} = \frac{2-2\bar{\gamma}}{2-\bar{\gamma}} = 2(1 - \frac{\sqrt{\gamma_L^2+\theta^2}-\gamma_L}{\theta^2})$$

Proof. We first demonstrate that the strategies of both players are admissible. First, observe that if $\frac{1}{2}(1-\theta^2) \leq \gamma_L \leq \frac{2-2\theta}{2-\theta}$ then $\theta \leq \bar{\gamma} \leq 1$. These inequalities also imply that $f_{H2} = f_{L1} \geq 0$ and $f_{H3} = f_{L2} \geq 0$. It is also obvious that $f_{H1} \geq 0$. To prove that all probabilities are less than 1, we establish that $f_{H1} + f_{H2} + f_{H3} \leq 1$. This inequality implies that $f_{L1} + f_{L2} \leq 1$.

$$f_{H1} + f_{H2} + f_{H3} - 1 = \frac{\gamma_H - \sqrt{\gamma_L^2 + \theta^2}}{\theta} \leq 0 \text{ if } \theta \geq \sqrt{\gamma_H^2 - \gamma_L^2}$$

Finally, we demonstrate that both random variables have the correct expected values.

$$E[\Gamma_L] = f_{L1}\frac{\theta+\bar{\gamma}}{2} + f_{L2} = \frac{\bar{\gamma}-\theta}{2-\bar{\gamma}}(\frac{\theta+\bar{\gamma}}{2}) + \frac{2-2\bar{\gamma}}{2-\bar{\gamma}} = \gamma_L$$

$$E[\Gamma_H] = f_{H1}\theta + E[\Gamma_L] = \gamma_H$$

Next, we establish that the proposed strategies are mutual best responses. According to Lemma 1, any admissible best response to Γ_H, denoted $\hat{\Gamma}_L$ must have the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1-\phi_M-\phi_H-\phi_{H1} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H1} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists at endpoints of G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta+\epsilon$ leads to all ties at θ breaking in favor of player L. In order for this strategy to be admissible, it must be that

$$\phi_M\bar{g}_M + \phi_H\bar{g}_H + \phi_{H1} = \gamma_L \iff \bar{g}_M = \frac{\gamma_L - \phi_H\bar{g}_H - \phi_{H1}}{\phi_M}$$

The expected payoff of using such a strategy against Γ_H is given by:

$$\begin{aligned} & \phi_M(1-f_{H2}-f_{H3}+f_{H2}\int_\theta^{\bar{\gamma}}\frac{x-\theta}{\bar{\gamma}-\theta}\; g_M(x)\; dx) + \phi_H(1-f_{H3}) + \phi_{H1}(1-\frac{f_{H3}}{2}) \\ = \;& \phi_M(1-f_{H2}-f_{H3}+f_{H2}\frac{\bar{g}_M-\theta}{\bar{\gamma}-\theta}) + \phi_H(1-f_{H3}) + \phi_{H1}(1-\frac{f_{H3}}{2}) \\ = \;& \phi_M(1-f_{H2}-f_{H3}+f_{H2}\frac{\frac{\gamma_L-\phi_H\bar{g}_H-\phi_{H1}}{\phi_M}-\theta}{\bar{\gamma}-\theta}) + \phi_H(1-f_{H3}) + \phi_{H1}(1-\frac{f_{H3}}{2}) \\ = \;& \phi_M\frac{2(\bar{\gamma}-\theta)(1-f_{H3})-2\bar{\gamma}f_{H2}}{2(\bar{\gamma}-\theta)} + \phi_H\frac{2(\bar{\gamma}-\theta)(1-f_{H3})-2\bar{\gamma}f_{H2}-2f_{H2}(\bar{g}_H-\bar{\gamma})}{2(\bar{\gamma}-\theta)} \\ & + \phi_{H1}\frac{2(\bar{\gamma}-\theta)(1-f_{H3})-2\bar{\gamma}f_{H2}}{2(\bar{\gamma}-\theta)} + \frac{2\gamma_L f_{H2}}{2(\bar{\gamma}-\theta)}. \end{aligned}$$

Observe first that the coefficient on ϕ_H is less than the coefficient on either ϕ_M or ϕ_H, hence, for any best response, $\phi_H = 0$. Furthermore,

$$2(\bar{\gamma}-\theta)(1-f_{H3}) - 2\bar{\gamma}f_{H2} = 2(\bar{\gamma}-\theta)(1-\frac{2-2\bar{\gamma}}{2-\bar{\gamma}}) - 2\bar{\gamma}\frac{\bar{\gamma}-\theta}{2-\bar{\gamma}} = 0.$$

Thus, the payoff of any admissible random variable does not depends on ϕ_{H1}, ϕ_M, G_M. Thus any random variable with the structure $\hat{\Gamma}_L$ and $\phi_H = 0$ is a best response to Γ_H. In particular, Γ_L is a best response to Γ_H.

Next, we show that Γ_H is a best response to Γ_L. According to Lemma 1, any admissible best response to Γ_L, denoted $\hat{\Gamma}_H$ must have the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_M - \phi_H - \phi_{H1} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H1} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists in G_H). In order for this strategy to be admissible, it must be that

$$\phi_M \bar{g}_M + \phi_H \bar{g}_H + \phi_{H1} = \gamma_H \iff \bar{g}_M = \frac{\gamma_H - \phi_H \bar{g}_H - \phi_{H1}}{\phi_M}.$$

The expected payoff of using such a strategy against Γ_L is given by:

$$\phi_M(1 - f_{L1} - f_{L2} + f_{L1} \int_\theta^{\bar{\gamma}} \frac{x - \theta}{\bar{\gamma} - \theta}\ g_M(x)\ dx) + \phi_H(1 - f_{L2}) + \phi_{H1}(1 - \frac{f_{L2}}{2})$$

Because of the equalities $f_{L1} = f_{H2}, f_{L2} = f_{H3}$ this equation becomes

$$\begin{aligned} & \phi_M(1 - f_{H2} - f_{H3} + f_{H2}\frac{\bar{g}_M - \theta}{\bar{\gamma} - \theta}) + \phi_H(1 - f_{H3}) + \phi_{H1}(1 - \frac{f_{H3}}{2}) \\ = \ & \phi_M(1 - f_{H2} - f_{H3} + f_{H2}\frac{\frac{\gamma_L - \phi_H \bar{g}_H - \phi_{H1}}{\phi_M} - \theta}{\bar{\gamma} - \theta}) + \phi_H(1 - f_{H3}) + \phi_{H1}(1 - \frac{f_{H3}}{2}) \\ = \ & \phi_M \frac{2(\bar{\gamma} - \theta)(1 - f_{H3}) - 2\bar{\gamma} f_{H2}}{2(\bar{\gamma} - \theta)} + \phi_H \frac{2(\bar{\gamma} - \theta)(1 - f_{H3}) - 2\bar{\gamma} f_{H2} - 2 f_{H2}(\bar{g}_H - \bar{\gamma})}{2(\bar{\gamma} - \theta)} \\ & + \phi_{H1} \frac{2(\bar{\gamma} - \theta)(1 - f_{H3}) - 2\bar{\gamma} f_{H2}}{2(\bar{\gamma} - \theta)} + \frac{2\gamma_H f_{H2}}{2(\bar{\gamma} - \theta)}. \end{aligned}$$

Thus, from the previous equation, it follows that in any best response $\phi_H = 0$. Furthermore, the payoff of any admissible strategy is independent of ϕ_M, ϕ_{H1}, G_M, thus any admissible strategy with $\phi_H = 0$ is a best response. As Γ_H satisfies these criteria it is a best response. ■

Claim 7: If $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\frac{1}{2}(1 + \theta^2) \leq \gamma_H \leq \frac{2 - 2\theta + \theta^2}{2 - \theta}$ then the following strategies constitute an equilibrium:

$$\Gamma_H = \begin{cases} \theta & \text{with probability} \quad 1 - f_{H1} - f_{H2} \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad f_{H1} \\ 1 & \text{with probability} \quad f_{H2} \end{cases}$$

$$\Gamma_L = \begin{cases} 0 & \text{with probability} \quad 1 - f_{L1} - f_{L2} \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad f_{L1} \\ 1 & \text{with probability} \quad f_{L2} \end{cases}$$

$$\bar{\gamma} = 2 - \gamma_H - \sqrt{\gamma_H^2 - \theta^2}$$

$$f_{H1} = \frac{\bar{\gamma} - \theta}{2 - \bar{\gamma}}$$

$$f_{H2} = \frac{2-2\bar{\gamma}}{2-\bar{\gamma}}$$

$$f_{L1} = \frac{2\gamma_L(\bar{\gamma}-\theta)}{(2-\bar{\gamma})^2-\theta^2}$$

$$f_{L2} = \frac{4\gamma_L(1-\bar{\gamma})}{(2-\bar{\gamma})^2-\theta^2}$$

$$E[u_p] =$$

Proof. We first demonstrate that the strategies of both players are admissible. As $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2} \leq \gamma_H, \bar{\gamma}$ is a real number. Observe that $\frac{1}{2}(1+\theta^2) \leq \gamma_H \leq \frac{2-2\theta+\theta^2}{2-\theta} \rightarrow \theta \leq \bar{\gamma} \leq 1$. Under these conditions, clearly both f_{H1}, f_{H2} are positive. Furthermore, $f_{H1} + f_{H2} - 1 = -\frac{\theta}{2-\bar{\gamma}} < 0$. Observe next that,

$$\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2} \rightarrow \bar{\gamma} \leq 2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2}$$

This inequality will play a significant role again later in the proof. For now, note that

$$2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2} - (\theta + 2(1-\gamma_L)) = \gamma_L - \theta - \sqrt{\gamma_L^2 + \theta^2} < 0$$

The right hand side equals zero when $\theta = 0$ and has a negative derivative in θ. Thus,

$$\bar{\gamma} \leq 2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2} \rightarrow \bar{\gamma} \leq \theta + 2(1-\gamma_L)$$

Observe that $\theta \leq \gamma_H \rightarrow 2 - \bar{\gamma} = \gamma_H + \sqrt{\gamma_H^2 - \theta^2} \geq \theta$. Hence $f_{L1}, f_{L2} \geq 0$. Next, observe that $f_{L1} + f_{L2} - 1 = \frac{\bar{\gamma}-(\theta+2(1-\gamma_L))}{2-\bar{\gamma}-\theta} \leq 0$ as described above. Therefore Γ_H, Γ_L are random variables. They are admissible if they satisfy the constraints on the expected values.

$$E[\Gamma_H] = (1 - f_{H1} + f_{H2})\theta + f_{H1}\frac{\theta+\bar{\gamma}}{2} + f_{H2} = \frac{\theta^2 + (2-\bar{\gamma})^2}{2(2-\bar{\gamma})} = \gamma_H$$

$$E[\Gamma_L] = f_{L1}\frac{\theta+\bar{\gamma}}{2} + f_{L2} = \gamma_L$$

Note also that $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\frac{1}{2}(1+\theta^2) \leq \gamma_H \leq \frac{2-2\theta+\theta^2}{2-\theta}$ means that $\gamma_L \leq \frac{2-2\theta}{2-\theta}$.

Next, we establish that the proposed strategies are mutual best responses. According to Lemma 1, any admissible best response to Γ_H, denoted $\hat{\Gamma}_L$ must have the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_M - \phi_H - \phi_{H1} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H1} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists at endpoints of G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta + \epsilon$ leads to all ties at θ breaking in favor of player L. In order for this strategy to be admissible, it must be that

$$\phi_M \bar{g}_M + \phi_H \bar{g}_H + \phi_{H1} = \gamma_L \iff \bar{g}_M = \frac{\gamma_L - \phi_H \bar{g}_H - \phi_{H1}}{\phi_M}.$$

The expected payoff of using such a strategy against Γ_H is given by:

$$\begin{aligned}
& \phi_M(1 - f_{H1} - f_{H2} + f_{H1} \int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\, g_M(x)\ dx) + \phi_H(1 - f_{H2}) + \phi_{H1}(1 - \frac{f_{H2}}{2}) \\
= \;& \phi_M(1 - f_{H1} - f_{H2} + f_{H1} \frac{\bar{g}_M - \theta}{\bar{\gamma}-\theta}) + \phi_H(1 - f_{H2}) + \phi_{H1}(1 - \frac{f_{H2}}{2}) \\
= \;& \phi_M(1 - f_{H1} - f_{H2} + f_{H1} \frac{\frac{\gamma_L - \phi_H \bar{g}_H - \phi_{H1}}{\phi_M} - \theta}{\bar{\gamma}-\theta}) + \phi_H(1 - f_{H2}) + \phi_{H1}(1 - \frac{f_{H2}}{2}) \\
= \;& \phi_M \frac{(\bar{\gamma}-\theta)(1-f_{H2}) - \bar{\gamma} f_{H1}}{(\bar{\gamma}-\theta)} + \phi_{H1} \frac{(2-f_{H2})(\bar{\gamma}-\theta) - 2f_{H1}}{2(\bar{\gamma}-\theta)} \\
& + \phi_H(\frac{(2-f_{H2})(\bar{\gamma}-\theta) - 2f_{H1}}{2(\bar{\gamma}-\theta)} - \frac{f_{H1}(\bar{g}_H - \bar{\gamma})}{\bar{\gamma}-\theta}) + \frac{\gamma_L f_{H1}}{\bar{\gamma}-\theta}.
\end{aligned}$$

Clearly, the coefficient on ϕ_H is less than the coefficient on ϕ_{H1}. Thus, in any best response, $\phi_H = 0$. It is also easy to check that the coefficient on ϕ_M, ϕ_{H1} are equal to zero. Thus the payoff to using a strategy of type $\hat{\Gamma}_L$ is independent of ϕ_M, ϕ_{H1}, G_M. Thus any random variable in class $\hat{\Gamma}_L$ is a best response, provided $\phi_H = 0$. As Γ_L satisfies these criteria, it is a best response.

Next, we show that Γ_H is a best response to Γ_L. According to Lemma 1, any admissible best response to Γ_L, denoted $\hat{\Gamma}_H$ must have the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_M - \phi_H - \phi_{H1} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H1} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists in G_H). In order for this strategy to be admissible, it must be that

$$\phi_M \bar{g}_M + \phi_H \bar{g}_H + \phi_{H1} = \gamma_H \iff \bar{g}_M = \frac{\gamma_H - \phi_H \bar{g}_H - \phi_{H1}}{\phi_M}$$

The expected payoff of using such a strategy against Γ_L is given by:

$$\phi_M(1 - f_{L1} - f_{L2} + f_{L1} \int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\, g_M(x)\ dx) + \phi_H(1 - f_{L2}) + \phi_{H1}(1 - \frac{f_{L2}}{2})$$

By symmetry with the previous calculations this simplifies to:

$$\begin{aligned}
& \phi_M \frac{(\bar{\gamma}-\theta)(1-f_{L2}) - \bar{\gamma} f_{L1}}{(\bar{\gamma}-\theta)} + \phi_{H1} \frac{(2-f_{L2})(\bar{\gamma}-\theta) - 2f_{L1}}{2(\bar{\gamma}-\theta)} \\
& + \phi_H(\frac{(2-f_{L2})(\bar{\gamma}-\theta) - 2f_{L1}}{2(\bar{\gamma}-\theta)} - \frac{f_{L1}(\bar{g}_H - \bar{\gamma})}{\bar{\gamma}-\theta}) + \frac{\gamma_H f_{L1}}{\bar{\gamma}-\theta}
\end{aligned}$$

As in the previous calculation, $\phi_{H1} = 0$ for any best response. Next observe that

$$2((\bar{\gamma}-\theta)(1-f_{L2})-\bar{\gamma}f_{L1}) = (2-f_{L2})(\bar{\gamma}-\theta)-2f_{L1} = \frac{2(\bar{\gamma}-\theta)}{(2-\bar{\gamma})^2-\theta^2}$$

$$= \frac{2(\bar{\gamma}-\theta)}{(2-\bar{\gamma})^2-\theta^2}(-\bar{\gamma}^2+(4-2\gamma_L)+\theta^2+4\gamma_L-4)$$

This is larger than zero, provided $\bar{\gamma} \leq 2-\gamma_L-\sqrt{\gamma_L^2+\theta^2}$, which was demonstrated previously. Hence, any random variable in class $\hat{\Gamma}_H$ is a best response, provided $\phi_H = 0$. As the strategy Γ_H satisfies these criteria, it is a best response. ■

Claim 8: If $\gamma_L \leq \frac{2-2\theta}{2-\theta}$ and $\gamma_H \geq \frac{2-2\theta+\theta^2}{2-\theta}$ then it is a Nash Equilibrium for player L to use a fully revealing strategy, and for player H to use a quasi-revealing strategy.

Proof. A *quasi-revealing* strategy for player i is the following binary random variable G_i:

$$G_i = \begin{cases} \theta & \text{with probability} \quad \frac{1-\gamma_i}{1-\theta} \\ 1 & \text{with probability} \quad \frac{\gamma_i-\theta}{1-\theta} \end{cases}$$

This is indeed a random variable if $\gamma_i \geq \theta$. It is also admissible:

$$\theta(\frac{1-\gamma_i}{1-\theta})+\frac{\gamma_i-\theta}{1-\theta} = \gamma_i$$

We refer to this strategy as quasi-revealing because, like the fully revealing signal, it is binary and a good signal realization reveals the proposal to be good for sure. Unlike the fully revealing signal, a bad realization does not reduce the principal's posterior to 0, but only to θ.

A fully revealing strategy is always admissible. A quasi-revealing strategy is well-defined iff $\gamma_H \geq \theta$. By assumption, $\gamma_H \geq \frac{2-2\theta+\theta^2}{2-\theta} = \theta+2\frac{(1-\theta)^2}{2-\theta} \geq \theta$, thus a quasi-revealing strategy is admissible for player H.

We first establish that a quasi-revealing strategy for player H is a best reply to a fully revealing strategy for player L. According to Lemma 1, any best reply to a fully revealing strategy can be represented in the following way:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1-\phi_M-\phi_H \\ G_M & \text{with probability} \quad \phi_M \\ 1 & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$. If a mass point exists at 1, it is not part of G_M. In order for such a strategy to be admissible it must be that

$$\phi_M \bar{g}_M + \phi_H = \gamma_H$$

Thus, player H best response is characterized by the solution to the following maximization:

$$\begin{array}{cc} \max_{\phi_M,\phi_H,g} & \phi_M(1-\gamma_L)+\phi_H(1-\frac{\gamma_L}{2}) \\ \text{s.t.} & \phi_M\bar{g}_M+\phi_H=\gamma_H, \;\; \phi_M+\phi_H \leq 1 \\ & \phi_M \geq 0, \;\; \phi_H \geq 0, \;\; \bar{g}_M \geq \theta \end{array}$$

Thus for any best response it must be that $\phi_M \bar{g}_M + \phi_H = \gamma_H$ and $\phi_M + \phi_H = x \leq 1$ which implies

$$\phi_H = \frac{\gamma_H - \bar{g}_M x}{1 - \bar{g}_M}, \;\; \phi_M = \frac{x - \gamma_H}{1 - \bar{g}_M}.$$

In addition,

$$\phi_H \geq 0 \text{ and } \phi_M \geq 0 \iff \gamma_H \leq x \leq \frac{\gamma_H}{\bar{g}_M}$$

The payoff of playing this strategy is simply

$$u(x, g_i) = \frac{x - \gamma_H}{1 - \bar{g}_M}(1 - \gamma_L) + \frac{\gamma_H - \bar{g}_M x}{1 - \bar{g}_M}(1 - \frac{\gamma_L}{2})$$

Observe that if some combination of $(x, \bar{g}_M)$ is feasible, then any combination with the same value of x but smaller value of $\bar{g}_M$ is also feasible. Because $\frac{du}{d\bar{g}_M} = -\frac{1}{2}\gamma_H \frac{x-\gamma_H}{(1-\bar{g}_M)^2} < 0$ a best response must have $\bar{g}_M = \theta$. Thus,

$$u(x, \theta) = \frac{x - \gamma_H}{1 - \theta}(1 - \gamma_L) + \frac{\gamma_H - \theta x}{1 - \theta}(1 - \frac{\gamma_L}{2}) = \frac{2(1 - \gamma_L) - (2 - \gamma_L)\theta}{2(1 - \theta)} x + \frac{\gamma_H \gamma_L}{2(1 - \theta)}$$

Therefore, provided the coefficient on x is positive, the best response will be to set x to its largest feasible value $min[1, \frac{\gamma_H}{\theta}]$. However, $\gamma_H \geq \frac{2-2\theta+\theta^2}{2-\theta} \to \gamma_H \geq \theta$. Thus, if the coefficient on x is positive, the best response of H is to choose $x = 1$, which gives $\phi_M = \frac{1-\gamma_H}{1-\theta}, \phi_H = \frac{\gamma_H - \theta}{1-\theta}$, the quasi-revealing signal. Therefore, if

$$\frac{(2 - \gamma_L)\theta - 2(1 - \gamma_L)}{2(1 - \theta)} \geq 0 \iff \gamma_L \leq \frac{2 - 2\theta}{2 - \theta}$$

The best reply of player H to a fully revealing signal on the part of L is a quasi-revealing signal. Next, consider the best reply of player L to a quasi-revealing signal from player H.

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_M - \phi_H \\ G_M & \text{with probability} \quad \phi_M \\ 1 & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$. Clearly, choosing G_M with a mass point at θ leads to ties with positive probability. Choosing a mass point at $\theta + \epsilon$ leads all ties to break in favor of player L, causing a discrete jump in payoff when a tie occurs, at expense of a marginal reduction in payoff otherwise. Thus, no mass point exists on θ in a best response. If a mass point exists at 1, it is not part of G_M. In order for such a strategy to be admissible it must be that

$$\phi_M \bar{g}_M + \phi_H = \gamma_H$$

Thus, player H best response is characterized by the solution to the following maximization:

$$\begin{array}{cc} \max_{\phi_M, \phi_H, g} & \phi_M(1 - \gamma_L) + \phi_H(1 - \frac{\gamma_L}{2}) \\ \text{s.t.} & \phi_M \bar{g}_M + \phi_H = \gamma_H, \;\; \phi_M + \phi_H \leq 1 \\ & \phi_M \geq 0, \;\; \phi_H \geq 0, \;\; \bar{g}_M > \theta \end{array}$$

Note that, because ties are dominated, the last inequality is strict. Thus for any best response it must be that $\phi_M \bar{g}_M + \phi_H = \gamma_H$ and $\phi_M + \phi_H = x \leq 1$, which implies

$$\phi_H = \frac{\gamma_H - \bar{g}_M x}{1 - \bar{g}_M}, \ \ \phi_M = \frac{x - \gamma_H}{1 - \bar{g}_M}$$

In addition,

$$\phi_H \geq 0 \text{ and } \phi_M \geq 0 \iff \gamma_H \leq x \leq \frac{\gamma_H}{\bar{g}_M}$$

The payoff of playing this strategy is simply

$$u(x, g_i) = \frac{x - \gamma_H}{1 - \bar{g}_M}(1 - \gamma_L) + \frac{\gamma_H - \bar{g}_M x}{1 - \bar{g}_M}(1 - \frac{\gamma_L}{2})$$

Observe that if some combination of $(x, \bar{g}_M)$ is feasible, then any combination with the same value of x but smaller value of $\bar{g}_M$ is also feasible. Because $\frac{du}{d\bar{g}_M} = -\frac{1}{2}\gamma_H \frac{x - \gamma_H}{(1 - \bar{g}_M)^2} < 0$ agent L would always like to set $\bar{g}_M$ to be as small as possible, but still above θ. Thus, because of an open set problem, if $\phi_M > 0$ the best response is not uniquely defined. Observe, however, that the payoff of using any strategy with $\phi_M > 0$ is strictly less than the payoff the player would expect if all ties broke in favor of L, and $\bar{g}_M = \theta$, but if $\phi_M = 0$, then the issue of ties is irrelevant. The payoff of agent L if all ties break in his favor is given by the following:

$$\bar{u}(x, \theta) = \frac{x - \gamma_L}{1 - \theta}(1 - \gamma_H) + \frac{\gamma_L - \theta x}{1 - \theta}(1 - \frac{\gamma_H}{2}) = \frac{2(1 - \gamma_H) - (2 - \gamma_H)\theta}{2(1 - \theta)}x + \frac{\gamma_H \gamma_L}{2(1 - \theta)}.$$

Therefore, provided the coefficient on x is negative, player L best response is well-defined, as it requires $x = \gamma_L$ which means that L plays the fully revealing signal and $\phi_M = 0$. Therefore, if

$$\frac{(2 - \gamma_H)\theta - 2(1 - \gamma_H)}{2(1 - \theta)} \leq 0 \iff \gamma_H \geq \frac{2 - 2\theta}{2 - \theta}$$

Since $\gamma_H \geq \frac{2 - 2\theta + \theta^2}{2 - \theta} \geq \frac{2 - 2\theta}{2 - \theta}$, the required inequality holds. The best reply of player L to player H providing a fully-revealing signal is to provide a quasi-revealing signal. ■