Seeking Rents in the Shadow of Coase*

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December 6, 2005

Abstract

The possibility of trading property rights is generally seen as an opportunity to improve the allocation of resources in society. However, ex post trade my provide stronger incentives to invest in rent-seeking activities to secure such rights in the first place. When property rights are not yet allocated, opening markets to trade may thus result in dissipation costs due to rent-seeking that overcome the allocative benefits of trade. We show this point in a two-stage game. In the first stage, parties invest in securing property rights as in Tullock's well-known rent-seeking game. In the second stage, these rights may or may not be traded in a Coasean market. The final outcome is evaluated in terms of dissipation and misallocation costs and the trade and no-trade scenarios are compared.

JLE Classification: C72, D72

Keywords: Rent-seeking, asymmetric prize, misallocation, dissipation, lost treasure

*The authors would like to thank___ for helpful comments. ___ provided skilfull editorial assistance.

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1 Introduction

Imagine that two knights are taking part in a tournament whose prize is the hand (and the love) of a princess. The knights are of equal strenght, valor, and courage, but while one of them only aims at the kingdom, the other also secretly loves the princess. The literature on rent-seeking has analyzed these types of games and pointed to the fact that the participants will dissipate socially valuable resources in the attempt to win the prize. However, focused as it has been on contests with a commonly valued prize, this literature has failed to realize that there is an additional social cost associated with rent-seeking tournaments: the princess may marry the wrong knight.

When parties attach different values to a prize, a missallocation cost of rent-seeking arises from the fact that the prize may end up in the hands of the low-valuing party. In addition, although their strenght is equal, they will exert different levels of effort, as their marginal benefits differ. Contests to win esteem, advance in social ranks, or obtain a non-tradable permit are of this type. However, in some cases the allocative inefficiency of rent-seeking contests can be corrected ex post. Although in our example the knights cannot subsequently reallocate the princess, other sought-after rents can be traded ex post. In the case of tradable rights, the low-valuing party is likely to sell the prize to the other if he wins the contest. The race to obtain rights over the new "*.eu*" Internet-domain names resembles this scenario, in which an initial misallocation can be corrected through ex post Coasean barganing. After an initial face (the so called *Sunrise period*) in which special interests are considered, such domanin names will be allocated on a strict 'first come, first served' basis.[1]

Ex post reallocation is desirable for low-valuing parties (e.g. a teenager who registers the domain *www.shadowofcoase.eu*), since it allows them to sell at a price that is potentially higher than their valuation, thereby in fact increasing the value of the rent for them. High-valuing parties (e.g. the authors of this article) may or may not benefit from the prospect of ex post reallocation. On the one hand, ex post reallocation gives them an opportunity to purchase the right when they fail in the rent-seeking contest. On the other hand, they will have to face more fierce competition from low-valuing parties.[2] The opportunity for ex post reallocation of the rent affects the total social cost of a rent-seeking contest in two different ways. First, it eliminates the risk of misallocations. Second, by making the prize appealing also for low-valuing players, it increases the stakes in the game, occasioning greater rent dissipation. The overall impact of ex post bargaining on social cost is ambiguous, and greatly depends on the parties' bargaining power. From the study of the parties' rent-seeking incentives, it emerges that, when the parties' rent valuations are sufficiently divergent, equilibrium levels of effort may be decreasing in their marginal returns to effort. This result runs contrary to the conventional wisdom in the literature according to which higher returns to effort induce an increase in rent-seeking efforts and in the total rent dissipation.

[1] See the information available at *www.eurid.eu*.

[2] In our example, more fierce competition may take the form of strategic registration of all possible domain names somewhat related to the one at stake. For example: *www.shadow-of-coase.eu*, or *www.shadow_of_coase.eu*, or also *www.seekingrentsintheshadowofcoase.eu* and so forth.

This paper brings together insights and results from rent-seeking and property rights theory. The key contribution is to show that the possibility for ex post Coasean bargaining affects the rent-seeking incentives for the original appropriation of property rights and, vice versa, rent-seeking outcomes affect the process of Coasean bargaining. The rent-seeking literature (Tullock, 1967; Krueger, 1974; Posner, 1975; Bhagwati, 1982)[3] generally considers parties competing for the appropriation of a commonly-valued rent. Asymmetries between the parties, when introduced, are modeled in terms of different returns to rent-seeking effort. We add to this literature by allowing the parties' valuations to diverge.[4] Further, we bring the Coase theorem to bear on our analysis (Coase, 1959 and 1960). According to the Coase theorem, ex post contractual negotiation will correct any possible initial misallocation of resources. It has been observed that rules of first possession (such as 'first come, first served' and 'finders, keepers') generate rent-seeking incentives in the initial allocation of property rights (Barzel, 1968; Lueck, 1995). We show that the possibility of ex post Coasean reallocation of the appropriated resources, while solving problems of misallocation, may exacerbate rent dissipation due to increased rent-seeking efforts. The increase in rent dissipation may more than offset the allocative gains. In this case, Coasean bargaining results in a social loss. Under some conditions, foreclosing the opportunity for Coasean barganing may actually improve social welfare.[5]

In the following section we present the basic model of rent-seeking with asymmetric rent valuations. In section 3 we analyze rent-seeking under perfect information, and in section 4 we extend the analysis to the case of imperfect information. In section 5, we introduce the possibility for ex post Coasean bargaining in the perfect and imperfect information framework. Section 6 concludes the paper with ideas for possible future extensions.

2 The model

We consider a rent-seeking contest between two parties, who have equal marginal returns to effort. Let B_i, with $i = 1, 2$, denote the parties' rent-seeking expenditures and V_i denote the value of the rent. Following the conventional Tullock framework,[6] a party's payoffs is given by

$$U_i(B_i, B_j) = \frac{B_i^r}{B_i^r + B_j^r} V_i - B_i \tag{1}$$

[3] For a review of the literature see Buchanan, Tollison and Tullock (1980); Congleton and Tollison (1995); Lockard and Tullock (2000); Rowley, Tollison and Tullock (1988); Tollison (2003).

[4] Other papers that study rent-seeking games with heterogeneous private valuations are Baye, Kovenock and De Vries (1993, 1996), Amann and Leininger (1996), Krishna and Morgan (1997), Che and Gale (1998), Gavious, Moldovanu and Sela (2002), and Onderstal (2005). In contrast to our work, these papers assume a completely discriminatory contest, i.e., the highest bidder wins with probability 1.

[5] Some previous contribution also emphasize the perverse effects of trading possibilities. For example, in a context different from ours, Jacklin (1987) shows that the presence of liquid equity markets undermines the role of banks as reducing liquidity risk.

[6] The splitting of the prize may in reality be a function of the parties' efforts of a different form from Tullock's specification. Employing the traditional Tullock framework has the advantage of making our results easily comparable with existing literature on rent-seeking, while offering the possibility to explicitly calculating equilirbium values, which would not be possible were we to use a general implicit function.

with $i \neq j$. The payoff function can be interpreted as an expected payoff function where greater efforts by one party increase his probability of winning the entire rent, rather than increasing the share of the rent that will be appropriated.[7] As usual, r is an index of the parties' marginal productivity of effort.[8] In the following, we will focus on situations where the parties' participation constraint is always fulfilled. This is guaranteed when the players face constant or decreasing marginal returns to effort ($r \leq 1$), thus playing pure strategies.[9]

Players differ in their valuation of the rent V_i; for simplicity and without loss of generality, we assume that there are two types of players: H (high-valuing party, denoted by subscript $_H$) and L (low-valuing party, denoted by subscript $_L$), $V_H > V_L > 0$. Previous contributions (for a survey see Lockard and Tullock, 2001) study the pattern of the parties' rent-seeking efforts as a function of the rent value, but restrict the analysis to situations where the parties have an identical valuation of the rent, be it either V_H or V_L. In the present analysis, we extend the analysis considering the important – and yet previously unexplored – cases in which the parties exhibit different valuations of the rent.

As known from the previous literature, if parties were symmetrical and attached the same value to the rent, they would play the same strategies and consequently split the rent in equal shares in equilibrium. When parties' valuations differ, their strategies and shares of the rent are also expected to differ.

In the following we investigate the effect of these asymmetries on the parties strategies. We subsequently consider the allocative function of rent-seeking contests and the interesting problems associated with the efficient final allocation of the rent.

With parties with equal valuations of the rent, the efficiency of rent-seeking outcomes is generally evaluated in terms of total rent dissipation through the parties' effort. When the parties have different valuations of the rent, a new source of inefficiency may come about, given the fact that rents are not necessarily appropriated by players that value them the most. Put differently, when parties have heterogeneous valuations of the rent, losses from inefficient misallocations of the rent should be considered in addition to the traditional problems of rent dissipation. The misallocation losses can be viewed as an oppurtunity cost due to the fact that the rent could have been put to a higher-value use.

Accordingly, we define two loss variables: the rent-dissipation, given by the sum of the parties' efforts,

$$D = B_i + B_j \tag{2}$$

and the rent-misallocation, given by the share of the rent appropriated by the low-valuing player (with $V_j \leq V_i$ and relabel otherwise)

$$M = \frac{B_j^r}{B_i^r + B_j^r} (V_i - V_j) \tag{3}$$

[7] Thus, hereinafter when referring to the parties' share of the rent, we could alternatively refer to the parties' probability to win the entire rent.

[8] Alternatively, r can be seen as the discriminative power of the auction.

[9] Note that this is a sufficient but not a necessary condition.

When parties have heterogeneous valuations of the rent, the misallocation loss would be given by the difference between the parties' valuations multiplied by the share of the rent appropriated by the low-valuing party. Obviously, in the special case of parties with equal valuations of the rent, $V_i = V_j$, the misallocation is equal to zero, irrespective of whether both parties are low-valuing or high-valuing players.

In the following two Sections, we model a rent-seeking contest between parties with different valuations of the rent, distinguishing two different settings. First, we consider a complete information case, where the parties know each other's type. Second, we consider the incomplete information case, where parties know their own type but do not know the type of the other player.

We subsequently extend the results of complete and incomplete information contests to consider a scenario in which parties can enter an additional stage of the game, where rents can be reallocated (sold) by a low-valuing player to a high-valuing player. Through such Coasean bargaining, rent misallocations are corrected. But optimal game strategies may be altered, with remaining rent-dissipation losses.

3 Analysis of rent-seeking with complete information

3.1 Parties' equilibrium efforts and payoffs

When there is complete information, each party maximizes his payoff in (1) according to the following FOC:

$$\frac{\partial U_i}{\partial B_i} = r\frac{B_i^{r-1}B_j^r}{\left(B_i^r + B_j^r\right)^2}V_i - 1 = 0 \tag{4}$$

It is well known that, if parties have equal valuations of the rent, the equilibrium effort levels are $B_i = B_j = \frac{r}{4}V_i$, and the total rent dissipation is $D = \frac{r}{2}V_i$. Given the parties' equal valuations of the rent, there is no possible misallocation resulting from the game. When parties' have heterogeneous valuations, the outcome of the game depends also on the difference between the parties' valuations. To simplify notations, we introduce a variable measuring the level of asymmetry between the parties' valuations, $\gamma \equiv \frac{V_L}{V_H}$. The value of γ ranges $0 < \gamma < 1$, reaching a value of 1 when the parties have equal valuations of the rent, and approaching 0 when the parties have highly asymmetric valuations.

Proposition 1 *With complete information, the parties' equilibrium levels of efforts are directly proportional to the parties' valuations of the rent:* $\frac{B_L^*}{B_H^*} = \gamma$*. The equilibrium payoffs are instead less than proportional to the parties' valuations:* $\frac{U_L^*}{U_H^*} < \gamma$*.*

In equilibrium, the party with higher valuation makes larger rent-seeking expenditures compared to the other party. The ratio of the parties' efforts is equal to the ratio of their valuations. The high-valuing party has greater incentives to invest in rent-seeking and, thus, will appropriate a larger share of the rent. Additionally, he gives greater value to the rent. The combination of these two effects gives the high-valuing party a more-than-proportional advantage on his opponent. The high-valuing player, while investing $\frac{1}{\gamma}$ times more than his opponent, earns proportionally much more than that: his payoff is in fact more than $\frac{1}{\gamma}$ times larger than that of the other party.

To illustrate, let us consider the simple case of constant returns to scale: $r = 1$. In this case the ratio of the shares of the rent $\left(\frac{B_i^r}{B_i^r + B_j^r}\right)$ received by the parties is γ, which gives an advantage to the high-valuing player. Further, the high-valuing individual has the additional advantage of valuing the received rent more than the other. This brings the ratio of gross payoffs (rent shares multiplied by the partie' valuation of of the rent) to γ^2. Finally, when calculating the net payoffs, the advantage of the high-valuing player becomes even greater, because the ratio of the gross payoff is more favorable to the high-valuing party than the ratio of the efforts. Therefore, the expended effort has a lower incidence on the net payoff for the high-valuing party than for the low-valuing party. It is in fact very easy to verify that the ratio of the net payoffs becomes γ^3. This brings to light the interesting result that the ability of the high-valuing party to secure a higher payoff grows esponentially with his valuation.

3.2 Comparative statics

Performing some simple comparative-statics analysis, it is possible to study how the parties' efforts and payoffs vary when the parameters of the game, r and γ, change.

Corollary 2 *With complete information, we have* $\frac{\partial B_H^*}{\partial \gamma} > 0$ *and* $\frac{\partial B_L^*}{\partial \gamma} > 0$, $\frac{\partial U_H^*}{\partial \gamma} < 0$ *and* $\frac{\partial U_L^*}{\partial \gamma} > 0$.

This corollary yields that both parties' efforts increase as their respective evaluations approaching each other (γ goes towards 1). From a different perspective, when their valuations are further apart, parties participate in an asymmetric rent-seeking contest, investing less in rent-seeking effort. Not surprisingly, it follows that the high-valuing party gains (his expected payoff increases) when the evaluations get further apart, while the low-valuing party loses in the same scenario.

Corollary 3 *With complete information, we have* $\frac{\partial B_H^*}{\partial r} <> 0$ *and* $\frac{\partial B_L^*}{\partial r} <> 0$*, the sign of these derivatives goes from negative to positive as* r *decreases and* γ *increases and vice versa. Likewise, we have* $\frac{\partial U_H^*}{\partial r} <> 0$*; finally* $\frac{\partial U_H^*}{\partial r} < 0$.

Studying how the parties' equilibrium levels of efforts vary when their strenght r changes yields an interesting result that runs against conventional wisdom in the public choice literature. It is commonly believed that when the parties' marginal produtivity of effort rises, they will exert more effort, up to the point that their payoffs decrease towards zero. This finding has motivated the literature trying to model Tullock's game in terms of mixed strategies. When the parties' valuations differ, we find a counter-intuitive result. When the parties' valuations are sufficiently far apart (γ sufficiently close to 0) their levels of effort may decrease as a result of an increase in r (instead of increasing), and the payoff of the high-valuing party may increase (instead of decreasing), while the payoff of the other party always decreases.

These findings are explained by the fact that an increase in r makes competition more fierce. When γ is sufficiently low, the low-valuing party may prefer to give way to his opponent (thereby decreasing his effort and losing in terms of decreasing payoff). Consequently the high-valuing party can respond reducing his effort, while at the same time gaining a larger payoff due to the partial withdrawal of his opponent.

3.3 Dissipation and misallocation

The social cost of rent-seeking is given by the sum of rent dissipation and rent misallocation. Using the results of the previous section it is possible to calculate these costs, expressed in (2) and (3), and to assess the effects of changes in r and γ on total rent-seeking costs.

Proposition 4 *With complete information the total rent dissipation is* $0 < D^* < \frac{1}{2}V_H$, *with* $\frac{\partial D^*}{\partial \gamma} > 0$ *and* $\frac{\partial D^*}{\partial r} <> 0$; *the sign of* $\frac{\partial D^*}{\partial r}$ *goes from negative to positive as* r *decreases and* γ *increases and vice versa. The rent misallocation is* $0 < M^* < \frac{1}{2}V_H$, *with* $\frac{\partial M^*}{\partial \gamma} <> 0$ *and* $\frac{\partial M^*}{\partial r} < 0$; *the sign of* $\frac{\partial M^*}{\partial \gamma}$ *goes from negative to positive as* r *increases and* γ *decreases and vice versa. The total social loss is* $0 < D^* + M^* < \frac{1}{2}V_H$, *with* $\frac{\partial}{\partial \gamma}(D^* + M^*) <> 0$ *and* $\frac{\partial}{\partial r}(D^* + M^*) <> 0$.

When the parties' rent valuations approach each other, the rent dissipation grows, a result that follows directly from the fact that each party's effort surges with γ. However, contrary to previous literature, even when parties play pure strategies the rent-dissipation may actually decrease when the parties's productivity of effort increases; in fact we have seen above that when parties have higly asymmetric valuations, their efforts may decrease in r (productivity of effort), and hence total rent dissipation may also decrease.

Differences between the parties' valuations, however, create potential misallocations (the rent may be appropriated by the low-valuing player), occasioning an allocative loss. In our formulation, rent misallocation occurs whenever party L appropriates the rent, while the gravity of the allocative loss depends on the difference between the parties' valuations, as in (3).

The misallocation may either increase or decrease when γ varies, depending on the parties' strenght, r. Two opposite forces are responsible for this result. On the one hand, if the party valuations become closer, the low-valuing party's effort tends to approach that of the other party. Thus, with more homogeneous valuations the probability of a misallocation becomes greater. On the other hand, the gravity of the misallocation plummets, since the parties' valuations differ less. For low values of γ the former effects dominates ($\frac{\partial M^*}{\partial \gamma} > 0$), while for larger values of γ the latter effect dominates ($\frac{\partial M^*}{\partial \gamma} < 0$).

The misallocation unambiguously decreases as the parties become stronger ($\frac{\partial M^*}{\partial r} < 0$). In fact, when the parties' strenght improves, the gravity of the misallocation remains unchanged, while the probability that the low-valuing party wins decreases. This is due to the fact that with an increase in r, the incentives of the high-valuing party to invest in rent-seeking increase more than for his competitor, due to his larger stakes in the game). Obvioulsy, when the parties' valuations are the same, the misallocation loss disappears.

Total rent-seeking losses are given by $D^* + M^*$. Such a total loss has two components: dissipation, D^*, increasing in γ, and misallocation, M^*, which initially increases and then decreases in γ. The final effect is that for lower levels of γ total rent-seeking losses increase as the parties' valuations approach each other ($\frac{\partial}{\partial \gamma}(D^* + M^*) > 0$). With higher values of γ, parties' valuations become more homogeneous and total rent-seeking costs start decreasing, as the reduction in misallocation losses more than compensates for the increase in dissipation ($\frac{\partial}{\partial \gamma}(D^* + M^*) < 0$). At the limit, when the parties' valuations converge, the total social loss is reduced to the sole dissipation ($\frac{r}{2}V_H$).

The result is opposite to the former if we consider the variation of the total rent-seeking losses in r. When parties have heterogeneous valuations, total rent-seeking losses decrease in r ($\frac{\partial}{\partial r}(D^* + M^*) < 0$). This is due to the fact that both D (dissipation) and M (misallocation) decrease in this case. When the parties' valuations converge, total rent-seeking losses increases, as the increase in dissipation dominates the decrease in misallocation ($\frac{\partial}{\partial r}(D^* + M^*) > 0$).

Given the fact that D^* and M^* mostly vary in opposite directions, the maximum social loss is not the sum of the maximum D^* plus the maximum M^* but only $\frac{1}{2}V_H$. This is an interesting result, showing that, even when we account for the misallocation costs created by heterogeneous rent valuations, total rent-seeking losses do not exceed rent-dissipation losses obtained in the standard Tullock game.

4 Analysis of rent-seeking with incomplete information

In this section, we relax the assumption of complete information used in the previous analysis. Players do not observe their opponent's rent valuation before making their effort decisions, but only know their own valuation. We assume that low-valuing and high-valuing players are equally likely. A party with value $V_i = V_L, V_H$ exerts effort B_i and obtains an expected payoff $U_i(V_i, B_i, B_L, B_H)$, which depends on the type and effort expended by the other player. The other player will choose effort B_L or B_H according to whether he is a low- or high-valuing individual. This generates expected payoffs equal to:

$$U_i(V_i, B_i, B_L, B_H) = \frac{1}{2}\frac{B_i^r}{B_i^r + B_L^r}V_i + \frac{1}{2}\frac{B_i^r}{B_i^r + B_H^r}V_i - B_i. \tag{5}$$

The first and second terms on the right-hand side of (5) represent the equally-likely rent appropriations, depending on whether the other player has a low or high valuation of the rent. The parties maximize their payoffs taking into account the probability that their opponent is of a certain type according to the following FOC

$$\frac{\partial U_i(V_i, B_i, B_L, B_H)}{\partial B_i} = \frac{1}{2}\left(\frac{rB_i^{r-1}B_L^r}{(B_i^r + B_L^r)^2} + \frac{rB_i^{r-1}B_H^r}{(B_i^r + B_H^r)^2}\right)V_i - 1 = 0 \tag{6}$$

We will use ‡ to denote equilibrium values with incomplete information.

Proposition 5 *With incomplete information, the parties' equilibrium levels of efforts are directly proportional to the parties' valuations of the rent:* $\frac{B_L^\ddagger}{B_H^\ddagger} = \gamma$*. The equilibrium payoffs are instead less than proportional to the parties' valuations:* $\frac{U_L^\ddagger}{U_H^\ddagger} < \gamma$*.*

These results are similar to those derived for the case of complete information. Parties with higher valuation make larger rent-seeking expenditures and earn larger payoffs. While the ratio of the parties' efforts is equal to the ratio of their valuations, the high-valuing party a more-than-proportional advantage on his opponent with respect to payoffs.

4.1 Comparative statics

Performing some simple comparative-statics analysis, as in the previous section, it is possible to study how the parties' efforts and payoffs vary when the parameters of the game, r and γ, change.

Corollary 6 *With incomplete information, we have* $\frac{\partial B_H^\ddagger}{\partial \gamma} > 0$ *and* $\frac{\partial B_L^\ddagger}{\partial \gamma} > 0$*,* $\frac{\partial U_H^\ddagger}{\partial \gamma} < 0$ *and* $\frac{\partial U_L^\ddagger}{\partial \gamma} > 0$*.*

The effect of changes in the difference between the parties' valuations of the rent are the same whether information is complete or incomplete.

Corollary 7 *With incomplete information, we have* $\frac{\partial B_H^\ddagger}{\partial r} > 0$ *and* $\frac{\partial B_L^\ddagger}{\partial r} > 0$*. Moreover,* $\frac{\partial U_H^\ddagger}{\partial r} <> 0$ *__. Finally* $\frac{\partial U_L^\ddagger}{\partial r} < 0$*. TO BE COMPLETED*

These results reveal some interesting differences between rent-seeking under complete and incomplete information. With complete information, higher levels of r could induce parties' to expend less resources in the rent-seeking activity. This result run contrary to the traditional Tullock's framework. When information is incomplete, this counterintuitive result disappears. Higher levels of r always induce parties' to expend more resources in rent-seeking. Another difference between the complete and incomplete information outcomes concerns the equilibrium payoffs. With incomplete information, as r increases parties will normally earn smaller payoffs due to their larger expenditures. However, the high-valuing party may still benefit from an increase in the marginal returns from effort when the increase in the high-valuing party effort more than offsets the increase in the low-valuing party effort. This occurs when his valuation is substantially higher than the valuation of his opponent and the marginal return to effort is sufficiently low.

4.2 Dissipation and misallocation

We proceed to study the social costs of rent-seeking in the incomplete information setting.

Proposition 8 *With incomplete information the total rent dissipation is* $0 < D^\ddagger < \frac{1}{2}V_H$*, with* $\frac{\partial D^\ddagger}{\partial \gamma} > 0$ *and* $\frac{\partial D^\ddagger}{\partial r} > 0$*. The rent misallocation is* $0 < M^\ddagger < \frac{1}{2}V_H$*, with* $\frac{\partial M^\ddagger}{\partial \gamma} <> 0$ *___ and* $\frac{\partial M^\ddagger}{\partial r} < 0$*. The total social loss is* $0 < D^\ddagger + M^\ddagger < \frac{1}{2}V_H$*, with* $\frac{\partial}{\partial \gamma}\left(D^\ddagger + M^\ddagger\right) <> 0$ *___ and* $\frac{\partial}{\partial r}(D^\ddagger + M^\ddagger) <> 0$*___. TO BE COMPLETED*

These results are similar to those derived for the complete information case, with the exception of variation of the dissipation D with respect to changes in r. This result is directly related to what was observed under proposition 5, given the fact that dissipation is given by the sum of the parties' efforts.

5 Rent-seeking with Coasean bargaining

As shown in the previous analysis, the presence of asymmetric rent valuations creates the risk of allocative inefficiencies. In some situations, possible misallocations of the rent can however be corrected through ex post Coasean bargaining. Low-valuing winners can transfer the rent to the high-valuing opponents.

The results of the previous analysis should thus be revisited in light of the possibility of ex post reallocations of the rent. We consider the general case in which the winner of the appropriated rent and the party who seeks the ex post reallocation of the rent split the contractual surplus according to their respective bargaining power, α and $1-\alpha$, with $0 \leq \alpha \leq 1$. Whenever a reallocation of the rent takes place, the price paid by the high-valuing party to the low-valuing party is $P \equiv \alpha V_H + (1-\alpha) V_L$. In this Section, we study how the opportunity for such ex post bargaining affects the parties' payoffs and ex ante incentives. Several interesting results are derived from this analysis.

Intuitively, the possibility of Coasean bargaining transforms an asymmetric valuation into a symmetric valuation problem. The opportunity to transfer the rent to the high-valuing contestant, induces low-valuing contestants to take into account the valuation of the other party.

The expected value of a rent reallocation varies between the complete and incomplete information settings. In a complete information setting, the low-valuing individual knows that he will be able to reallocate the rent to his high-valuing opponent. In an incomplete information setting, a low-valuing player only has a $\frac{1}{2}$ probability to face a high-valuing contestant, and will take this probability into account when deciding his effort level. We shall thus consider these two scenarios in turn.

5.1 Coasean bargaining with complete information

In a complete information setting, low-valuing parties will choose their effort level considering the potential price obtainable if the rent is won and subsequently transferred to high-valuing contestants. If the parties play pure strategies, the possibility of ex post Coasean barganing affects their payoffs as follows:

$$\begin{aligned} U_L(B_H, B_L) &= \frac{B_L^r}{B_H^r + B_L^r} P - B_L \\ U_H(B_H, B_L) &= \frac{B_H^r}{B_H^r + B_L^r} V_H + \frac{B_L^r}{B_H^r + B_L^r} (V_H - P) - B_H \end{aligned}$$

Rearranging, we have:

$$U_L(B_H, B_L) = \frac{B_L^r}{B_H^r + B_L^r} P - B_L \tag{7a}$$

$$U_H(B_H, B_L) = \frac{B_H^r}{B_H^r + B_L^r} P - B_H + (V_H - P) \tag{7b}$$

The above formulations of the parties' payoffs highlight the interesting features of rent-seeking with Coasean bargaining. The parties' payoffs remain asymmetric, because the parties' actual gain from the game depends on their respective bargaining power. Unless the low-valuing party has full bargaining power and is able to extract the entire surplus from the other party, the high-valuing party will still have a larger payoff from the game. This is due to the fact that the high-valuing party appropriates the difference between his valuation and the price paid to his opponent.

However, despite this persistent asymmetry, the parties' incentives to invest in rent-seeking effort become symmetric. This is an interesting result because parties with asymmetric payoffs play symmetrically. The reason for this is that the high-valuing party will always obtain his high-valued rent V_H, whether through direct appropriation or ex post reallocation. Holding V_H as a constant, the difference for a high-valuing player between success and failure only depends on the price to be paid to the low-valuing party if he successfully appropriates the rent. If the high-valuing party succeeds in the rent-seeking game, he avoids to make payment P. The same is true for the low-valuing party, for whom succeeding in the rent-seeking game means securing a payment of P from his opponent.

The symmetry between the parties' incentives is also evident from the fact that the FOC for (7a) and (7b) are identical. Both parties participate in the rent-seeking game with the prospect of appropriating a rent that can be sold (or should be bought) for the price P. The surplus V_H and P is always appropriated by the high-valuing player, regardless of who appropriates the rent in the first place. This yields that the parties' equilibrium levels of efforts are the same and can be calculated (as in a traditional Tullock game), by reference to the price P rather than the different valuations of the contested rent (a double asterisk denotes equilibrium values with Coasean barganing):

$$B_L^{**} = B_H^{**} = \frac{r}{4} P \tag{8}$$

A consequence of the equivalence between the parties' incentives is that the total rent dissipation can also be calculated as in a traditional Tullock game and expressed as a function of price P as follows:

$$D^{**} = B_L^{**} + B_H^{**} = \frac{r}{2} P \tag{9}$$

Proposition 9 *With complete information, when rents can be reallocated ex post at price P, the parties' equilibrium levels of efforts are the same $B_H^{**} = B_L^{**}$, with $\frac{\partial B_L^{**}}{\partial \gamma} = \frac{\partial B_H^{**}}{\partial \gamma} = 0$ and $\frac{\partial B_L^{**}}{\partial r} = \frac{\partial B_H^{**}}{\partial r} > 0$ and $\frac{\partial B_L^{**}}{\partial P} = \frac{\partial B_H^{**}}{\partial P} > 0$. The equilibrium payoffs are instead different $U_L^{**} < U_H^{**}$, with $\frac{\partial U_L^{**}}{\partial \gamma} = \frac{\partial U_H^{**}}{\partial \gamma} = 0$, $\frac{\partial U_L^{**}}{\partial r} = \frac{\partial U_H^{**}}{\partial r} < 0$ and $\frac{\partial U_H^{**}}{\partial P} < \frac{\partial U_L^{**}}{\partial P} < 0$. The rent dissipation is $0 < D^{**} < \frac{1}{2} V_H$, with $\frac{\partial D^{**}}{\partial \gamma} = 0$, $\frac{\partial D^{**}}{\partial r} > 0$ and $\frac{\partial D^{**}}{\partial P} > 0$. The rent misallocation is $M^{**} = 0$.*

The opportunity for ex post reallocations affects the ex ante parties' incentives to expend in rent-seeking. In the presence of an opportunity for ex post reallocation of the contested rent, the parties behave as rent-seekers for the expected price P, price at which the rent will be reallocated ex post. As a consequence, the social loss of rent-seeking depends on the bargaining power of the low-valuing party. Low-valuing parties with greater barganing power α are able to extract a higher price. Therefore, as the balance of the barganing power shifts from the high- to the low-valuing party, parties exert more effort and the social loss of rent-seeking increases.

It is interesting to compare the outcomes obtained under Coasean bargaining with the results of the previous section.

Corollary 10 *$B_L^{**} > B_L^*$ and $U_L^{**} > U_L^*$; $B_H^{**} <> B_H^*$ and $U_H^{**} <> U_H^*$; $D^{**} <> D^*$ and $D^{**} <> D^* + M^*$.*

Coasean bargaining unambiguously improves the position of the low-valuing party, who exerts more effort and is able to appropriate a larger share of the rent, through ex post negotiations with his high-valuing opponent. A variation in the relative bargaining power of the parties will only quantitatively affect this result, determining the price to be paid for the ex post reallocation.

The position of the high-valuing party, instead, crucially depends on the parties' barganing power and resulting price. If the price is low, also the the high-valuing party will receive a larger payoff when Coasean bargaining is possible. However, if the price is high, the effect of Coasean bargaining on the high-valuing party's payoff may be positive or negative. In fact, the possibility of ex post reallocation may in fact have two effects. First, it makes it possible for the rent to be purchased ex post by the high-valuing party. This lowers the high-valuing party's need to invest in rent-seeking to appropriate the rent in the first place. Second, it makes the low-valuing party behave more aggressively. If the high-valuing party has a sufficiently strong bargaining position, the price will be low and the first effect will dominate the second, likely improving his payoff. However, if the low-valuing party has a strong bargaining power, the price will be high and possibly close to the full rent valuation of the prospective buyer. In this case, the second effect prevails and we have the paradoxical result that giving the high-valuing party the opportunity to buy the rent ex post worsens his position, causing a decrease in his payoff.

By the same token, Coasean bargaining may increase or decrease rent dissipation, depending on the parties' barganining power. Consider two limit cases: When the high-valuing party has the bargaining power ($\alpha = 0$), the price is equal to V_L and parties compete as if they were both low-valuing parties. Hence, the opportunity for ex post reallocation of the rent reduces the equilibrium level of dissipation. On the contrary, when all barganing power is allocated to the low-valuing party ($\alpha = 1$), the price is equal to V_H and both parties compete for the same higher value. Evidently, in this case the dissipation is larger in the Coasean environment. Although misallocation costs are totally eliminated by Coasean bargaining, the possibility for ex post reallocation may actually increase the total social cost of rent-seeking.

We can identify an iso-dissipation boundary where the rent dissipation under Coasean bargaining is equal to the rent dissipation without Coasean bargaining. To illustrate, consider the case of constant returns to scale, $r = 1$. We can easily identify a threshold level of α (the bargaining power of the low-valuing party), that will generate equal levels of dissipation:

$$\alpha^D = \frac{\gamma}{1+\gamma}$$

Likewise, we can identify the threshold level of α, such that the rent dissipation under Coasean bargaining is equal to the total social loss without Coasean bargaining (rent dissipation plus rent misallocation):

$$\alpha^{D+M} = 3\frac{\gamma}{1+\gamma}$$

When $\alpha < \alpha^{D+M}$, Coasean bargaining leads to a reduction in total rent-seeking losses. Ex post reallocation of the rent eliminates misallocation costs and for values of $\alpha > \alpha^D$ these savings more than compensate the possible increase in rent dissipation. On the contrary, when $\alpha > \alpha^{D+M}$, the opportunity for ex post reallocation of the rent increases the total social cost.

5.2 Coasian bargaining with incomplete information

In the incomplete information setting, we assume that after the rent-seeking game, the parties' valuations are revealed. The parties know ex ante that they will engage in ex post bargaining if the rent is initially appropriated by a low-valuing party competing with a high-valuing party. In all other scenarios where parties have the same valuation or where the rent is appropriated by the high-valuing player, the rent will not be reallocated. The parties' payoffs can thus be rewritten in expected terms, taking into account the various possibilities, including the possible ex post reallocation at price P:

$$U_L(V_L, B_i, B_L, B_H) = \frac{1}{2}\frac{B_i^r}{B_i^r + B_L^r}V_L + \frac{1}{2}\frac{B_i^r}{B_i^r + B_H^r}P - B_i \quad \text{(10a)}$$

$$U_H(V_H, B_j, B_L, B_H) = \frac{1}{2}\frac{B_j^r}{B_j^r + B_H^r}V_H + \frac{1}{2}\left(V_H - \frac{B_L^r}{B_j^r + B_L^r}P\right) - B_j. \quad \text{(10b)}$$

Proposition 11 *With incomplete information, when rents can be reallocated ex post at price P, the parties' equilibrium levels are $B_H^{\ddagger\ddagger} > B_L^{\ddagger\ddagger}$ with $\frac{B_L^{\ddagger\ddagger}}{B_H^{\ddagger\ddagger}} > \gamma$. $U_L^{\ddagger\ddagger}$??__, $U_H^{\ddagger\ddagger}$??__, $D^{\ddagger\ddagger}$???. The rent misallocation is $M^{\ddagger\ddagger} = 0$. TO BE COMPLETED*

Corollary 12 *$B_L^{\ddagger\ddagger} > B_L^{\ddagger}$, and $D^{\ddagger\ddagger} <> D^{\ddagger} + M^{\ddagger}$. $B_H^{\ddagger\ddagger}$??__$U_L^{\ddagger\ddagger}$??__, $U_H^{\ddagger\ddagger}$??__ TO BE COMPLETED*

The results of the bargaining under incomplete information case are qualitatively similar to those derived under complete information. The opportunity for ex post Coasean reallocations has an indeterminate effect on the overall social cost of rent-seeking.

6 Conclusions

New dimensions of the rent-seeking problem emerge when rent-seekers have different valuations of the sought-after rent. In this paper, we have analyzed these new dimensions, showing how rent-seeking incentives are affected by such valuation asymmetries. Unlike winner-takes-all auctions, rent-seeking does not guarantee the efficient allocation of the rents. This paper highlights an interesting interrelationship between these misallocation costs and other rent-seeking costs. This paper derives a result that runs contrary to the conventional wisdom in the literature: when the parties' rent valuations are sufficiently divergent, equilibrium levels of effort may be decreasing in the marginal returns to effort.

When parties have unequal valuations of the rent, rent-seeking contests may serve as valuable mechanisms to force parties to reveal their preferences through investment choices. Higher valuing parties will fight more aggressively and will thus have higher probabilities to appropriate the rent. In this sense, rent-seeking contests may play a valuable allocative role. Given the mechanics of rent-seeking contests, however, this allocative role remains imperfect and misallocations are possible.

In this paper we thus extend the analysis to study the effect of ex post reallocation on the parties incentives and total rent dissipation. According to the Coase theorem, ex post contractual negotiation corrects possible initial misallocation of resources. Whenever the sought-after prize is transferable, the opportunity for ex post reallocation eliminates any possible misallocation. We show that the possibility of ex post Coasean reallocation of the appropriated resources, while solving problems of misallocation, may exacerbate rent dissipation due to an increase in the stakes of the game. The increase in rent dissipation may more than offset the allocative gains, rendering the opportunity for Coasean bargaining socially undesirable.

Further studies should consider the relationship between asymmetries in the parties' valuation of the rent and other forms of asymmetries, such as differences in the parties' returns to effort or rent-seeking-costs. These results shed light on important policy questions, and provide an important foundations for the design of rent-seeking contests.

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7 Appendix

Proof of proposition 1. Each party chooses his level of effort B_i in order to maximaze his payoff from participation in (1). Straightforward calculations on the FOCs in (4) yield the following result:

$$B_L^* = r\frac{\gamma^r}{(1+\gamma^r)^2}V_L \tag{11a}$$

$$B_H^* = r\frac{\gamma^r}{(1+\gamma^r)^2}V_H \tag{11b}$$

from which it follows that in equilibrium we have $\frac{B_L^*}{B_H^*} = \frac{V_L}{V_H} = \gamma$. The SOCs are

$$\frac{\partial^2 U_L}{\partial B_L^2} = -\frac{rB_L^r B_H^r}{B_L^2 (B_H^r + B_L^r)^3} V_L (r(B_L^r - B_H^r) + B_H^r + B_L^r) < 0 \tag{12a}$$

$$\frac{\partial^2 U_H}{\partial B_H^2} = -\frac{rB_L^r B_H^r}{B_H^2 (B_H^r + B_L^r)^3} V_H (r(B_H^r - B_L^r) + B_H^r + B_L^r) < 0 \tag{12b}$$

It is evident that (12a) holds true iff $r(B_L^r - B_H^r) + B_H^r + B_L^r > 0$, which can be rewritten as $r < \left|\frac{B_H^r + B_L^r}{B_H^r - B_L^r}\right|$. Likewise (12b) holds true iff $r < \left|\frac{B_L^r + B_H^r}{B_L^r - B_H^r}\right|$. Both conditions are verified in our model, where $r \le 1$. Thus, (B_L^*, B_H^*) is the unique Nash equilibrium of the game. Substituting (11) into (1), we obtain:

$$U_L^* = \frac{\gamma^{2r} + (1-r)\gamma^r}{(1+\gamma^r)^2}V_L \tag{13a}$$

$$U_H^* = \frac{1 + (1-r)\gamma^r}{(1+\gamma^r)^2}V_H \tag{13b}$$

which yields $\frac{U_L^*}{U_H^*} = \frac{\gamma^{2r}+(1-r)\gamma^r}{1+(1-r)\gamma^r}\gamma$. Noting that $0 < \frac{\gamma^{2r}+(1-r)\gamma^r}{1+(1-r)\gamma^r} < 1$, we have $\frac{U_L^*}{U_H^*} < \gamma$. The parties' participation constraints are satisfied iff $\gamma^{2r} + (1-r)\gamma^r > 0$ (for L) and $1 + (1-r)\gamma^r > 0$ (for H). Both conditions are verified since $r \le 1$. *QED*

Proof of corollary 2. $\frac{\partial B_H^*}{\partial \gamma} = r^2\gamma^{r-1}\frac{1-\gamma^r}{(\gamma^r+1)^3}V_H > 0$, since $\gamma < 1$; likewise we have $\frac{\partial B_L^*}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left(r\frac{\gamma^r}{(1+\gamma^r)^2}\gamma V_H\right) = r\gamma^r\frac{1+r(1-\gamma^r)+\gamma^r}{(\gamma^r+1)^3}V_H > 0$. This result enables us to discuss changes in the parties' relative valuations as separate from changes in the absolute value of the prize. If the latter increses without affecting the ratio γ, both parties' efforts obviously surge. In our model, this may occur when both V_H and V_L increase in the same proportion: for example, both of them double. In addition, we have $\frac{\partial U_H^*}{\partial \gamma} = -r\gamma^{r-1}\frac{1+r(1-\gamma^r)+\gamma^r}{(\gamma^r+1)^3}V_H < 0$; $\frac{\partial U_L^*}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left(\frac{\gamma^r(1+\gamma^r)-r\gamma^r}{(1+\gamma^r)^2}\gamma V_H\right) = \gamma^r\frac{(1+\gamma^r)^2 - r^2(1-\gamma^r)}{(\gamma^r+1)^3}V_H > 0$, in fact $(1+\gamma^r)^2 - r^2(1-\gamma^r) > 0$ iff $r < \frac{(1+\gamma^r)^2}{1-\gamma^r}$, which is always verified given that we have $r \le 1$ and $\frac{(1+\gamma^r)^2}{1-\gamma^r} > 1$. *QED*

Proof of corollary 3. $\frac{\partial B_H^*}{\partial r} = \gamma^r\left(\frac{1+\gamma^r+r(\ln\gamma)(1-\gamma^r)}{(\gamma^r+1)^3}\right)V_H$; $\frac{\partial B_L^*}{\partial r} = \frac{\partial}{\partial r}\left(r\frac{\gamma^r}{(1+\gamma^r)^2}\gamma V_H\right) = \gamma^r\left(\frac{1+\gamma^r+r(\ln\gamma)(1-\gamma^r)}{(\gamma^r+1)^3}\right)\gamma V_H$. These derivatives are positive (negative) iff $1+\gamma^r+r(\ln\gamma)(1-\gamma^r)$ is positive (negative). Both outcomes are possible, as it is easy to show by setting $r = 1$ and letting γ alternatively approach 1 and 0, respectively.

Since we have $\frac{\partial}{\partial r}\left(1+\gamma^r+r\left(\ln\gamma\right)\left(1-\gamma^r\right)\right) = \ln\gamma - r\left(\ln^2\gamma\right)\gamma^r < 0$, $\frac{\partial}{\partial\gamma}\left(1+\gamma^r+r\left(\ln\gamma\right)\left(1-\gamma^r\right)\right) = \frac{r}{\gamma}-\frac{r^2}{\gamma}\left(\ln\gamma\right)\gamma^r > 0$ the sign of $\frac{\partial B_H^*}{\partial r}$ and $\frac{\partial B_L^*}{\partial r}$ goes from negative to positive as r decreases and γ increases and vice versa.

Moreover, we have $\frac{\partial U_H^*}{\partial r} = -\gamma^r\frac{1+\gamma^r+(\ln\gamma)(1+\gamma^r+r(1-\gamma^r))}{(\gamma^r+1)^3}V_H$, which is positive (negative) iff $1+\gamma^r+\left(\ln\gamma\right)\left(1+\gamma^r+r\left(1-\gamma^r\right)\right)$ is negative (positive). As above, it is easy to show that both outcomes are possible by setting $r=1$ and considering γ close to 0 and 1, respectively. Concerning the other party we have instead $\frac{\partial U_L^*}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\gamma^r(1+\gamma^r)-r\gamma^r}{(1+\gamma^r)^2}\gamma V_H\right) = -\gamma^r\frac{1+\gamma^r-(\ln\gamma)(1+\gamma^r-r(1-\gamma^r))}{(\gamma^r+1)^3}\gamma V_H < 0$, which follows from $\ln\gamma < 0$ and $r \leq 1$. *QED*

Proof of proposition 4. Substituting (11) in (2) we have

$$D^* = B_L^* + B_H^* = r\frac{\gamma^r}{\left(1+\gamma^r\right)^2}\left(1+\gamma\right)V_H \tag{14}$$

with $\frac{\partial D^*}{\partial\gamma} = \frac{\partial B_H^*}{\partial\gamma}+\frac{\partial B_L^*}{\partial\gamma} > 0$ and $\frac{\partial D^*}{\partial r} = \frac{\partial B_H^*}{\partial r}+\frac{\partial B_L^*}{\partial r} <> 0$. From $0<\gamma<1$ and $0<r\leq 1$ it follows that $0 < D^* < \frac{1}{2}V_H$.

Substituting (11) in (3) we have

$$M^* = \frac{B_L^{*r}}{B_H^{*r}+B_L^{*r}}(V_H-V_L) = \frac{\gamma^r}{1+\gamma^r}(1-\gamma)V_H$$

with $\frac{\partial M^*}{\partial\gamma} = \frac{r\gamma^r(1-\gamma)-\gamma^{r+1}(1+\gamma^r)}{\gamma(\gamma^r+1)^2}V_H$, which is positive (negative) iff $r\left(1-\gamma\right)-\gamma\left(1+\gamma^r\right)$ is positive (negative). Both outcomes are possible as is it easy to verify by setting $r=1$ and considering γ close to 0 and 1, respectively. Since we have $\frac{d}{dr}\left(r\left(1-\gamma\right)-\gamma\left(1+\gamma^r\right)\right) = 1-\gamma-\gamma\left(\ln\gamma\right)\gamma^r > 0$ and $\frac{d}{d\gamma}\left(r\left(1-\gamma\right)-\gamma\left(1+\gamma^r\right)\right) = -r-\gamma^r-r\gamma^r-1 < 0$ the sign of $\frac{\partial M^*}{\partial\gamma}$ goes from negative to positive as r increases and γ decreases and vice versa. Moreover, we have $\frac{\partial M^*}{\partial r} = \left(\ln\gamma\right)\frac{\gamma^r}{(1+\gamma^r)^2}\left(1-\gamma\right)V_H < 0$ and $0 < M^* < \frac{1}{2}V_H$. M^* approaches zero when the parties' valuations converge (γ goes towards 1) and approaches $\frac{1}{2}V_H$ when r approaches 0 and the parties' valuations diverge (γ goes towards 0).

The total social loss due to rent-seeking is given by D^*+M^*:

$$D^*+M^* = \frac{\gamma^r}{\left(1+\gamma^r\right)^2}\left(r\left(1+\gamma\right)+\left(1-\gamma\right)\left(1+\gamma^r\right)\right)V_H \tag{15}$$

with

$$\frac{\partial}{\partial\gamma}\left(D^*+M^*\right) = \gamma^{r-1}\frac{r\left(1+\gamma^r\right)-\gamma\left(1+\gamma^r\right)^2+r^2\left(1+\gamma\right)\left(1-\gamma^r\right)}{\left(\gamma^r+1\right)^3}V_H$$

and

$$\frac{\partial}{\partial r}\left(D^*+M^*\right) = \gamma^r\frac{\left(1+\gamma^r\right)\left(1+\gamma\right)+\left(\ln\gamma\right)\left(\gamma^r+1\right)\left(1-\gamma\right)+r\left(\ln\gamma\right)\left(1-\gamma^r\right)\left(1+\gamma\right)}{\left(\gamma^r+1\right)^3}V_H$$

The former (latter) derivative can be either positive (negative) or negative (positive), as it is easy to verify by means of the numerical examples already used: $r=1$ and γ close to 0 or 1, respectively.

$0 < D^* + M^* < \frac{1}{2}V_H$. The total social los approaches 0 when r approaches 0 and γ approaches 1, or vice versa; it approaches $\frac{1}{2}V_H$ when both r and γ approach 0 or 1. *QED*

Proof of proposition 5. Straightforward calculations on the FOCs in (6) yield the following result

$$\frac{1}{2}V_L\left(\frac{r}{4B_L}+\frac{rB_L^{r-1}B_H^r}{(B_L^r+B_H^r)^2}\right) = 1 \tag{16a}$$

$$\frac{1}{2}V_H\left(\frac{r}{4B_H}+\frac{rB_H^{r-1}B_L^r}{(B_L^r+B_H^r)^2}\right) = 1 \tag{16b}$$

Substituting $B_L = \zeta B_H$ in (16a) and (16b) yields (after some straightforward manipulation)

$$B_L^{\ddagger} = \frac{1}{2}r\left(\frac{1}{4}+\frac{\zeta^r}{(1+\zeta^r)^2}\right)V_L$$

$$B_H^{\ddagger} = \frac{1}{2}r\left(\frac{1}{4}+\frac{\zeta^r}{(1+\zeta^r)^2}\right)V_H$$

so that it is readily established that $\zeta \equiv \frac{V_L}{V_H} = \gamma = \frac{B_L^{\ddagger}}{B_H^{\ddagger}}$. Finally, observe that for $r \leq 1$, and $B_i > 0$ we have

$$\frac{\partial^2 U_i(V_i,B_i,B_L,B_H)}{\partial B_i^2}$$

$$= V_i\frac{rB_i^{r-2}}{2}\left(\frac{B_H^r}{(B_H^r+B_i^r)^3}\left((r-1)B_H^r-(r+1)B_i^r\right)+\frac{B_L^r}{(B_L^r+B_i^r)^3}\left(B_L^r(r-1)-B_i^r(r+1)\right)\right)<0$$

Therefore, the second-order condition is satisfied as well.

Substuting into (5) we obtain that the parties' payoffs in equilibrium are

$$U_i(V_i,B_i,B_L,B_H) = \frac{1}{2}\frac{B_i^r}{B_i^r+B_L^r}V_i+\frac{1}{2}\frac{B_i^r}{B_i^r+B_H^r}B_i - B_i$$

at the equilibrium levels of effort we have:

$$U_L^{\ddagger} = \frac{1}{4}V_L+\frac{1}{2}\frac{B_L^r}{B_L^r+B_H^r}V_L - B_L$$

$$U_H^{\ddagger} = \frac{1}{4}V_H+\frac{1}{2}\frac{B_H^r}{B_L^r+B_H^r}V_H - B_H.$$

Substituting, we have:

$$\frac{U_L^{\ddagger}}{U_H^{\ddagger}} = \frac{(\gamma^r+1)\gamma\left[\frac{1}{4}V_H-B_H\right]+\frac{1}{2}\gamma^{r+1}V_H}{(\gamma^r+1)\left[\frac{1}{4}V_H-B_H\right]+\frac{1}{2}V_H}$$

$$< \frac{(\gamma^r+1)\left[\frac{1}{4}V_H-B_H\right]+\frac{1}{2}V_H}{(\gamma^r+1)\left[\frac{1}{4}V_H-B_H\right]+\frac{1}{2}V_H}\gamma = \gamma$$

Proof of corollary 6

It is easy to show that the function $t(\gamma, r) \equiv \left\{\frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2}\right\}$ is strictly increasing in γ. As $B_H = \frac{1}{2} r V_H t(\gamma, r)$, $B_L = \frac{1}{2} r \gamma V_H t(\gamma, r)$, $U_H^\ddagger = \frac{1}{4} V_H + \frac{1}{2} \frac{1}{\gamma^r + 1} V_H - \frac{1}{2} r V_H \; t(\gamma, r)$ the comparative statics for B_H, B_L, and $U_H^\ddagger$ follow trivially. Moreover,

$$\begin{aligned} U_L^\ddagger &= \frac{1}{4} V_L + \frac{1}{2} \frac{B_L^r}{B_L^r + B_H^r} V_L - B_L \\ &= \gamma V_H \left[\frac{1}{4}\left(1 - \frac{1}{2} r\right) + \frac{1}{2} \frac{\gamma^r (1 + \gamma^r - r)}{(1+\gamma^r)^2} \right] \\ &= \gamma V_H \left[\frac{1}{4}\left(1 - \frac{1}{2} r\right) + \frac{1}{2} t(\gamma, r)(1 + \gamma^r - r) \right] \end{aligned}$$

which is increasing in γ.

Proof of corollary 7

Straightforward calculations yield

$$\begin{aligned} \frac{\partial B_H}{\partial r} &= \frac{1}{2} V_H \left\{ \frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2} + r \frac{1 - \gamma^r}{(1+\gamma^r)^3} \gamma^r \log \gamma \right\} \\ &\geq \frac{1}{2} V_H \left\{ \frac{1}{4} + \frac{1}{2} \gamma^r + \gamma^r \log \gamma^r \right\} \\ &> 0 \end{aligned}$$

Because $B_L = \gamma B_H$, $\frac{\partial B_L}{\partial r} > 0$ immediately follows.

It is easy to show that the function $t(\gamma, r) \equiv \left\{\frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2}\right\}$ is strictly decreasing in r. Recall from the proof of corollary 6 that

$$U_L^\ddagger = \gamma V_H \left[\frac{1}{4}\left(1 - \frac{1}{2} r\right) + \frac{1}{2} t(\gamma, r)(1 + \gamma^r - r) \right]$$

so that $U_L^\ddagger$ is decreasing in r. Moreover,

$$\begin{aligned} \frac{\partial U_H^\ddagger}{\partial r} &= -\frac{1}{2} \frac{\gamma^r \log \gamma}{(\gamma^r + 1)^2} V_H - \frac{\partial B_H}{\partial r} \\ &= -\frac{1}{2} V_H \left\{ \frac{\gamma^r \log \gamma}{(\gamma^r + 1)^2} + \frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2} + r \frac{1 - \gamma^r}{(1+\gamma^r)^3} \gamma^r \log \gamma \right\} \\ &= -\frac{1}{2} V_H \left\{ \frac{\gamma^r \log \gamma}{(\gamma^r + 1)^2} \left[1 + r \frac{1 - \gamma^r}{1 + \gamma^r} \right] + \frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2} \right\} \end{aligned}$$

It is readily verified that

$$\frac{\partial U_H^\ddagger}{\partial r} < 0$$

for $\gamma = 1$. Moreover, if $r = 0$, and γ close to 0

$$\frac{\partial U_H^\ddagger}{\partial r} = -V_H \left\{ \frac{\log \gamma}{8} + \frac{1}{4} \right\} > 0.$$

Proof of proposition 8

$D^\ddagger$ is the expected value of the sum of the parties' efforts:

$$\begin{aligned} D^\ddagger &= \frac{1}{4}\left(B_L^\ddagger + B_L^\ddagger\right) + \frac{1}{4}\left(B_H^\ddagger + B_H^\ddagger\right) + \frac{1}{4}\left(B_L^\ddagger + B_H^\ddagger\right) + \frac{1}{4}\left(B_H^\ddagger + B_L^\ddagger\right) \\ &= B_L^\ddagger + B_H^\ddagger = \frac{1}{2} r\left(1+\gamma\right) V_H \left\{\frac{1}{4} + \frac{\gamma^r}{\left(1+\gamma^r\right)^2}\right\} \end{aligned}$$

The comparative statics results with respect to $D^\ddagger$ immediately follow from the comparative static results for $B_H^\ddagger$ and $B_L^\ddagger$. Since $D^\ddagger$ increase both in r and in γ, it can be easily showed that $0 < D^\ddagger < \frac{1}{2}V_H$. $M^\ddagger$ is the probability that a low-valuing party wins against a high-valuing party multiplied by the difference between the high and the low valuation:

$$\begin{aligned} M^\ddagger &= \frac{1}{2}\frac{\left(B_L^\ddagger\right)^r}{\left(B_L^\ddagger\right)^r + \left(B_H^\ddagger\right)^r}\left(V_H - V_L\right) \\ &= \frac{1}{2}\left(1-\gamma\right) V_H \frac{\gamma^r}{1+\gamma^r} \end{aligned}$$

For $r = 1$,

$$\frac{\partial M^\ddagger}{\partial \gamma} = \frac{1}{2}\frac{1-2\gamma-\gamma^2}{\left(1+\gamma\right)^2} V_H$$

which is strictly positive (negative) for γ close to 0 (1). In addition, for $r = 0$,

$$M^\ddagger = \frac{1}{2}\left(1-\gamma\right) V_H$$

so that $\frac{\partial M^\ddagger}{\partial \gamma} < 0$ for r close to 0. Furthermore, we have $\frac{\partial M^\ddagger}{\partial r} = \frac{1}{2}\left(1-\gamma\right) V_H \frac{\gamma^r}{(\gamma^r+1)^2}\ln\gamma < 0$. From the former results, it is easy to show that $0 < M^\ddagger < \frac{1}{2}V_H$. Since $D^\ddagger$ and $M^\ddagger$ reach their maximum for opposite values of r and γ, we have $0 < D^\ddagger + M^\ddagger < \frac{1}{2}V_H$.The comparative statics for $D^\ddagger + M^\ddagger$ with respect to γ follows easily by substituting $r = 0$ and $r = 1$. Finally,

$$\frac{\partial (D^\ddagger + M^\ddagger)}{\partial r} = \frac{1}{2}V_H\left[\left(1+\gamma\right)\left(\frac{1}{4} + \frac{\gamma^r}{\left(1+\gamma^r\right)^2} + \frac{\left(1-\gamma^r\right)\gamma^r \log\gamma^r}{\left(1+\gamma^r\right)^3}\right) + \frac{\left(1-\gamma\right)\gamma^r\log\gamma}{\left(1+\gamma^r\right)^2}\right].$$

This expression is strictly positive for r close to 1 and negative for r close to 0.

Proof of proposition 9. The following claims derive from straighforward manipulation of (8): $B_L^{**} = B_H^{**}$, $\frac{\partial B_L^{**}}{\partial \gamma} = \frac{\partial B_H^{**}}{\partial \gamma} = 0$, $\frac{\partial B_L^{**}}{\partial r} = \frac{\partial B_H^{**}}{\partial r} > 0$ and $\frac{\partial B_L^{**}}{\partial P} = \frac{\partial B_H^{**}}{\partial P} > 0$. Substituting (8) into (7), we have $U_H^{**} = U_L^{**} + V_H - P$, which yields $U_L^{**} < U_H^{**}$; we further have $\frac{\partial U_L^{**}}{\partial \gamma} = \frac{\partial U_H^{**}}{\partial \gamma} = 0$, $\frac{\partial U_L^{**}}{\partial r} = \frac{\partial U_H^{**}}{\partial r} = -\frac{P}{4} < 0$, and $\frac{\partial U_H^{**}}{\partial P} = -\left(\frac{r}{4}+1\right) < \frac{\partial U_L^{**}}{\partial P} = -\frac{r}{4} < 0$. For the rent dissipation $D^{**} = \frac{r}{2}P$ we have $\frac{\partial D^{**}}{\partial \gamma} = 0$, $\frac{\partial D^{**}}{\partial r} = \frac{P}{2} > 0$ and $\frac{\partial D^{**}}{\partial P} = \frac{r}{2} > 0$. D^{**} approaches V_H as r approaches 1 and P approaches V_H; D^{**} approaches 0 as r approaches 0. It is self-evident that the rent misallocation is $M^{**} = 0$. *QED*

Proof of corollary 10. Using (8) and (11a) we have $B_L^{**} = \frac{r}{4}P > r\frac{\gamma^r}{(1+\gamma^r)^2}V_L = B_L^*$, because $P \geq V_L$ and $\frac{1}{4} > \frac{\gamma^r}{(1+\gamma^r)^2}$. Using (7a), (8), and (13a), we obtain $U_L^{**} = \frac{2-r}{4}P > \frac{\gamma^{2r}+(1-r)\gamma^r}{(1+\gamma^r)^2}V_L = U_L^{**}$, because $V_L \leq P$ and $\frac{2-r}{4} > \frac{\gamma^{2r}+(1-r)\gamma^r}{(1+\gamma^r)^2}$. Using (8) and (11b) we obtain $B_H^{**} = \frac{r}{4}P < r\frac{\gamma^r}{(1+\gamma^r)^2}V_H = B_H^*$ if

$P < \frac{4\gamma^r}{(1+\gamma^r)^2} V_H$ and $B_H^{**} \geq B_H^*$ otherwise (note that $\frac{4\gamma^r}{(1+\gamma^r)^2} < 1$). Using (7b), (8), and (13b), we obtain $U_H^{**} = V_H - \frac{2+r}{4} P > \frac{1+(1-r)\gamma^r}{(1+\gamma^r)^2} V_H = U_H^*$ if $P < \frac{4\gamma^r}{(1+\gamma^r)^2} \frac{3-r+\gamma^r}{2+r} V_H$ and $U_H^{**} \leq U_H^*$, otherwise.

Concerning the social loss, using (14) we have $D^{**} = \frac{r}{2} P < r \frac{\gamma^r}{(1+\gamma^r)^2} (1+\gamma) V_H = D^*$ if $P < \frac{2\gamma^r}{(1+\gamma^r)^2} (1+\gamma) V_H$ and $D^{**} \geq D^*$, otherwise. Using (15) we have $D^{**} = \frac{r}{2} P < \frac{\gamma^r}{(1+\gamma^r)^2} (r(1+\gamma) + (1-\gamma)(1+\gamma^r)) V_H = D^* + M^*$ if $P < \frac{2\gamma^r}{r(1+\gamma^r)^2} (r(1+\gamma) + (1-\gamma)(1+\gamma^r)) V_H$ and $D^{**} \geq D^* + M^*$, otherwise. *QED*

Proof of proposition ??. Differentiating (10)

$$\begin{aligned}
\frac{\partial U_L(V_L, B_i, B_L, B_H)}{\partial B_i} &= \frac{1}{2} \frac{r B_i^{r-1} B_L^r}{(B_i^r + B_L^r)^2} V_L + \frac{1}{2} \frac{r B_i^{r-1} B_H^r}{(B_i^r + B_H^r)^2} P - 1 \\
\frac{\partial U_H(V_H, B_j, B_L, B_H)}{\partial B_j} &= \frac{1}{2} \frac{r B_j^{r-1} B_H^r}{\left(B_H^r + B_j^r\right)^2} V_H + \frac{1}{2} \frac{r B_j^{r-1} B_L^r}{\left(B_L^r + B_j^r\right)^2} P - 1
\end{aligned}$$

The FOCs in equilibrium, where a player with valuation equal to V_L (V_H) exerts effort $B_i = B_L$ ($B_j = B_H$), are:

$$B_L = \frac{r}{8} V_L + \frac{1}{2} r \frac{B_L^r B_H^r}{(B_L^r + B_H^r)^2} P \tag{17}$$

$$B_H = \frac{r}{8} V_H + \frac{1}{2} r \frac{B_L^r B_H^r}{(B_H^r + B_L^r)^2} P \tag{18}$$

Hence, $B_H^{\ddagger\ddagger} = B_L^{\ddagger\ddagger} + \frac{r(1-\gamma)}{8} V_H$ and $D^{\ddagger\ddagger} = B_L^{\ddagger\ddagger} + B_H^{\ddagger\ddagger} = \frac{r(1+\gamma)}{8} V_H + r \frac{B_L^r B_H^r}{(B_L^r + B_H^r)^2} P$. The parties' equilibrium levels of efforts may be written as:

$$B_L^{\ddagger\ddagger} = \frac{r}{8} V_L + \frac{1}{2} r p_L^{\ddagger\ddagger} \left(1 - p_L^{\ddagger\ddagger}\right) P \tag{19}$$

$$B_H^{\ddagger\ddagger} = \frac{r}{8} V_H + \frac{1}{2} r p_L^{\ddagger\ddagger} \left(1 - p_L^{\ddagger\ddagger}\right) P \tag{20}$$

where

$$p_L^{\ddagger\ddagger} = \frac{\left(B_L^{\ddagger\ddagger}\right)^r}{\left(B_L^{\ddagger\ddagger}\right)^r + \left(B_H^{\ddagger\ddagger}\right)^r} \tag{21}$$

is the equilibrium probability that a low-valuing party wins against a high-valuing party.

The set of equations (19), (20), and (21) is not tractable from an analytical point of view. Still, some interesting comparisons can be made between situations with and without an after market.

Finally, observe that for $r \leq 1$, all $B_i > 0$, and $B_H > B_L$,

$$\frac{\partial^2 U_i(V_i, B_i, B_L, B_H)}{\partial B_i^2} < 0.$$

Therefore, the second-order condition is satisfied as well.

What remains to be checked is whether the system of equations (17) and (18) has a solution. First, we fix B_L so that B_H is given by (18). Note that

$$B_H = B_L + \frac{r}{8} (V_H - V_L).$$

Let

$$\Phi(x) \equiv x + \frac{r}{8}\left(V_H - V_L\right).$$

Note that Φ is a continuous function. Now, we check that equation (17) has a solution, i.e., there is a $y \geq 0$ for which

$$\Psi(y) \equiv \frac{rV_L}{8} + \frac{1}{2} rP \frac{y^r \Phi(y)^r}{\left(y^r + \Phi(y)^r\right)^2} - y = 0.$$

Observe that $\Psi(0) > 0$ and $\Psi\left(\frac{r}{8}\left[V_H + P\right]\right) < 0$. Hence, by the intermediate value theorem, there is a $y > 0$ for which $\Psi(y) = 0$. Therefore, the system of equations (17) and (18) has at least one solution.[10] This solution, however, is not tractable in an analytical way. *QED*

Proof of corollary ??. Recall that for the ratio of the equilibrium bids it holds true that

$$\frac{B_L^{\ddagger\ddagger}}{B_H^{\ddagger\ddagger}} > \gamma.$$

Because this ratio determines the winning probability of a low-valuing against a high-valuing party, we can compare $B_L^{\ddagger}$ and $B_L^{\ddagger\ddagger}$. Recall that

$$\begin{aligned} B_L^{\ddagger} &= \frac{1}{2} r \left(\frac{1}{4} + \frac{\gamma^r}{\left(1+\gamma^r\right)^2} \right) V_L \\ &= \frac{1}{2} r \left(\frac{1}{4} + p_L^{\ddagger}\left(1 - p_L^{\ddagger}\right) \right) V_L \end{aligned}$$

where $p_L^{\ddagger}$ is the equilibrium probability that a low-valuing party wimns against a high-valuing party when there is no possibility of ex post reallocation of the rent. We have observed that that

$$\frac{B_L^{\ddagger\ddagger}}{B_H^{\ddagger\ddagger}} > \gamma = \frac{B_L^{\ddagger}}{B_H^{\ddagger}}$$

which implies that

$$\frac{1}{2} > p_L^{\ddagger\ddagger} > p_L^{\ddagger}$$

so that

$$p_L^{\ddagger\ddagger}\left(1 - p_L^{\ddagger\ddagger}\right) > p_L^{\ddagger}\left(1 - p_L^{\ddagger}\right)$$

and in turn

$$B_L^{\ddagger\ddagger} > B_L^{\ddagger}.$$

Analogously, $D^{\ddagger\ddagger}$ can be written as

$$D^{\ddagger\ddagger} = B_L^{\ddagger\ddagger} + B_H^{\ddagger\ddagger} = \frac{r}{8}\left(V_L + V_H\right) + rp_L^{\ddagger\ddagger}\left(1 - p_L^{\ddagger\ddagger}\right) P.$$

When $\alpha \geq \frac{1}{2}$, $P \geq \frac{1}{2}\left(V_H + V_L\right)$, so that

$$D^{\ddagger\ddagger} \geq \frac{1}{2} r \left(V_L + V_H\right) \left\{ \frac{1}{4} + p_L^{\ddagger\ddagger}\left(1 - p_L^{\ddagger\ddagger}\right) \right\}$$

[10] The solution is not necessarily unique.

Recall that

$$
\begin{aligned}
D^{\ddagger} &= \frac{1}{2}r\left(V_L+V_H\right)\left\{\frac{1}{4}+\frac{\gamma^r}{\left(1+\gamma^r\right)^2}\right\} \\
&= \frac{1}{2}r\left(V_L+V_H\right)\left\{\frac{1}{4}+p_L^{\ddagger}\left(1-p_L^{\ddagger}\right)\right\}
\end{aligned}
$$

Because

$$
p_L^{\ddagger\ddagger}\left(1-p_L^{\ddagger\ddagger}\right)>p_L^{\ddagger}\left(1-p_L^{\ddagger}\right)
$$

it immediately follows that$D^{\ddagger\ddagger}>D^{\ddagger}$.

Finally, we prove that $D^{\ddagger}+M^{\ddagger}>D^{\ddagger\ddagger}$ for α close to 0. We can write

$$
D^{\ddagger}+M^{\ddagger}-D^{\ddagger\ddagger}>D^{\ddagger}-D^{\ddagger\ddagger}
$$

Moreover, for $\alpha=0$,

$$
\begin{aligned}
D^{\ddagger}-D^{\ddagger\ddagger} &= r\left[\frac{1}{2}\left(V_L+V_H\right)p_L^{\ddagger}\left(1-p_L^{\ddagger}\right)-p_L^{\ddagger\ddagger}\left(1-p_L^{\ddagger\ddagger}\right)P\right] \\
&= r\left[\frac{1}{2}\left(V_L+V_H\right)p_L^{\ddagger}\left(1-p_L^{\ddagger}\right)-p_L^{\ddagger\ddagger}\left(1-p_L^{\ddagger\ddagger}\right)V_L\right] \\
&> r\left[\frac{1}{2}\left(V_L+V_H\right)p_L^{\ddagger}\left(1-p_L^{\ddagger}\right)-\frac{1}{4}V_L\right] \\
&= rV_H\left[\frac{1}{2}\left(1+\gamma\right)\frac{\gamma^r}{\left(1+\gamma^r\right)^2}-\frac{1}{4}\gamma\right] \\
&> rV_H\left[\frac{1}{2}\left(1+\gamma\right)\frac{\gamma}{\left(1+\gamma\right)^2}-\frac{1}{4}\gamma\right] \\
&> 0.
\end{aligned}
$$

The first inequality follows from

$$
p_L^{\ddagger\ddagger}\left(1-p_L^{\ddagger\ddagger}\right)<\frac{1}{4},
$$

and the second from the fact that

$$
\frac{\gamma^r}{\left(1+\gamma^r\right)^2}
$$

is decreasing in r. *QED*