A New Approach to Aggregation of Individual Choice

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Abstract:

This paper starts with a glance at the literature on public choice about voting methods, and tries to shed light into a few problems that have been ignored. One of the criterion that is being used in calling a voting method "good" or setting its efficiency is the picking of Condorcet winner when it exists. The paper shows that using condorcet method is not a good system of reference for the efficiency of rules. Its drawbacks start from its construction, and then to its unpleasant effects. A few cases are shown when condorcet winner is not the social optimum while other rules like Borda and the new candidate (rule) do pick the optimum. Furthermore, Condorcet method is in itself misleading and unable to use the signal for filling in the preferences of individuals with their intensities.

Having shown that the criterion usually used is not a good criterion, the paper struggles to find criteria for finding a good criterion. Having shown the lack of the possibility of proving the existence of the "best" rule (as being optimum based on social optima) and that a criterion is missing, the paper focuses by finding a new approach to social aggregation decision which does not use a criterion (since it is non-existent so far in our knowledge set). Instead it strives to establish itself through the process of its creation by filling some of the gaps and drawbacks that Condorcet and Borda method had. Further applications of the model show that even though there is lack of proof of its omni-validity, there can be cases in which it does do better off than other rules, thus establishing it as a good candidate in terms of outcome, but better in terms of its approach.

1. Introduction

A lot of thought and analysis has been dedicated to voting systems. Despite the abundant thought poured into this subject a lot of questions remain vague and unresolved. Finding the alternative that is most suitable for overall society remains a very complex process and still subject to change.

Literature dedicated to analyze those issues dates back to centuries ago with the French enlightment philosopher, Marquis de Condorcet (1745-1794), French mathematician Jean-Charles de Borda (1733-1799), and a continuous string of others before and after them. Later on, Buchanan and Tullock analyzed decision aggregation through an economic standpoint. In *Calculus of Consent*, they showed that there is a trade-off between external costs and decision making costs, thus the minimized sum of them should define the optimal majority number K. The importance of their work is not only the latter economic approach of voting, but also the result that there could not be a unique universal rule, but this optimal majority K can be different for different fields, committees, and so forth.

The aim of this paper is to extend the analysis of voting methods and their efficiency. This analysis will expand in introducing a new method of voting which aim is not only to be give outcomes close to social optima but also to drive away from the deterministic and unidimensionality of existing methods.

The first section will analyze one of the most important existing voting method. I aim to show the drawbacks of Condorcet voting mechanism, since it has been used as a model in expressing other methods' efficiency in being "good" rules[1]. This sections shows how Condorcet method adopts a "myopic" approach, can be very "efficient" in ignoring the intensity of preference, and also cases are brought to show that it does not necessarily pick the social optima choice.

[1] Muller

The second section will explain the new method of voting with a different approach. As an inspiration served the incompleteness of Condorcet and the quasi-cardinal approach of Borda. The third section introduces a new voting method and examples that show cases when it can pick Borda rule, while it is different than condorcet winner, and yet be a social optima; it can pick Condorcet winner, while different from Borda winner; and it can pick social optima when other two rules do not. The model will always have one winner, and in the contrary if more a winners, then society will be indifferent to them (as a whole).

2. Criteria for a Good Criterion

Maybe the title is a bit misleading, for this section does not introduce criteria for finding good criteria for voting system. However, it does show that maybe there does not exist such criterion, at least not one that public choice partisans (scholars) have unraveled yet.

There are a few main conditions are generally accepted as to be essential for a fair voting rule, besides Pareto Criterion, Monotonicity[2] such as:

1. No dictatorship (assuming more than one person's vote matters)
2. Pick Condorcet Winner when it exists
3. A rule that encourages honest revelation of preferences (ordinal rather than cardinal)
4. Eliminate or minimize the incentive of strategic voting

This section will focus on the second criteria arguing that Condorcet Winner is not always a social optimum and thus it fails to serve as a point of reference for other voting methods to be compared with[3].

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[2] Matt Corks, *Evaluating Voting Methods*.
He explains Monotonicity as "If alternative *a* is declared the winner under a voting method, and one or more voters change their preferences in a way favour to *a* (making no other changes), then *a* should still win."
[3] Public Choice III, a table is shown in which the efficiency of other voting methods are displayed in reference to how often they pick a Condorcet winner when one exists (pp.150).

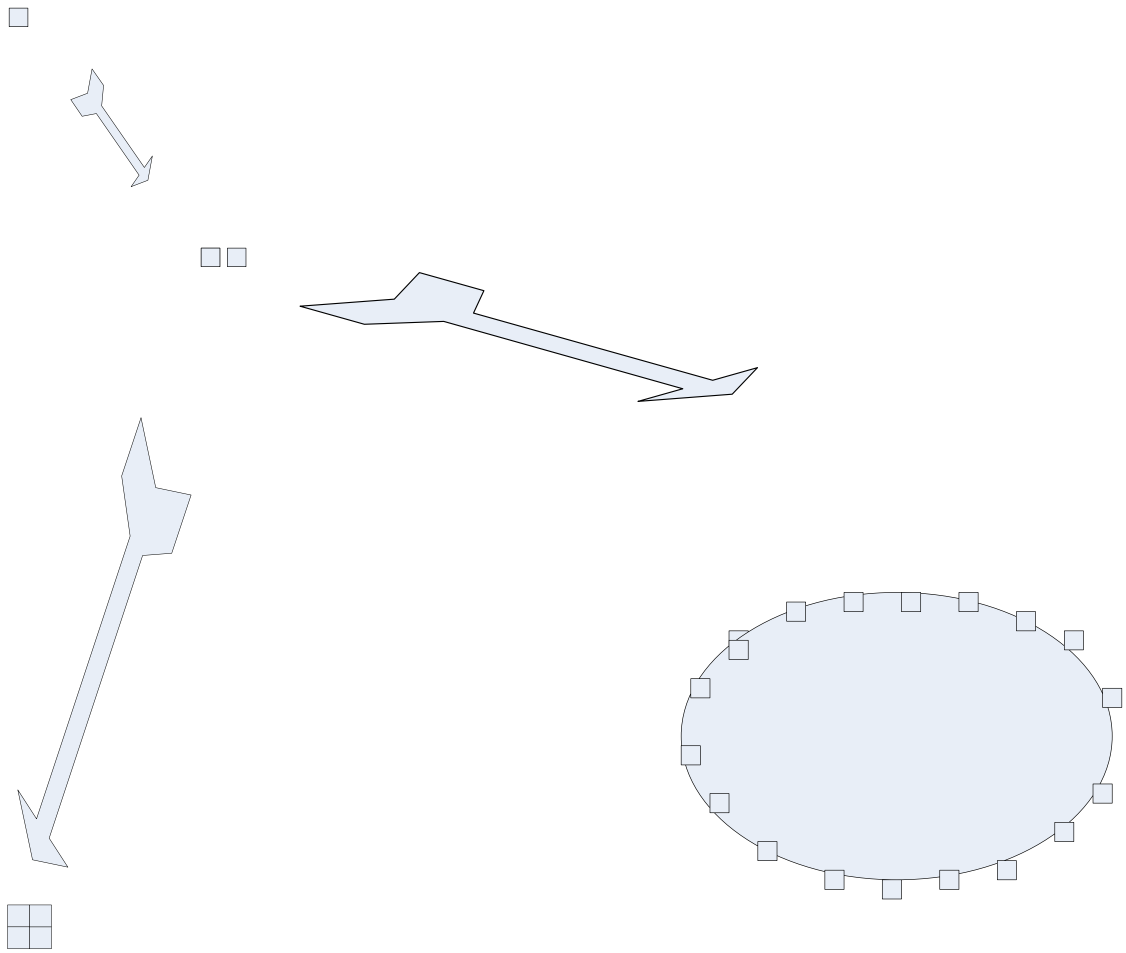
2.1 "Myopic" View of Condorcet Rule

Starting with the definition, "The **Condorcet winner** of an election is the candidate who, when compared in turn with each of the other candidates, is preferred over the other candidate"[4].

The first problem that can be observed with pairwise comparison is that each time of a comparison all other information is being deleted except the piece about the two candidates being compared. This *myopic* view consists in the investigation of the matter by observing each time a local neighborhood (2 candidates) while ignoring the global "truth" every iteration.

This technique may lead to a distortion of the "truth", if such a truth exists. An analogy would be, as shown in the figure below, that if we look too closely to the local neighborhood, only a dot (point) or two can be observed. But those can be individual points, or element of a square, ellipse, sphere, anything. By merely limiting the observation to a fragment (local neighborhood), in this case, nothing can be inferred from the system, or the set that it is part of, unless observed "globally".

[4] Encyclopedia



This approach seems to ignore the full picture and look separately at bits of it while neglecting the interconnection among them. This may explain the cases in which pairwise comparison shows social intransitivity despite of individual transitivity which leads to the “existence” of cycles. This casts the doubt of whether the way condorcet method is set is the source of cycles, or do they really exist?

For instance other rules (e.g. Borda) will *always* pick a winner and never show cycles. This is not to say that those rules are best because they never lack a winner, or that do not reveal cycles, but it is an insight that may suggest that rules when created by humans and when implemented, may create cycles even when they do not exist while looking at the whole picture, but may not show the cycles even when they exist.[5]

2.2 Voting Rules vs. Intensity of Preference

I would to a certain degree agree with the claim that Borda rule, and probably my new voting rule, is a very deterministic approach and *ignores*[6] the intensity of the preferences. However, we should be careful in the words we use and how we shape our thoughts, since there are two questions that I ponder:

1. The approach is deterministic compared to what?
2. Assuming the intensity of preference is essential, do we have a better alternative to reveal, "measure", and "compare" it?

I will start with the second question. I do agree with Bentley's ideas in that "intense preferences should count more, because democracy does not get "quantities" in policy right." In the article, "Interest Groups" by Michael Munger, the following example is displayed:

Assume there are 25 people, from whom 15 are voting for policy A, and the rest of 10 individuals vote for policy B.

# Voters\ Preferences	15	10
1	A	B
2	B	A

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[5] As will be shown later in a few examples, even when a Condorcet winner is missing, there is a social optimum, meaning that cycles displayed by one rule do not necessarily mean that they exist. On the other hand Gordon Tullock in his article "*The General Irrelevance of the General Impossibility Theorem*", would show an example of a type of "concealed cycle", a cycle that will not be seen due to the sequence of alternatives chosen and maybe the lack of proposal of one of them (11-12). However, I am not going to extend upon this argument because he still looks at a pairwise comparison (and a kind of agenda setting since the sequence of pairwise voting matters), although looking at the whole picture in the example, it is clear that D is social optimum.

[6] Better said as later will be shown, "it does not take full account"

Bentley's example

Thus, according to both Borda and Condorcet rule, the winner will be A. However, as Bentley argues, in this example because differences of intensity of preference among the two groups (15 vs. 10) the aggregated force that the 10 voters exert is higher than the 15 voters do. Thus, taking in consideration this intensity of preferences, policy B should be the winner.

My claim is that once we include other policies to be considered and voted upon, this intensity of preferences will start to be discovered in a discrete way[7]. For instance, I would suggest, that in this example if we add policy C, with the following conditions:

a) C is the least preferred for the 15 voters
b) C is more preferred than A for the 10 voters. From Bentley's example those 10 voters do dislike A very intensely, and we could find a policy C that would be more preferred than A, but less preferred than B.

With the introduction of C upon the above assumptions, the votes will look as the following:

Adjusted Example

# Voters\ Preferences	15	10
1	A	B
2	B	C
3	C	A

Condorcet Winner: A
Borda's Winner: B
New Rule's Winner: B[8]

Thus, this case shows that picking Condorcet Winner is not a good criterion for an efficient rule, because it fails more than the other rules to consider the new information added, which can reveal partially the intensity of preferences. Because Condorcet rule consists on pairwise

[7] Unless we chose infinitely set of alternatives for voters to choose, in which case the full continuous function will be revealed (assuming honest and non-strategic voting). However, this is quite costly, unimplementable, and maybe even irrelevant. Thus, a discrete revelation is the objective (a few alternatives, but more than 2).
[8] Appendix 1

comparison, it deletes a lot of information that can *signal* how much one is more preferred than the other[9].

As later the paper will introduce with the new rule, for now its implementation will give the above results showing that Borda's and the new rule absorb and try to make use of the added information given by the voters through C, and the relevance of intensity of preferences. In this case, it can recognize that although 10 voters are the minority, they dislike A a lot more compared to B and C, and thus like intensively more B to A than the majority likes A to B, and thus Borda does pick B as its winner.

I am not trying to show by this example that Borda's rule, nor the new rule, are the best rules. My null hypothesis is that

Ho: Condorcet rule and picking the Condorcet winner when it exists is a good criteria for a rule

And I am trying to reject Ho, by showing that Condorcet winner is not always the social optimum, actually the other rules that we are trying to compare and place them a rating on the basis of Condorcet, can do better than the "model" compared with. Also, if we want to move into preference intensity, Condorcet again is more probable to ignore this intensity based on the structure of its functioning (pairwise comparison).

By rejected Ho, the path is opened in searching for new rules through a different approach, and evaluating them through different criteria. However, this should not be understood that I am reversing the logical threat by claiming that my rule is the best just by some examples that can beat both Borda and Condorcet. These examples are necessary to show that this rule is a good candidate, but not sufficient to declare it a "winner".

[9] It always leaves the level to Preference 1 and 2.

2.3 Condorcet Winner Not Always the Social Optima

Theoretically, it has been argued that Condorcet method is fragile when its rules are scrutinized, by deleting and ignoring important information. In this section, it will be shown also an example that Condorcet Winner is not necessarily the social optimum, by the example in section 4.2. and fig 3. In a society of 4 voters choosing among 3 alternatives, condorcet winner is not a social optimum, while Borda and new rule winner correspond to the social optimum conditioned upon the set of alternatives of being able to be voted on (3 choices).

3. New Voting Rule

Having appreciated but as well observed the drawbacks of pairwise comparison, an inspiration emerged to create a new approach that can use the full information but at the same time setting it in a multidimensional space in order to avoid Borda's method of expressing the votes through a unit of measurement[10] and collapsing it in one dimension for comparison.

3.1 Mathematical Model

Assumptions:

In this paper, I will use the general assumption of lack intransitivity in the individual level in so agreeing with Gordon Tullock in his article in response to May, "*The Irrationality of Intransitivity*". Once recognizing transitivity as a parameter of rationality in each individual, and as a good criterion of rationality in general, it follows that it should be a criterion of defining a good social aggregation decision. However, as it will be shown later cycles are present due to this intransitivity while social aggregation.

[10] Setting difference of 1 through each preference order (what I mean by unit of measurement)

Another assumption made in this paper is single-peak preferences[11]. Thus, individual's degree of satisfaction falls off as each moves away from his/her optimum (local and global maximum is the same) in any direction. Also the utility function is not only assumed uni-peaked but also symmetric, assuming separability and salience[12], leading to indifference curves being perfect circles, for simplifications of the spatial model analyzed later on the paper. As Munger writes, perfect circled indifference curves are present when preferences are separable and issues salient (54-59). This assumption allows for calculating the social optimum as the minimum distance among the ideal positions of voters and the actual alternative picked.

Model:

Imagine a three dimensional-space in which some points are randomly *flying* around. The space has a reference point O and three dimensions which are pulling those points towards their directions respectively. If they are pulled with an equal force from all directions (3), the points will be in the same position equally distanced from those dimensions (perpendicularity). However, if a point is pulled more by axis X, it will tend to be closer to that particular axis. Thus, the different forces exercised simultaneously, their magnitude and their direction will locate the points in different positions.

We can implement this imagination in creating a new voting method. The dimensions x, y, z will symbolize the preferences (order 1, 2, 3) respectively and the points will be regarded as the alternatives (candidates) for which individuals will have to vote. Once the voters have revealed their ordinal preference (in this case on 3 preferences) over m candidates[13], the winner has to be calculated. Each candidate will be represented by a point in our space, which coordinates will be derived from the number of votes. For instance, if candidate A has received in total:

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[11] Definition: "For any triple of alternatives {x,y,z} one alternative at least will never be ranked as third of the set by any criterion" (Arrow 44).

[12] Hinich and Munger, "Analytical Politics" (indifference curves as circles, ellipses, etc)

[13] m < number of voters n (if m=n, the method gives not result in cases that each voter picks a different combination of preference. In this case we can end up with vectors (1,1,1), and thus give no solution).

# Voters\ Preferences	**10**	**5**	**10**
1	**A**	B	B
2	B	**A**	C
3	C	C	**A**

10 votes as most preferred (order 1)

5 votes as second most preferred (order 2)

10 votes as least preferred from the allowed orders (order 3)

then A will be represented by a point (or vector from the origin) with coordinates **(10, 5, 10)**. The same procedure will be followed for all candidates. Once all the vectors are in the space, the winner will be the one which vector will be the closest to the 1-order dimension (x axis). A way to identify the closeness is through the angle that the vector creates with the aforementioned axis. Further, this angle will be the smallest for the smallest sin(a).[14]

So far, the result comes as a result of smallest angle of the vector with the first preference, but ignores its relation to the 3rd preference (least preferred). The rule has to make sure that alternative that it chooses is the winner (closest to 1-preference axis) but to avoid it being also the loser (closest to the last-preference axis). Thus, the winner should not only be the most preferred socially (closest to the 1-axis), but also it should be the least of the less preferred (the least of the loser) for the society (further away from 3-axis). The interaction between the two should give the result. Thus, the winner should have the smallest ratio of sin (alpha)/sin (gamma).[15]

3.2 Abstract View (Inspiration)

If we lived in a pure simplistic and innocent world, with perfect information, sincere revelation of individual preferences, probably this issue of voting methods would be approached in a

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[14] Coordinates are all positive (since votes >0) thus only the first spatial quadrant will be used, and a = [0, 90], for which sin(a)= [0,1] and the smaller the angle the smaller sin(a).

[15] Alfa = angle of vector of alternative with 1st preference axis;
Gama = angle of vector of alternative with last-preference axis;

different way. Assuming that there can be no pure and honest mapping of two or three alternatives from asking individuals directly, we can try to find a way to allow for some signal of voters' preference and use that information. It will never be the pure "truth", but it is an effort to get closer to it.

The axes symbolize a "source" of attracting forces. They are symmetrically distributed in the multidimensional space (in 2D the angle is 90degree, 3D, etc..). The result of the force that attracts the "point" in space depends on: its magnitude and direction. The magnitude is set by the number of votes that the "point" (candidate) receives for that particular "set" (1 preference, 2 preference, etc). However, the direction depends on the position of the axes, which are determined already in the space chosen before people have even voted, and regardless of the people who vote and their preferences and utilities. This could be seen as a shortcoming of the model. It is crucial to realize that this "problem" could not be solved. The system is set and as more alternatives are provided, which increase the # of preference orders (e.g., 3 candidates with 3 preference options are mapped in 3 orders)[16], the dimensions of the system, and thus the forces that pull these "points", the "efficiency" of the model improves[17].

This space system is necessary to be set. We are trying to determine the positions of the "points" (issues/candidates) thus finding an outcome based on certain rules (system of reference in our case the space is part of the rules). At first, the outcome is the unknown, and we take a certain set of rules as known we can find and outcome O.

If				
Outcome	(O):	unknown		
Rule	(R):	known	=>	Find O(R)

If				
Outcome	(O):	unknown		
Rule	(R):	unknown	=>	Undefined O(R)

[16] There has to be an equal number of preference options to the number of candidates. If there are more candidates then options of preference, 4 candidates and able to order them only in 1,2,3, then the outcomes will be distorted.
[17] # of voters has to be >= to # of alternatives (appendix)

Assuming R (set of rules chosen in this case) and implementing it, a certain outcome O is found. However, outcome O can be closer to the social optimum, the higher the degree of multidimensional space. The higher the number of choices people have to map, the more accurate is the mapping of intensity of utility of choices to the system.

3.3 Application (few examples)

Example 1: ***Public Choice III, pp. 149:***

Preference	Voters				
	1	1	1	1	1
1	A	A	A	B	B
2	B	B	B	C	C
3	C	C	C	A	A

Condorcet winner: A
New Method winner: A
Borda Winner: B

Calculations: Appendix 1b

Example 2: ***Perspectives on Public Choice, pp. 191[18]:***

Preference	Voters					
	30	1	29	10	10	1
1	A	A	B	B	C	C
2	B	C	A	C	A	B
3	C	B	C	A	B	A

Peter =A
Paul =B
Jack =C

[18] Young, Peyton H. "Group Choice and Individual Judgements" (section: Condorcet's Critique of Borda: Independence of Irrelevant Alternatives).

Condorcet:	Cycle (A>B>C but C>A) (> preferred)
Borda Winner:	A
New Method Winner:	B

Calculations: Appendix 1c

4. New Rule vs. Other Rules

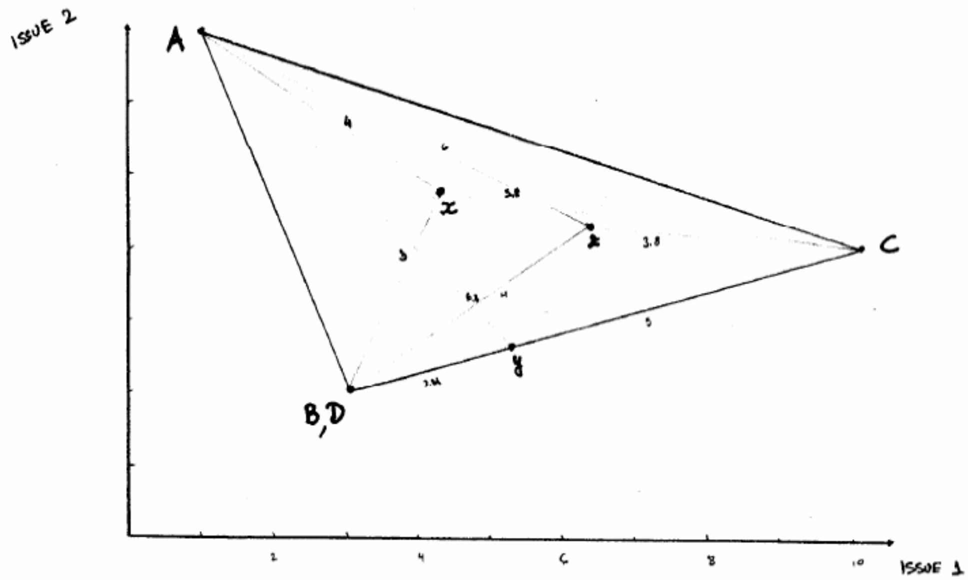
Having observed the new rule, its procedure and a few examples while having the votes, this section will make the link with the spatial model.

4.1 *Social Optima (Spatial Model)*

This example includes 4 voters choosing on 3 alternatives. We assume that we know the "truth" from the spatial model, and map into the decisions that voters will make. Using the decisions, different winners are picked dependent on the rule. The most socially beneficial outcome corresponds to Borda's winner and new rule, but not to condorcet which has a semi-cycle.

Fig 3.

FIG 3



xA = 4	zA = 6	yA = 6.3
xB = xD = 3	zB = zD = 4	yB = yD = 2.2
xC = 5.8	zC = 3.8	yC = 5

Assumptions:

1) Preferences are single peaked
2) Separable & salience ⇒ perfect circle shaped indifference curves
⇓
This allows to use distance as indication & comparison of satisfaction accross "ideal" points of individuals and accross alternatives.

A: x R z R y
(4 < 5.9 < 6.3)

B,D: y R x R z
(2.4 < 3 < 4)

C: z R y R x
(3.8 < 5 < 5.9)

⇒

# Voters / Order Preference	A (1)	B,D (2)	C (1)
1	x	y	z
2	z	x	y
3	y	z	x

(Appendix 3: calculations)

FIG 3

Condorcet Method:

x vs. y	y vs. z	z vs. x
1 3	2 2	1 3
⇓	⇓	
y R x	y I x	x R z

⇒ Cycle

⇒ Condorcet Winner is missing

Borda Method:

x Points = 2 + 1 = 3

y Points = 2·2 + 1 = 5 ⇒ Winner: y

z Points = 2 + 1 = 3

Social Optima:

Find min Σ (distance of alternative proposed to each ideal point of voters)

x : xA + 2 xB + xC = 4 + 6 + 5.9 = 15.9

(xB = xD)

y : yA + 2 yB + yC = 6.3 + 4.4 + 5 = 15.7 → Social Optimum y

z : 6 + 8 + 5.8 = 25.6

New Rule:

$\vec{x}\begin{pmatrix}1\\2\\1\end{pmatrix}$ $\vec{y}\begin{pmatrix}2\\1\\1\end{pmatrix}$

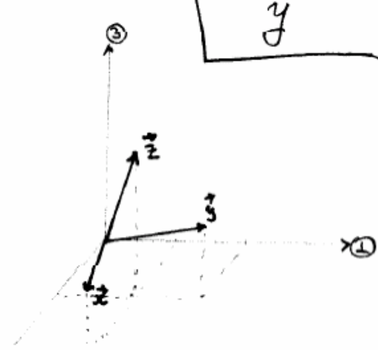
Calculations

$\sin(\gamma_x) = 0.9$

$\sin(\gamma_y) = 0.6$

$\sin(\gamma_z) = 0.9$

→ Winner y



4.2 *Spatial Model and Inferences*

Using the outcomes from spatial model (finding the point of smallest distance) =>
Find Optima => Using optima to match a rule that will pick it

r: rule
$r \in R$ (Set of Rules) R = Limited

1. Spatial Model => Social Optima => $r1, r2\ldots, \in R$ (Limited)

2. r_i => (maps into) Spatial Model => Social Optima (Infinite)

While trying to connect the spatial model with the voting methods, I used the first approach, which can give some results that can be observed and analyzed, while the second approach leads to infinite solutions. The second approach shows that for each rule r_i that is picked, there can be shown an infinite instances (examples) in the spatial model, for which r_i picks the social optima.

The problem with second approach can be best shown in following figure:

Fig4

If A => infinite optimum solutions

⇨ How to differentiate from A vs. B?

If B => infinite optimum solutions [19]

[19] Taken it futher, it inspires more questions such as: is infinity higher than infinity? If you substract something from infinity, it still remains infinity, etc… However, a line is an infinite set of points. A plane is an infinite set of points (and infinite set of lines). But place and line are not the same, even though are both a set of infinite objects

Using the first approach, in the examples above the spatial model is assumed to be known (as the truth), and then we derive what we see in reality (the votes of individuals in orders of 1, 2, 3, and so on). Using the "data", we calculate the winners through different voting rules, and then we go back to the "truth" the check which one would have a better result. This is not to say that we can prove what the best rule is, but at least we can disprove a rule being set as a model and point of reference, and observe deeply the matter.

5. Conclusion

This paper's objectives were to glance at the literature on public choice about voting methods, and depart from majority rule by focusing on other methods. Then it shows that there is still a lack of criteria for comparing and then choosing a voting method. So far, condorcet winner was used a model when a winner exists, however, it is not clear what are the reasons that this has been accepted, for even when Condorcet winner exists, it is not necessarily the social optimum. Further more Condorcet method is in itself misleading and unable to use the signal for filling in the preferences of individuals with their intensities. Having, showed that a criteria is missing, the paper focuses by finding a new approach to social aggregation decision which does not use a criteria (since it is non-existent so far in our knowledge set). Instead it strives to establish itself through the process of its creation by filling some of the gaps and drawbacks that Condorcet and Borda method had. Further applications of the model are shown to show that even though there is lack of proof so show its omni-validity, it shows that some cases in which it does do better off than other rules, thus establishing it as a good candidate in terms of outcome, but better in terms of its approach.

(even the same one: point). But in our case of the voting methods, we cannot see far enough as to make the distinction of planes and points, in order to discriminate among those infinite social optimal results that we get.

Appendix 1

Preference		
	Voters	
	10	15
1	a	b
2	b	c
3	c	a

	x	10	a =	10	s =	10	
a	y	0	h =	18.02776	sin(gamma)	0.5547	
	z	15	l =	15			1.5
			sin(alpha) =	0.83205			
	x	15	a =	18.02776	s =	18.02776	
b	y	10	h =	18.02776	sin(gamma)	1	
	z	0	l =	10			0.555
			sin(alpha) =	0.5547			
	x	0	a =	15	s =	15	
c	y	15	h =	18.02776	sin(gamma)	0.83205	
	z	10	l =	18.02776			1.202
			sin(alpha) =	1			

Appendix 1b

							sin(1-axis)/sin(3-axis)
	x	3	a =	3	s =	3	
A	y	0	h =	3.605551275	sin(gamma[20])	0.83205	0.666667
	z	2	l =	2			
			sin(alpha[21]) =	0.554700196			
	x	2	a =	3.605551275	s =	3.605551	
B	y	3	h =	3.605551275	sin(gamma)	1	0.83205
	z	0	l =	3			
			sin(alpha) =	0.832050294			
	x	0	a =	2	s =	2	
C	y	2	h =	3.605551275	sin(gamma)	0.5547	1.802776
	z	3	l =	3.605551275			
			sin(alpha) =	1.0			

[20] Gama= angle of vector with 3rd (LAST) Preference Axis
[21] Alfa= angle of vector with 1st Preference Axis

Appendix 1c

							sin(1-axis)/sin(3-axis)	
	x	31	a =	49.81967	s =	49.81967		
A	y	39	h =	51.0196	sin(gamma)	0.976481	0.81336539	
	z	11	l =	40.5216				
			sin(alpha) =	0.794236				
	x	39	a =	49.81967	s =	49.81967		
B	y	31	h =	51.0196	sin(gamma)	0.976481	0.66025659	
	z	11	l =	32.89377				
			sin(alpha) =	0.644728				
	x	11	a =	15.55635	s =	15.55635		
C	y	11	h =	61.01639	sin(gamma)	0.254954	3.8580173	
	z	59	l =	60.01666				
			sin(alpha) =	0.983615				

Appendix 2

Option 1

X	x	2	a =	2.236067977	s =	2.236068		
	y	1	h =	3	sin(gamma)	0.745356		
	z	2	l =	2.236067977				
			sin(alpha) =	0.74535599				
Y	x	1	a =	2.236067977	s =	2.236068		
	y	2	h =	3	sin(gamma)	0.745356	1.264911	
	z	2	l =	2.828427125				
			sin(alpha) =	0.942809042				
Z	x	2	a =	2.828427125	s =	2.828427		
	y	2	h =	3	sin(gamma)	0.942809	0.790569	
	z	1	l =	2.236067977				
			sin(alpha) =	0.74535599				

Option 2

X	x	2	a =	2.236067977	s =	2.236068	
	y	1	h =	3	sin(gamma)	0.745356	1
	z	2	l =	2.236067977			
			sin(alpha) =	0.745355992			
Y	x	1	a =	1.414213562	s =	1.414214	
	y	1	h =	3.31662479	sin(gamma)	0.426401	2.236068
	z	3	l =	3.16227766			
			sin(alpha) =	0.953462589			
Z	x	2	a =	3.605551275	s =	3.605551	
	y	3	h =	3.605551275	sin(gamma)	1.000	0.83205
	z	0	l =	3			
			sin(alpha) =	0.832050294			

Appendix 3

Preference

	Voters		
	1	2	1
1	X	Y	Z
2	Z	X	Y
3	Y	Z	X

						sin(1-axis)/sin(3-axis)	
	x	1	a =	2.236068	s =	2.236068	
X	y	2	h =	2.44949	sin(gamma)	0.912871	
	z	1	l =	2.236068			1
			sin(alpha) =	0.912871			
	x	2	a =	2.236068	s =	2.236068	
Y	y	1	h =	2.44949	sin(gamma)	0.912871	
	z	1	l =	1.414214			0.632456
			sin(alpha) =	0.57735			
	x	1	a =	1.414214	s =	1.414214	
Z	y	1	h =	2.44949	sin(gamma)	0.57735	
	z	2	l =	2.236068			1.581139
			sin(alpha) =	0.912871			

Appendix 4

In Fig 2, it is shown an example of a society with 5 voters (A, B, C, D, E) three alternatives to be voted on (x, y, z). Based on the assumptions[22], distances will indicate the order of preference of the alternatives for each candidate. In this particular example person E is indifferent between y and z. Thus, when E votes there could be two options:

Option 1

Preference	Voters				
	A	B	C	D	E
1	x	z	z	y	x
2	z	x	y	z	y
3	y	y	x	x	z

Option 2

Preference	Voters				
	1	1	1	1	1
1	x	z	z	y	x
2	z	x	y	z	z
3	y	y	x	x	y

Calculations showed in Appendix 2

Condorcet Winner: z
Borda Winner: z
New Rule: z

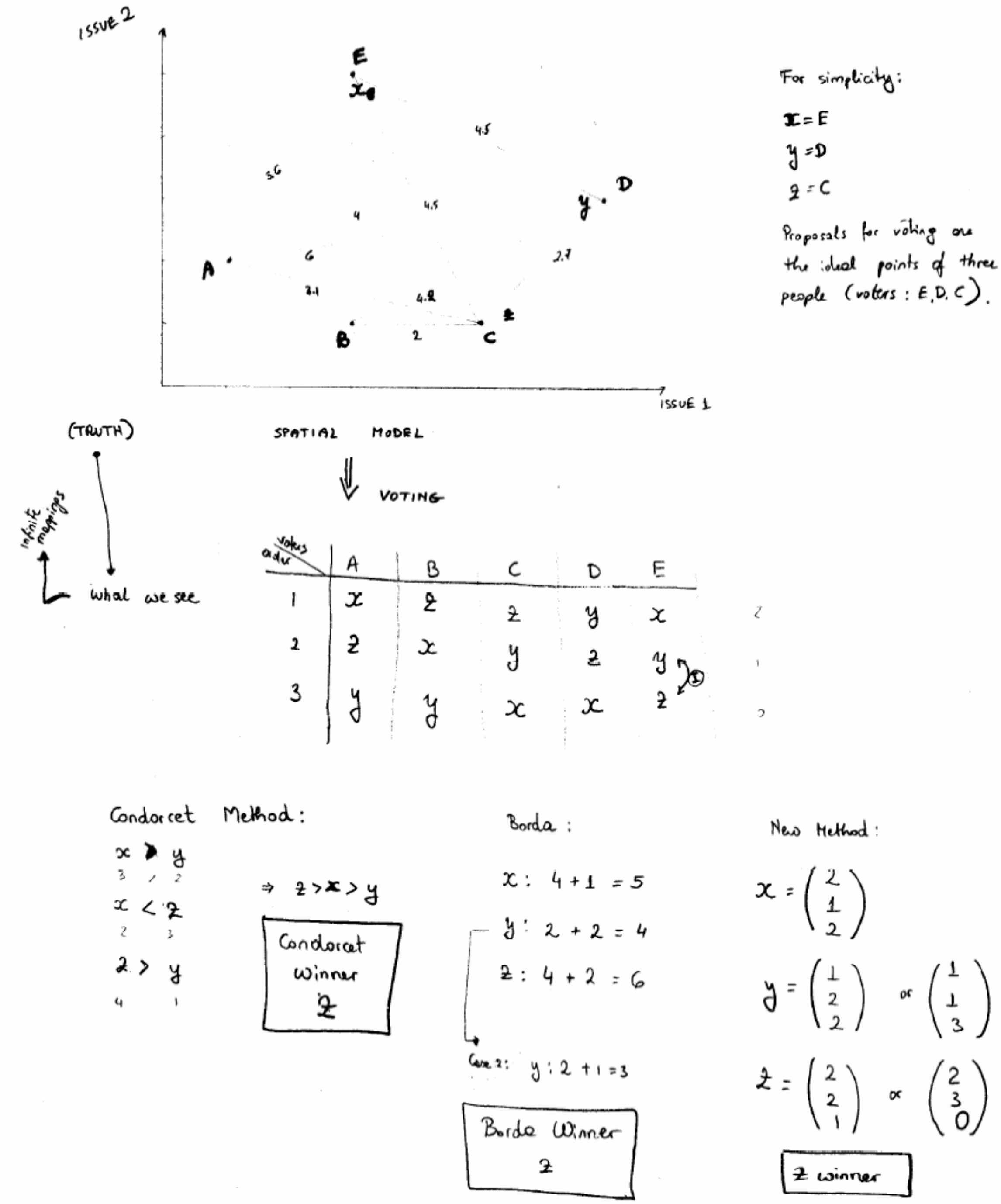
New Rule: x

The problem with the new rule is that in option 2 when the only difference consists in z being preferred more to y from E, the new rule shows that z ceases to be the winner in option 2. This is inconsistency, pushes forward for a closer look at the rule. So far, the result comes as a result of smallest angle of the vector with the first preference, but ignores its relation to the 3rd preference (least preferred). Thus, the winner should not only be the most preferred socially (closest to the 1-axis), but also it should be the least of the less preferred (the least of the loser) for the society (further away from 3-axis). The interaction between the two should give the result. Thus, the winner should have the smallest ratio of sin (alpha)/sin (gamma)[23]. Having implemented this change, all other results remain the same, but in this case, the result does become consistent, thus winner is z in both cases.

Fig 2

[22] Single Peaked preference, indifference curves are perfect circles (salience and separable).
[23] The winner should have a small (sin(alfa)) and a large (sin(gama)). The further they move to the aformentioned directions (respectively) the smaller the ratio is.

FIG 2



ISSUE 2
ISSUE 1
E
A
B
C
D
For simplicity:
x = E
y = D
z = C
Proposals for voting are the ideal points of three people (voters: E, D, C).
(TRUTH)
infinite mappings
what we see
SPATIAL MODEL
VOTING
votes
A B C D E
1 x z z y x
2 z x y z y
3 y y x x z
Condorcet Method:
x > y
x < z
z > y
⇒ z > x > y
Condorcet Winner z
Borda:
x: 4 + 1 = 5
y: 2 + 2 = 4
z: 4 + 2 = 6
y: 2 + 1 = 3
Borda Winner z
New Method:
z winner

Acknowledgements

I am grateful to Prof. Rowley for his encouragement on this paper, especially at a time I thought of giving up. And an enormous thank to some of my friends: Michael Parente, Sylvio Dochia, Nathaniel Paxson, Jennis Taylor, and Stela Pinguli that listened patiently to me and fed the light of curiosity and inspiration to this issue.

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