General Equilibrium Model of Abstention with Risk Averse Voters

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Abstract:

Voter participation and abstention is studied in a general equilibrium model with rational voters who differ in their degree of risk aversion. Though voters have common values in deterministic choices, they may vote for different candidates due to the different riskiness of their election programs. Voters choose to vote or abstain based on the cost of voting and their favorite candidate's perceived probability of election victory. In the equilibrium all voters maximize their expected utility and their expectations are rational; election results are consistent with the perceived probability. In some cases the election may not aggregate information well and a less favored candidate wins. We show necessary and sufficient conditions for the existence of an equilibrium in which votes are cast for both candidates. Abstention always arises among voters who do not have a strong preference for either candidate as long as the cost of voting is positive.

1 Introduction

Voter behavior in general and voter abstention in particular has been subject to intense debate among both the economists and the political scientists for several decades. While a number of approaches have helped deepen our understanding of why voters abstain, each of them seems to be plagued by serious shortcomings. Since a thorough review of the literature has been recently offered by Dhillon and Peralta (2002), we will focus on papers that provide results directly relevant to our model. The early decision-theoretic models, such as Downs (1957) and Tullock (1968) assume rational behavior of utility maximizing voters. But they have encountered an insurmountable obstacle in the "paradox of voting": why do voters bother voting if their impact in a large electorate is negligible?

This problem lead to a number of game-theoretic models, in which abstention arises as a strategic choice. In Feddersen and Pesendorfer (1996), uninformed voters choose to abstain even if voting is costless, and delegate the choice to the more informed voters. In essence, they remove the cost of potentially voting for the wrong candidate. In Feddersen and Pesendorfer (1999), a more general situation with differentially informed voters without common values yields a strong result that the election still aggregates the information correctly; despite abstention, a more preferred candidate wins. In a recent paper Borgers (2004) shows that the participation in small electorates may in fact be too high. He points out the negative externality a voter imposes on others if he chooses to vote: a number of votes cast increases, thus reducing the probability of all other votes to be pivotal. While game-theoretic models provide a detailed analysis of the probability of a pivotal vote, they result in multiple equilibria.

To address the shortcomings of both the decision-theoretic and the game-theoretic models a growing number of contributions resort to studying the behavior of voters under bounded rationality, such as Sieg and Schulz (1995) and Demichelis and Dhillon (2001), replacing full rationality with the behavior based on the adaptive learning. Other contributions, such as Ghirardato and Katz (2003) resort to non-traditional objective functions.

In our paper, we present a standard decision-theoretic model with risk averse voters choosing their favorite candidate and deciding whether to vote or to abstain so as to maximize their expected utility. A defining feature of this model is the rationality of voters' expectations; they vote or abstain based on the perceived chance of their favorite candidate winning the election. In equilibrium, their expectations are fulfilled and the proportion of votes each candidate receives is consistent with the perceived probability.

The model yields several notable results. First, we show the necessary and sufficient conditions for a unique interior equilibrium to exist. Second, as long as voting is costly, there are always voters with medium risk aversion who chose to abstain. And finally, we show examples where the election does not represent the preferences correctly: voters elect the "wrong" candidate who has the support of minority of the population, while the favorite candidate looses.

2 The Model

2.1 Population

The economy is populated by a continuum of voters who differ from each other in their degree of risk aversion. A voter's type, a, determines his coefficient of absolute risk aversion: his utility function is $u(c) = -exp [-ac]$, where c is the quantity consumed. Voters are uniformly distributed on an interval $(m - l, m + l)$, where $0 < m - l < m + l < \infty$; m is the average coefficient and l is the dispersion of risk aversion of population. Size of the population is normalized to 1. Voters with higher values of a are more risk averse while the ones with lower values of a are less risk averse.

2.2 Elections

Two candidates, an Incumbent and a Challenger, compete for votes. If the Incumbent wins, each voter is guaranteed x units of the consumption good. The Challenger, in contrast, promises to increase voters' consumption to $x + G$. Voters do not know Challenger's qualities completely. They believe that Challenger's policies will increase consumption to $x + G$ with probability q, but they may also result in a lower consumption $x - L$ with probability $1-q$. We assume that

A1: $0 < x - L < x < x + G < \infty$, and

A1: $q(x + G) + (1 - q)(x - L) > x$.

Assumption A2 states that the expected consumption if the Challenger wins is greater than the consumption under the Incumbent. It guarantees that there exists a voter $a^* \in (0,\infty)$ who is indifferent between the two candidates. All voters less risk averse than a* prefer the Challenger and those more risk averse prefer the Incumbent.

2.3 Expected Utility and Rational Expectations

Voting is costly. If the Incumbent wins the election, each agent who abstained will enjoy the utility $u^{inc} = - exp [- ax]$. Each voter who chose to vote incurred the cost c and his utility is $u = - exp [- a (x - c)] = exp [ac] u^{inc}$.

If the Challenger wins, the voter's expected utility is

$$u^{cha} = - q \exp [- a (x + G)] - (1 - q) \exp [- a (x - L)]$$

if he abstained and

$$u = - q \exp [- a (x + G - c)] - (1 - q) \exp [- a (x - L - c)] = \exp [ac] \; u^{cha}$$

if he voted. Notice that the cost of voting does not depend on which candidate the voter votes for.

Since voting is costly, some voters choose to abstain. A voter only votes if the benefit of voting in terms of expected utility (more than) compensates him for the loss of utility due to the incurred cost. A voter who prefers the Challenger ($u^{cha} > u^{inc}$) believes that the Challenger has a chance p^{cha} to win if all his supporters who maximize their expected utility by voting do so. The probability that the Challenger loses is $1 - p^{cha}$.

Each voter who prefers the Incumbent ($u^{cha} < u^{inc}$) believes that the Incumbent has a chance p^{inc} to win if all his supporters who maximize their expected utility by voting do so. The probability that the Incumbent loses is $1 - p^{inc}$. We assume that $p^{cha} + p^{inc} = 1$ and these probabilities are public knowledge. We also assume that if a voter chooses to abstain, he considers the chances of his favorite candidate winning the election to be zero.

If the voter votes for the Challenger, his expected utility is

$$EU^{cha,vote} = p^{cha} \{ - q \exp [- a (x + G - c)] - (1 - q) \exp [- a (x - L - c)]\}$$

$$- (1 - p^{cha}) \exp [- a (x - c)]$$

$$= exp\,[ac]\ \{\, p^{cha}\, u^{cha} + (1 - p^{cha})\, u^{inc} \}.$$

If this voter abstains instead, his expected utility is

$$EU^{cha,abst} = \ 0\, u^{cha} + (1 - 0)\, u^{inc} = u^{inc}$$

Correspondingly, if the voter votes for the Incumbent, his expected utility is

$$EU^{inc,vote} = p^{inc}\ exp\,[- a\,(x - c)]$$

$$-(1 - p^{inc}\,)\ \{- q\ exp\,[-\ a\,(x + G - c)] - (1 - q)\ exp\,[-\ a\,(x - L - c)]\}$$

$$= exp\,[ac]\ \{\, p^{inc}\, u^{inc} + (1 - p^{inc})\, u^{cha}\ \}.$$

If this voter abstains instead, his expected utility is

$$EU^{inc,abst} = \ 0\, u^{inc} + (1 - 0\,)u^{cha} = u^{cha}.$$

A supporter of the Challenger then decides to vote (abstain) if $EU^{cha,vote} > (<)\ EU^{cha,abst}$, while a supporter of the Incumbent then decides to vote (abstain) if $EU^{inc,vote} > (<)\ EU^{inc,abst}$.

Rational expectations then close the model: given that each voter believes p^{cha} and p^{inc} and that he chooses to vote or abstain so as to maximize his expected utility, the election will result in a proportion p^{cha} of votes cast for the Challenger and a proportion p^{inc} of votes cast for the Incumbent. The results of the election fulfill voters' expectations.

2.4 Equilibrium

First, we will examine the properties of the expected utility functions and introduce terminology that will be used to describe the equilibrium.

Recall that $a*$ is the voter who is indifferent between the two candidates. The existence of such a voter on the interval $(0,\infty)$ is guaranteed by the assumption A2. All voters a <(>) a* are less (more) risk averse and prefer the Challenger (Incumbent). Among supporters of both candidates there are voters who will choose to vote and others who will abstain.

Claim 1 If $qG - (1 - q)L > c$, and $p^{inc} = 1$, then there exists a voter $a^{cha} \in (0,a*)$ who supports the Challenger and is indifferent between voting and abstaining. All less risk averse voters $a \in (0, a^{cha})$ will vote for the Challenger and more risk averse voters $a \in (a^{cha}, a*)$ will abstain.

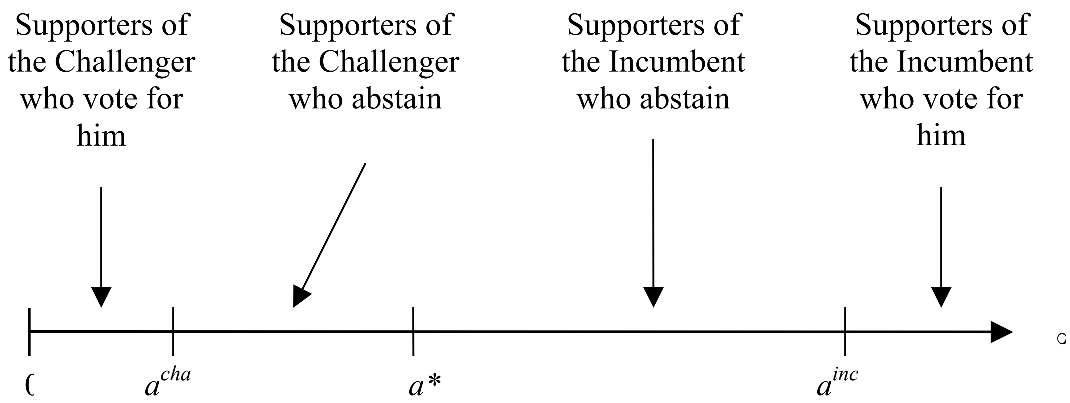
Proof: See Appendix

Claim 2 If $c < L$ and $p^{inc} = 1$, then there exists a voter $a^{inc} \in (a*, \infty)$ who supports the Incumbent and is indifferent between voting and abstaining. Less risk averse voters $a \in (a*, a^{inc})$ will abstain and most risk averse voters $a \in (a^{inc}, \infty)$ will vote for the Incumbent.

Proof: See Appendix

In other words, if the expected increase in consumption (when Challenger is elected) exceeds the cost of voting, there are voters with sufficiently low coefficient of risk aversion who will choose to vote for him. However, if p^{inc} or p^{cha} is 0, no one will vote for that candidate. The preferences and voting behavior of voters with different coefficients of risk aversion are presented in Figure 1.

Figure 1



It is important to realize that the types a^{cha}, a^{inc} of voters who are indifferent between voting and abstaining and the voter $a*$ who is indifferent between the two candidates depend solely on the exogenous parameters of the Incumbent's expected payoff $qG – (1 – q)L$ and the cost of voting c, as specified in Claims 1 and 2. Properties of the equilibrium will then depend on how the population $(m – l, m + l)$ is spaced out along the positive real line $(0, \infty)$. Equilibrium in this model is defined below.

Definition *The population is divided into a subset C of voters who vote for the Challenger, a subset I of voters who vote for the Incumbent and a subset A of voters who abstain. An equilibrium is a set* $(\{C,I,A\}, p^{cha}, p^{inc})$*, where* $\{C,I,A\}$ *is a partition of the population such that* $EU^{cha,vote} > EU^{cha,abst}$ *holds for all* $a \in C$, $EU^{inc,vote} > EU^{inc,abst}$ *holds for all* $a \in I$, *and* $EU^{inc,vote} < EU^{inc,abst}$ *or* $EU^{cha,vote} < EU^{cha,abst}$ *holds for all* $a \in A$. *Probabilities* p^{cha} *and* p^{inc} *satisfy* $p^{cha} = \int_C d\nu(a)$ *and* $p^{inc} = \int_I d\nu(a)$, *where* ν *is a uniform measure on* $(m – l, m + l)$.

Fundamental properties of the equilibrium are described in the following proposition.

Proposition

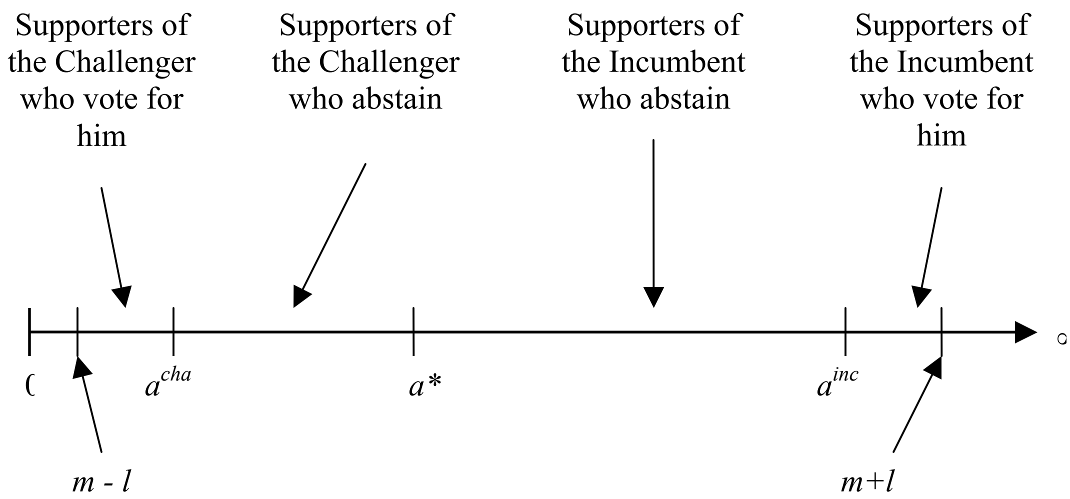
A. Let $p(qG - (1 - q)L) + (1 - p) > c$, $c < L$ *and the population parameters satisfy* $m - l < a^{cha}$ *and* $m + l > a^{inc}$. *Then there exists a unique equilibrium such that* $0 < p^{cha} < 1$, $p^{inc} = 1 - p^{cha}$. *The most risk averse and the least risk averse voters vote, while voters with medium risk aversion, centered around* a^*, *abstain from the election.*

B. Two trivial equilibria, $(p^{cha}, p^{inc}) = (1, 0)$ *and* $(p^{cha}, p^{inc}) = (0, 1)$ *exist in every population* $(m - l, m + l)$.

Proof: See Appendix.

The above proposition states that as long as $p(qG - (1 - q)L) + (1 - p) > c$ and $c < L$, the two trivial "corner" equilibria exist. For the unique interior equilibrium to exist, the population must contain supporters of both political camps who are actually voting. This is the case when both a^{cha} and a^{inc} are present in the population. An example of such a population is presented in Figure 2.

Figure 2



3 Examples

We discuss the properties of equilibria in detail in the following sequence of examples. The diagrams show the behavior of individual voters by means of the equivalent variation. Equivalent variation *VAR* measures the additional amount of the consumption good a voter who is forced to abstain would require in order to achieve the same level of utility he would enjoy if he voted. It follows that if the voter prefers to vote then $VAR > 0$. If the voter prefers to abstain, $VAR < 0$. If $VAR = 0$ the voter is indifferent between voting and abstaining.

Equivalent variation of the voter who supports the Challenger is then implicitly defined by $EU^{cha,vote} \equiv exp[-\ aVAR]\ EU^{cha,abst}$, while the equivalent variation of the voter who supports the Incumbent is defined by $EU^{inc,vote} \equiv exp[-\ aVAR]\ EU^{inc,abst}$.

3.1 Non-trivial equilibrium

Figure 3 shows the equilibrium in a model with the average coefficient of risk aversion $m = 8$ and the dispersion $l = 7.9$. The population then consists of all voters on the interval $(m - l, m + l) = (0.1, 15.9)$. The gain consumers enjoy if the Challenger wins and delivers on his promises is $G = 0.3$. If he fails, consumers lose $L = 0.1$. The probability that he succeeds is $q = 0.5$.

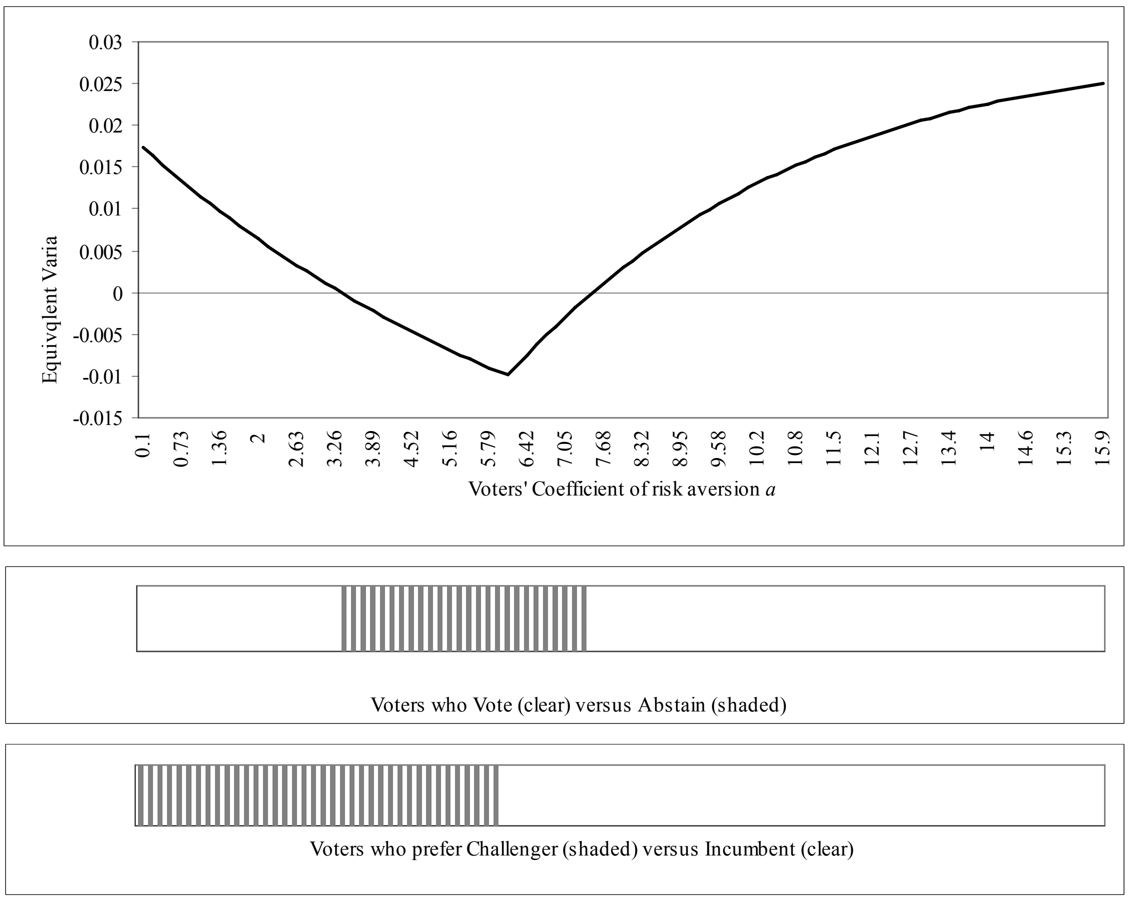
Figure 3 presents the equilibrium in three connected diagrams that share the horizontal axis showing the population in terms of voters' coefficients of risk aversion a. The less risk averse voters are closer to the origin, while the more risk averse ones have greater a. The top diagram shows the equivalent variation of each agent. The most risk averse and the least risk averse voters have positive equivalent variation VAR and vote. The voters in the middle, including a*, who id indifferent between the two candidates, have VAR < 0 and abstain from the election.

The middle diagram shows the shaded subset of voters who abstain (the rest of them vote), while the bottom diagram shows the shaded subset of voters who support the Challenger (the rest of them support the Incumbent).

In this example 25.74% of voters choose to abstain. Since voters' expectations are rational, the equilibrium probabilities $(p^{cha}, p^{inc}) = (0.28, 0.72)$ are reflected in the election results: the Challenger receives 28% of the vote and the Incumbent receives 72% of the vote. The important feature of this example is the fact that despite the abstention voters elected the "right" candidate. The Incumbent, who is the winner, has a support of

62.38% of voters, while the rest, 37.62% support the Challenger. This result, however, is not robust. We show a counterexample in 3.4 below.

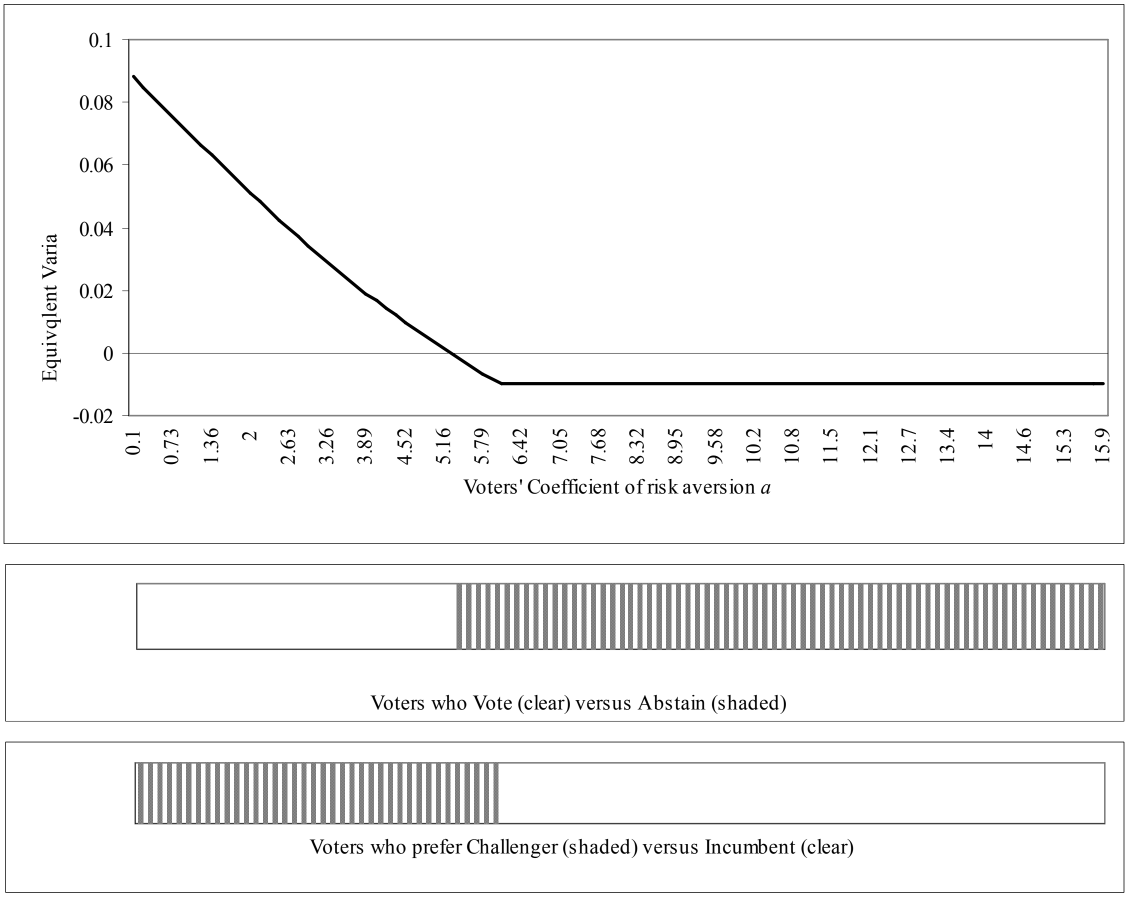
Figure 3 Example of an interior equilibrium with $(p^{cha}, p^{inc}) = (0.28, 0.72)$



3.2 Equilibrium with $p^{cha} = 1$

We use the above example to show the existence and properties of this trivial equilibrium in Figure 4. Notice that no supporters of the Incumbent vote and he receives 0% of votes. While some of the Challengers supporters abstain and some vote, he receives 100% of votes. In this example 67.32% of voters choose to abstain.

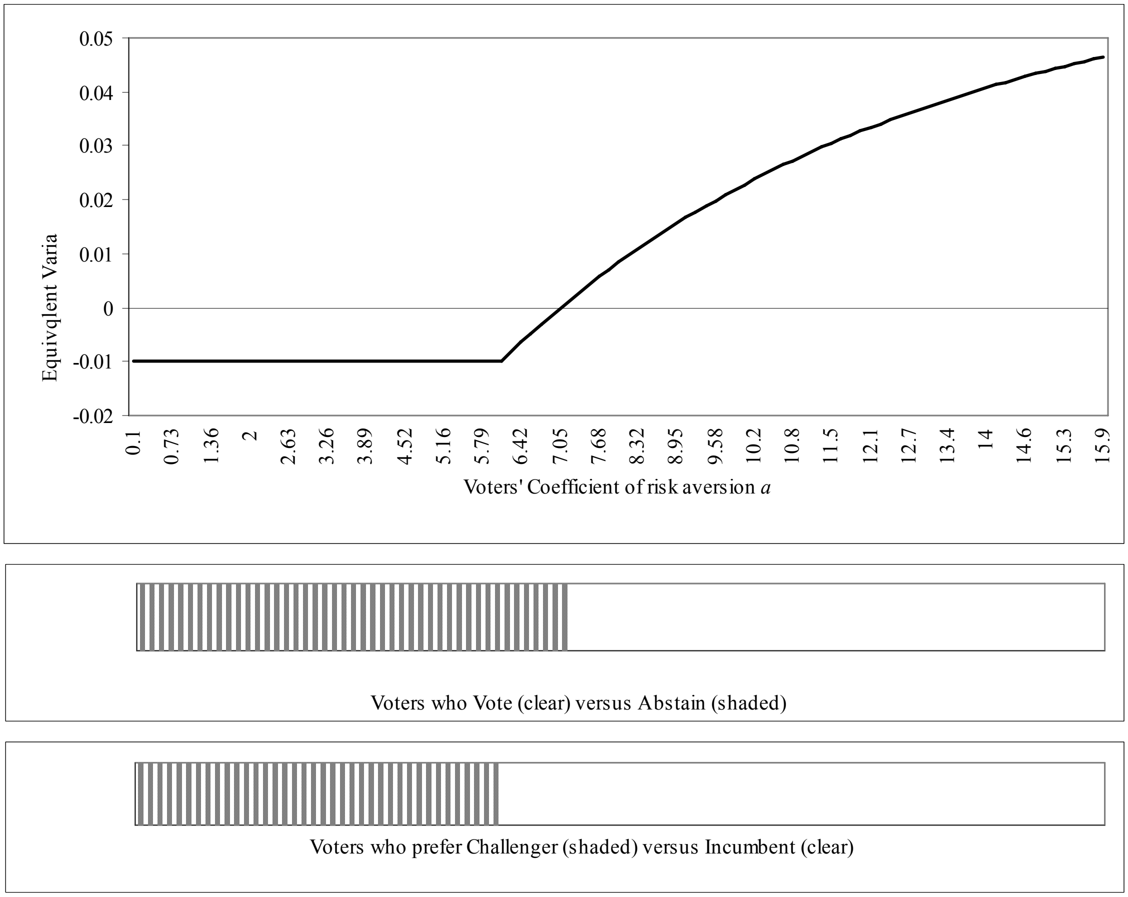
Figure 4 Example of a trivial equilibrium with $(p^{cha}, p^{inc}) = (1, 0)$



3.3 Equilibrium with $p^{inc} = 1$

Using the same example we show the existence and properties of this trivial equilibrium in Figure 5. Notice that this time no supporters of the Challenger vote and he receives 0% of votes. While some of the Incumbent's supporters abstain and some vote, he receives 100% of votes. In this example 44,55% of voters choose to abstain.

Figure 5 Example of a trivial equilibrium with $(p^{cha}, p^{inc}) = (0, 1)$



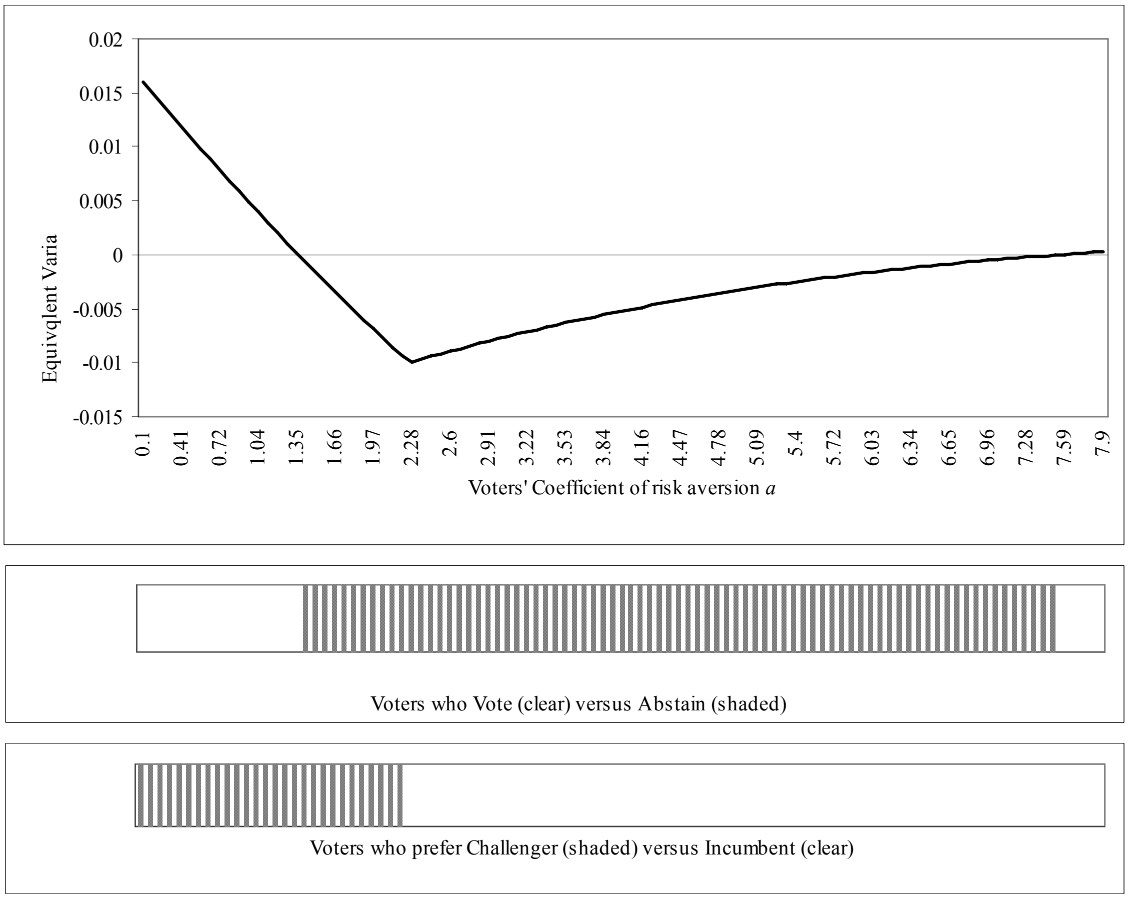
3.4 Non-trivial equilibrium with the "wrong" winner

Figure 6 shows the equilibrium in a model with the average coefficient of risk aversion $m = 4$ and the dispersion $l = 3.9$. The population then consists of all voters on the interval $(m - l, m + l) = (0.1, 7.9)$. The $G = 0.3$ and $L = 0.1$ are the same as before, but the probability that the Challenger delivers on his promises is $q = 0.34$.

In this example 78.22% of voters choose to abstain. Since voters' expectations are rational, the equilibrium probabilities $(p^{cha}, p^{inc}) = (0.762, 0.238)$ are reflected in the election results: the Challenger receives 76.2% of the vote and the Incumbent receives 23.8% of the vote. In this example however, voters elected the "wrong" candidate. The

Incumbent, who lost, has a support of 72.28% of voters, while only 27.72% of population supports the winner!

Figure 6 Example of an interior equilibrium with $(p^{cha}, p^{inc}) = (0.762, 0.238)$ and the "wrong" winner supported by only 27.72% of population



3.5 Other examples

While a number of other results with trivial equilibria can be generated, they are less interesting. Namely, if $m - l > a^*$, the entire population is relatively very risk averse and all voters support the Incumbent. Some of them vote, others abstain, but the result is unambiguous, $(p^{cha}, p^{inc}) = (0, 1)$. Conversely, if $m + l < a^*$, the entire population is relatively less risk averse and all voters support the Callenger. Some of them vote, others abstain, but the result is unambiguous, $(p^{cha}, p^{inc}) = (1, 0)$.

4 Conclusion

This model differs from the majority of models in recent literature in several aspects. It assumes that all voters have rational expectations, they have the same information, and their utility functions differ only in their coefficient of absolute risk aversion. The model shows several notable results. First, we show the necessary and sufficient conditions for a unique interior equilibrium to exist. Second, as long as voting is costly, abstention always arises among voters with medium risk aversion. And finally, we show examples where the election does not represent the preferences correctly: voters elect the "wrong" candidate who has the support of minority of the population, while the favorite candidate looses.

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Appendix

Proof of Claim 1

When $a \to 0$, a voter is approaching risk neutrality. Such a voter maximizes expected utility by choosing to vote, which offers higher expected payoff than abstention: $qG - (1 - q)L > c$.

Proof of Claim 2

If $c < L$, the most risk averse voters prefer to take the cost c in order to prevent an uncertain possibility of a greater loss L.

Proof of the Proposition

The voter a^* is indifferent between the two industries: $u^{cha} = u^{inc}$. But then both necessary conditions for voting $EU^{cha,vote} > EU^{cha,abst}$ and $EU^{inc,vote} > EU^{inc,abst}$ are violated, and a^* chooses to abstain.

If $p^{cha} = 1,$ the least risk averse supporters of the Challenger choose to vote, while all of the Incumbent’s supporters abstain. If $p^{inc} = 1,$ the most risk averse supporters of the

Incumbent choose to vote, while all of the Challenger.s supporters abstain. As p^{inc} increases and p^{inc} decreases, the percentage of votes for Challenger increases all the way to 100%, while the Incumbent's share of votes goes to 0, and vice versa.