

# The Implications of Tax Evasion for Economic Growth

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## Abstract

An endogenous growth model is examined, where individuals optimize consumption and tax evasion over time given the tax structure and tax compliance policy of the government. Applying methods of stochastic optimal control, the individual's problem is solved and the rate of economic growth is derived. Given the consumption and evasion decision government designs its tax policy. It is shown that a welfare maximizing government adjusts its tax rate upwards to ensure a sufficient provision of a public good. In effect, tax evasion has no impact on the growth rate.

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# 1 Introduction

Tax evasion is a phenomenon present in all societies using taxes to finance government expenditures. There is a huge body of literature on the estimate of its extent.<sup>1</sup> To give an impression of its magnitude: the U.S. American tax agency IRS (Internal Revenue Service) estimates that the American tax gap amounts to 17% (\$95.3 bn.) of taxes owed for 1992 (IRS (1996)).

This paper considers the consumption and tax evasion decision as a problem of intertemporal decision making under uncertainty. The methodological approach is stochastic optimal control, first introduced into the tax evasion literature by Lin and Yang (2001). Closed-form solutions are discussed for logarithmic utility functions and the implications for economic growth are examined. Given this evasion decision government designs its tax policy. It is shown that a welfare maximizing government adjusts its tax rate upwards to ensure a sufficient provision of public goods. In effect, tax evasion has no impact on the growth rate.

The paper is organized as follows: section 2 provides an overview on the related literature. The model is developed in section 3. In section 4 scope for further research is discussed.

## 2 Tax Evasion and Economic Growth

Although growth theory is a very active field of economic research, models explicitly taking into account the possibility of tax evasion are quite rare.

Roubini and Sala-i-Martin (1995) develop the first model, where government reacts to evasion: in countries with tax evasion government increases seignorage by repressing the financial sector and increasing inflation rates. This government policy tends to reduce the amount of services that the financial sector provides to the economy, therefore the results are lower growth rates. However, neither the evasion decision is endogenous nor is the interaction

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<sup>1</sup>See e.g. Slemrod (1985), Bajada and Schneider (2003).

between government and the financial sector modelled.

Caballé and Panades (1997) study how tax compliance policy affects the rate of economic growth in a (discrete-time) overlapping generations model with identical individuals with logarithmic utility, where tax financed public goods are productive and completely rival. They find that this effect is in general ambiguous, and depends on the importance of public inputs in the production process, because (if compliance is not perfect) stricter enforcement increases compliance, leading to two effects in opposite directions. On the one hand, private saving falls with falling disposable income. On the other hand, the rise of public inputs leads to higher investment, i.e. savings, because of the increased productivity of private capital.

Sengupta (1998) also investigates an overlapping generations model with identical individuals and no population growth. In particular, he investigates tax evasion in a differential income setup, with investment and wage income subject to different tax rates and different possibilities of evasion, where individuals can misreport investment income but not wage income. He shows that alterations in the audit and penalty rates affect individual's saving decisions depending on assumptions on preferences. He assumes that only the return to savings might be underreported.

Lin and Yang (2001) adapt part of the growth model of Barro (1990) in a continuous-time endogenous growth model with tax evasion. If public goods have consumptive character only, Barro (1990) finds that the growth rate is strictly decreasing in the tax rate. Lin and Yang (2001) show that for individuals with logarithmic preferences economic growth is increasing in tax evasion, because resources are diverted from the unproductive government sector to the productive private sector.

Ho and Yang (2002) develop a political economy model, where individuals differ in their skill of earning and concealing income. By taking into account the possibility of tax evasion they obtain a result contrary to Persson and Tabellini (1994). They show that the higher the tax rate the higher the distortionary effects but also the *lower* the benefit from redistribution. As its costs are higher the equilibrium redistribution will be lower and leads to

a lower rate of economic growth. This is the first model, where economic policy is derived as a political equilibrium in a model with tax evasion.

Chen (2003) investigates an endogenous growth model in continuous-time with a Cobb-Douglas production function with public capital financed by an income tax which can be evaded. He investigates the optimum decision of saving and evasion in an environment without uncertainty assuming that individuals hold assets of enough firms so that auditing for a fraction of income is certain by the law or large numbers. Government optimizes the tax rate, auditing probability and fine rate given the consumer's evasion decision. In general these policies have ambiguous effects, but for realistic parameter constellations he finds that growth declines with tax evasion.

### **3 A Dynamic Model of Tax Evasion and its Implications for Economic Growth**

The structure of the economy, household preferences, production possibilities, and government are defined. It is shown that for a given tax policy tax evasion spurs growth, because it leads to higher savings. In the following section it is shown that if the interaction between tax payers and government is modelled this conclusion need not hold as government changes its tax policy.

#### **3.1 Assumptions**

Consider a continuum of identical individuals of mass 1 with no population growth. Each individual is identified by a utility function defined on a private good  $c$  (consumption) and a pure public consumption good  $g$ , both considered as flows.

**Assumption 1 (Log-Utility)** *Assume that the utility function  $u$  has a con-*

stant coefficient of relative risk aversion of 1:

$$u(c, g) := \ln(c^{1-\theta}g^\theta) = (1 - \theta)\ln(c) + \theta\ln(g), \quad (1)$$

where  $\theta$  denotes the relative weight attached to consumption of the public good. Assume furthermore that the von Neumann-Morgenstern axioms of expected utility hold, i.e. individuals are expected-utility maximizers. Intertemporally, assume that utility is additive separable and state and time independent.

Consumers optimize their consumption and evasion stream over an infinite planning horizon  $T := [0, \infty)$ , where time is continuous, given restrictions imposed by the production possibility set and the tax and penalty system.<sup>2</sup>

To derive closed form solutions with ongoing growth, a model of endogenous growth as in Romer (1986) and Rebelo (1991) is used assuming that the production function is of the AK-type, i.e. exhibits constant returns to scale.<sup>3</sup>

**Assumption 2 (Production Possibilities)** *Let output per capita,  $y$ , be produced by the constant returns to scale production function:*

$$y = Ak, \quad (2)$$

where  $k$  denotes the per capita capital stock and  $A$  a technological parameter. Across time, the production function is assumed to be stationary.

Taxes are levied on income, and tax revenues are used to finance the public good. For simplicity, consider a linear tax and penalty framework:

**Assumption 3 (Tax and Penalty System)** *For positive income declarations, the tax system is fully specified by a constant income tax rate:  $0 \leq \tau \leq$*

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<sup>2</sup>This modifies the first growth models with consumer optimization of Ramsey (1928), Cass (1965) and Koopmans (1965).

<sup>3</sup>According to Barro and Sala-i-Martin (1995, p.39) this production function was first used by von Neumann (1937). Possible explanations for constant returns to scale are technological progress and human capital respectively, see e.g. Romer (1986), Rebelo (1991).

1. The penalty system is described by a constant fine rate  $0 \leq \zeta$  to be paid on the amount of evaded tax.<sup>4</sup> To be more precise, the tax schedule is given by the function  $T : \mathbb{R} \rightarrow \mathbb{R}_0^+$ ,  $y_d \mapsto T(y_d)$ :

$$T(y_d) := \begin{cases} 0, & y_d < 0, \\ \tau y_d, & 0 \leq y_d, \end{cases} \quad (3)$$

where  $y_d$  denotes declared income. The fine schedule (including tax repayment) is given by the random function  $\tilde{\zeta} : \mathbb{R} \times \Omega \rightarrow \mathbb{R}_0^+$ ,  $(y_e, \omega) \mapsto \tilde{\zeta}(y_e, \omega)$ :

$$\tilde{\zeta}(y_e, \omega) := \begin{cases} (1 + \zeta) \tau y_e, & 0 \leq y_e \wedge \omega = y_e, \\ \tau y_e, & y_e < 0 \wedge \omega = y_e, \\ 0, & \omega = 0, \end{cases} \quad (4)$$

where  $(\Omega, \mathcal{F}, \mu)$  is the probability space with event set  $\Omega := \{y_e, 0\}$ ,  $\sigma$ -algebra  $\mathcal{F} := \mathcal{P}(\Omega)$  and probability measure  $\mu(y_e) := \pi$ ,  $\mu(0) := 1 - \pi$ , for some detection probability  $\pi$ ,  $0 \leq \pi \leq 1$ .

**Remark 1** As there is no loss offset and overpayment of taxes is only repaid in case of audit, this specification of the tax and penalty structure ensures that a risk averse individual will always choose  $y_d \in [0, y]$ .

With identical individuals, and firms, this setup leads to a representative consumer-producer. Per capita output accrues to the consumer as income. At each point in time  $t$ , given income  $y(t)$ , the individual has to decide simultaneously how much income to declare,  $0 \leq y_d(t) \leq y(t)$ , and respectively how much to evade,  $y_e(t)$ , where  $y_d(t) + y_e(t) = y(t)$  always holds. Denote the share of evaded income at time  $t$  by  $e(t) := \frac{y_e(t)}{y(t)}$ . Tax evasion is therefore possible by underdeclaration of income. This approach is common in the tax evasion literature since the seminal paper by Allingham and Sandmo (1972).

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<sup>4</sup>This specification of the fine follows Yitzhaki (1974) and is the more realistic alternative to the one proposed first by Allingham and Sandmo (1972), where the penalty is imposed on evaded income.

Government does not know initial capital per capita  $k_0$  (therefore cannot infer the true income stream), investigates a fraction  $\pi$  of all individuals and detects evasion if and only if the tax cheater is subjected to a *random* audit. An individual takes the government's actions, i.e.  $\tau$ ,  $\pi$  and  $\zeta$  as given.<sup>5</sup> Individuals are assumed to be fully informed about the penalty structure and audit rates. Thus, from an individual's point of view, as auditing is random, disposable income after taxes and fines is a random variable:

$$\tilde{y}(t, \omega) = \begin{cases} y(t, y_e) = (1 - \tau)y(t) - \zeta\tau y_e(t), \\ y(t, 0) = y(t) - \tau y_d(t) = (1 - \tau)y(t) + \tau y_e(t). \end{cases} \quad (5)$$

At each point in time the individual trades off a lower tax payment by evading income with the risk of a fine in case of audit.

Denote by  $\bar{r} := 1 - \pi - \zeta\pi$  the expected return of one unit of evaded taxes. Then a well-known result due to Arrow (1970) can be applied:

**Remark 2** *In a static setup a risk averse individual will take risk, i.e. in the present context evade taxes, if and only if the expected return is positive, i.e.  $0 < \bar{r}$ .*

Tax evasion implies that the effective tax rate differs from the statutory rate.

**Lemma 1 (Expected Tax Rate)** *For given income  $y(t)$ , statutory tax rate  $\tau$  and enforcement parameters  $\pi$  and  $\zeta$ , and (endogenously determined) share of evaded income  $e(t)$ , the expected tax rate is:*

$$\tau_e(t) = (1 - \bar{r}e(t))\tau. \quad (6)$$

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<sup>5</sup>The possibility that the probability of audit may depend on the results of prior audits is excluded.

**Proof:** Fix  $t$ ,  $y(t)$ ,  $\tau$ ,  $\pi$ ,  $\zeta$ ,  $e(t)$  and remember  $\bar{r} := 1 - \pi - \pi\zeta$ . Then the expected tax and penalty payment is:

$$\begin{aligned}\mathbb{E} \left[ T(y_d(t)) + \tilde{S}(y_e(t), \omega) \right] &= (1 - \pi) \tau (1 - e(t)) y(t) + \pi (\tau y(t) + \zeta \tau e(t) y(t)) \\ &= \tau (1 - \bar{r} e(t)) y(t).\end{aligned}\quad (7)$$

Therefore income is taxed with the expected rate  $\tau(1 - \bar{r}e(t)) =: \tau_e(t)$ . ■

As a continuum of individuals is assumed, by the law of large numbers, this expected rate is equal to the effective rate of taxation in the whole economy.

**Assumption 4 (Balanced Government Budget)** *Contemporaneous tax revenues and penalties are used to finance the public good:*

$$g(t) = \tau_e(t)y(t), \quad \forall t \in T. \quad (8)$$

### 3.2 Individual Optimum

The first step of the analysis is the optimal allocation of capital across time in an uncertain environment from an individual's perspective. As disposable income is random, consumption and saving will depend on whether an individual was audited or not. Saving,  $s$ , augments the capital stock, and is therefore the driving force of economic growth. Taking into account uncertainty of audit the capital stock per capita in discrete-time follows a random difference equation:

$$\tilde{k}(t_{j+1}) = (1 - \delta) k(t_j) + \tilde{s}(t_j, \omega), \quad (9)$$

where

$$\tilde{s}(t_j, \omega) = \begin{cases} (1 - \tau + \bar{r}\tau e)y - c, & \omega = 0, \\ (1 - \tau - \zeta\tau e)y - c, & \omega = y_e, \end{cases} \quad (10)$$

and  $0 \leq \delta$  is a constant depreciation rate.

For continuous time, the capital stock follows a linear stochastic Itô differ-

ential equation with the same first and second moments as in discrete time:<sup>6</sup>

$$dk(t) = [(1 - \tau(t) + \bar{r}\tau e(t))y(t) - \delta k(t) - c(t)] dt + (\sigma e(t)y(t))^2 dz \quad (11)$$

where  $\sigma^2 := \pi(1 - \pi)[(1 + \zeta)\tau]^2$  and  $\{z(t), 0 \leq t \leq T\}$  is assumed to follow a one-dimensional standard Wiener process on the probability space  $(\Omega, \mathcal{F}, \mu)$ .

For the  $Ak$  production function the stochastic process for the capital stock (11) is given by:

$$dk(t) = \left[ \left( 1 - \tau - \frac{\delta}{A} + \bar{r}\tau e(t) \right) Ak(t) - c(t) \right] dt + (\sigma e(t)Ak(t))^2 dz(t). \quad (12)$$

Thus, the state of the economy is described by a stochastic differential equation involving variables, which can be adjusted by the individual so that his objective of maximal (expected discounted) utility is achieved.  $(1 - \tau(t) + \bar{r}\tau e(t))y(t) - \delta k(t) - c(t)$  denotes the average drift of the capital process, which is perturbed by a noisy term depending on whether an audit has occurred or not.

At any instant  $t$  the individual chooses  $\psi(t) := (c(t), e(t))$  in order to control (the moments of) the process. To ensure that the individual's objective functional is well-defined  $\psi(t)$  must at least be measurable. In the following, only Markov controls are considered.<sup>7</sup>

**Assumption 5 (Markov controls)** *Assume that the individual chooses a plan among feasible Markov controls, i.e. of controls that only depend on the current state.*

Consider now the decision of the taxpayer of how much to consume and save respectively and how many taxes to evade.

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<sup>6</sup>See Dixit and Pindyck (1994) for an introduction to stochastic processes and its applications.

<sup>7</sup>If the value function of problem (13) below is maximized using some other control, there is also a Markov control that does not perform worse under certain mild extra conditions (Oksendal (1998, Theorem 11.2.3, p.232)).

The individual's problem takes the form:

$$\max_{\psi(t)} \mathbb{E}_0 \left\{ \int_0^{\infty} \ln(c(t)) e^{-\rho t} dt \right\}, \quad (13)$$

s.t. (12)

$$0 \leq c(t) \leq (1 - \tau)Ak(t), \quad 0 \leq k(t), \quad k(0) = k_0, \quad (14)$$

$$0 \leq e(t) \leq 1, \quad (15)$$

as he takes the level of the public good as given. The Hamilton-Jacobi-Bellman (HJB) Equation for problem (13) is:

$$\begin{aligned} \rho I(k) = & \max_{\psi(t)} \left\{ \ln(c(t)) + I'(k) ([1 - \tau + \bar{r}\tau e(t)] y(t) - c(t)) \right. \\ & \left. + \frac{1}{2} I''(k) (\sigma e(t) y(t))^2 \right\}, \end{aligned} \quad (16)$$

where  $I(k) := I(k(0)) := \max_{c(t), e(t)} \mathbb{E}_0 \left\{ \int_0^{\infty} \ln(c(t)) e^{-\rho t} dt \right\}$ , s.t. (12),(14),(15), denotes the value function and the right hand side yields the necessary conditions for an interior optimum:

$$[c(t)]^{-1} - I'(k) = 0, \quad (17)$$

$$I'(k) \bar{r} \tau y(t) + I''(k) (\sigma y(t))^2 e(t) = 0, \quad (18)$$

By equation (17), in the optimum the marginal utility equals the marginal value of capital.

It follows:

$$c(t) = [I'(k)]^{-1}, \quad (19)$$

$$e(t) = -\frac{I'(k) \bar{r} \tau}{I''(k) \sigma^2 y(t)}. \quad (20)$$

Plugging into (16):

$$\begin{aligned}
\rho I(k) &= \ln ([I'(k)]^{-1}) + I'(k) \left( 1 - \tau - \frac{I'(k)(\bar{r}\tau)^2}{I''(k)\sigma^2 y(t)} y(t) - [I'(k)]^{-1} \right) \\
&\quad + \frac{1}{2} I''(k) \left( \sigma \frac{I'(k)\bar{r}\tau}{I''(k)\sigma^2 y(t)} y(t) \right)^2 \\
&= -\ln [I'(k)] - 1 + I'(k) (1 - \tau) Ak - \frac{[I'(k)]^2 (\bar{r}\tau)^2}{2I''(k)\sigma^2} \tag{21}
\end{aligned}$$

an implicit nonlinear differential equation of order 2, of which the solution can be written as:  $I(k) := \frac{\ln(k)+C}{\rho}$  (and  $I'(k) = \frac{1}{\rho k}$ ,  $I''(k) = -\frac{1}{\rho k^2}$ ), where  $C$  is a constant to be determined as follows:

$$\begin{aligned}
\ln(k) + C &= \ln(\rho k) - 1 + \frac{(1 - \tau) A}{\rho} - \frac{(\bar{r}\tau)^2}{2\sigma^2} Ak \\
&\Leftrightarrow C = \ln(\rho) - 1 + \frac{(1 - \tau) A}{\rho} + \frac{(\bar{r}\tau)^2}{2\rho\sigma^2} \tag{22}
\end{aligned}$$

Therefore, the value function is:

$$I(k) = \frac{1}{\rho} \left[ \ln(\rho k) - 1 + \frac{(1 - \tau) A}{\rho} + \frac{(\bar{r}\tau)^2}{2\rho\sigma^2} \right], \tag{23}$$

with derivative

$$I'(k) = \frac{1}{\rho k}. \tag{24}$$

Now the optimal plans for  $c(t)$  and  $e(t)$  can be determined.

**Proposition 1 (Individual Optimum for Log Utility)** *The optimal consumption and evasion plan are given by:*

$$c(t) = \rho k(t), \tag{25}$$

$$e(t) = \frac{\bar{r}\tau}{\sigma^2 A} = \frac{\bar{r}}{\pi(1 - \pi)(1 + \zeta)^2 \tau A}. \tag{26}$$

*Therefore, at each point in time it is optimal to consume a constant share of the current capital stock per capita. Also, evaded income is a constant*

share of the current capital stock per capita (remember:  $e(t) = \frac{y_e(t)}{y(t)}$ , thus  $y_e(t) = \frac{\bar{r}\tau A}{\sigma^2 A} k(t)$ ).

To ensure that the plans obey the restrictions of (14),(15) the following inequalities are assumed to hold:

$$0 \leq \rho \leq A, \quad (27)$$

$$0 \leq \bar{r} \leq \pi(1 - \pi)(1 + \zeta)^2 \tau A, \quad (28)$$

where  $0 \leq \bar{r}$  is satisfied if some tax evasion is profitable.

The comparative dynamics are straightforward:

**Proposition 2 (Comparative Dynamics)** *For the optimal consumption plan the following comparative dynamics hold (time arguments are left out for simplicity):*

$$\frac{\partial c}{\partial \rho} = k > 0, \quad (29)$$

$$\frac{\partial c}{\partial \tau} = 0. \quad (30)$$

*The effect of an increase in the tax rate is null, income and substitution effects exactly cancel out for logarithmic utility.*

*The comparative dynamics for the evasion rate are the following:*

$$\frac{\partial e}{\partial \tau} = -\frac{\bar{r}}{\pi(1 - \pi)(1 + \xi)^2 \tau^2 A} < 0, \quad (31)$$

$$\frac{\partial e}{\partial \bar{r}} = \frac{1}{\pi(1 - \pi)(1 + \xi)^2 \tau A} > 0. \quad (32)$$

In the whole economy because of the law of large numbers:

**Proposition 3 (Growth Rate)** *The (expected) growth rate of the economy is given by:*

$$\bar{\gamma}_k = (1 - \tau)A + \frac{\bar{r}^2}{\pi(1 - \pi)(1 + \zeta)^2} - \rho, \quad (33)$$

and therefore, for fixed tax and enforcement policy, is higher than in an economy without tax evasion if and only if tax evasion is profitable (and therefore optimal for individuals).

**Proof:** Given the problem of optimal saving (where no evasion of taxes is possible):

$$\begin{aligned} \max_{c(t)} U &:= \int_0^{\infty} \ln(c(t))e^{-\rho t} dt & (34) \\ \text{s.t. } \dot{k}(t) &= (1 - \tau)Ak(t) - c(t), \\ 0 &\leq c(t) \leq (1 - \tau)Ak(t), \quad 0 \leq k(t), \quad k(0) = k_0 \end{aligned}$$

in terms of the present value Hamiltonian  $\mathcal{H} := \ln(c(t))e^{-\rho t} - \lambda(t)[(1 - \tau)Ak(t) - c(t)]$  necessary conditions for an interior optimum are:

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{e^{-\rho t}}{c} + \lambda(t) = 0, \quad (35)$$

$$\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial k} = -\lambda(t)(1 - \tau)A, \quad (36)$$

a system of first order differential equations and the transversality condition. The solution of the second equation is given by:

$$\lambda(t) = C \exp(-(1 - \tau)At) \quad (37)$$

for some constant  $C$ . Inserting this into equation (35) one obtains:

$$\begin{aligned} \frac{1}{c(t)} &= C \exp([\rho - (1 - \tau)A]t) \\ \Leftrightarrow c(t) &= \frac{1}{C} \exp([(1 - \tau)A - \rho]t). \end{aligned} \quad (38)$$

Therefore the growth rate of consumption is  $(1 - \tau)A - \rho$ . The capital stock grows at the same rate:  $\gamma_k(t) = (1 - \tau)A - \rho$ , which is always smaller than  $\bar{\gamma}_k$ , if and only if  $\bar{r} > 0$ . ■

Note, that crucial for this result is the assumption that the government does

not change its tax policy and that  $\bar{\gamma}$  is strictly decreasing in the tax rate. For any given tax rate  $\tau$  in an economy without tax evasion there is a tax rate  $\tau' > \tau$  such that the growth rates are equal:  $\bar{\gamma}_k(\tau') = \gamma_k(\tau)$ , viz.  $\tau' = \tau + \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2 A}$ .<sup>8</sup>

### 3.3 Welfare Maximizing Government

Assume that the government maximizes welfare  $W$  in the economy without tax evasion. As all individuals are equal, welfare maximization is equivalent to maximization of utility of a representative individual, i.e. government solves:

$$\max_{\tau} W(\tau) := \int_0^{\infty} u(c, g) e^{-\rho t} dt, \quad (39)$$

taking into account its effect on the individual's consumption decision given by (27) (and the growth rate). By Assumption 4 all tax revenues are expensed for a public good, therefore  $g(t) = \tau y(t) = \tau A k(t)$ . By choosing the optimal tax rate, government trades off a higher tax revenue from given income with a lower growth rate because of reduced savings. For the logarithmic utility function, in particular, government's maximization problem is:

$$\max_{\tau} W = \int_0^{\infty} [(1 - \theta) \ln(\rho k(t)) + \theta \ln(\tau A k(t))] e^{-\rho t} dt, \quad (40)$$

where capital per capita follows the differential equation

$$\dot{k} = [(1 - \tau)A - \rho]k, \quad (41)$$

with initial condition

$$k(0) = k_0. \quad (42)$$

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<sup>8</sup>To be feasible, i.e.  $\tau' \leq 1$ ,  $\tau$  may not be too large, in particular  $\tau \leq 1 - \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2}$  has to hold.

At time  $t \in T$  capital per capita is therefore given by:

$$k(t) = k_0 e^{[(1-\tau)A-\rho]t}. \quad (43)$$

If one considers only the possibility to set a tax rate once and for all the optimization problem becomes:

$$\begin{aligned} \max_{\tau} W &= \int_0^{\infty} [(1-\theta) \ln(\rho k_0 e^{\gamma t}) + \theta \ln(\tau A k_0 e^{\gamma t})] e^{-\rho t} dt & (44) \\ \Leftrightarrow \max_{\tau} W &= \int_0^{\infty} [\gamma t + (1-\theta) \ln(\rho k_0) + \theta \ln(\tau A k_0)] e^{-\rho t} dt \\ \Leftrightarrow \max_{\tau} W &= \gamma \int_0^{\infty} t e^{-\rho t} dt + \theta \ln(\tau A k_0) \\ \Leftrightarrow \max_{\tau} W &= \frac{\gamma}{\rho} + \theta \ln(\tau A k_0), & (45) \end{aligned}$$

with first order condition:

$$\frac{\theta}{\tau} = \frac{A}{\rho} \quad (46)$$

so in the optimum:

$$\tau^* = \frac{\rho \theta}{A}. \quad (47)$$

The comparative statics are straightforward: If the weight attached to consumption of the public good increases, the tax rate should be higher as consumption of the public good is important; if the rate of time preference increases (the individual gets less patient), the growth rate is less important and taxation should be higher to supply a larger quantity of the public good immediately; if, however, the return of capital increases it allows government to lower the tax rate to obtain a sufficient level of public goods today and even achieve a higher growth rate.

Consider now, again, the economy with tax evasion. If government sets a tax

rate of

$$\tau' = \tau^* + \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2 A}, \quad (48)$$

it can induce the same growth rate.

But  $\tau'$  might not be optimal, because also current income is taxed at this higher rate. The solution of the following optimization problem shows that  $\tau'$  is indeed optimal.

$$\begin{aligned} \max_{\tau} \bar{W} &:= \int_0^{\infty} [(1-\theta) \ln(\rho k(t)) + \theta \ln(\tau(1-\bar{r}e)Ak(t))] e^{-\rho t} dt & (49) \\ \text{s.t. } & 0 \leq \tau \leq 1, \dot{k}(t) = [(1-\tau + \bar{r}\tau e)A - c]k(t), \forall t \in T, k(0) = k_0, \end{aligned}$$

where  $c = \rho k$ ,  $e = \frac{\bar{r}}{\pi(1-\pi)(1+\zeta)^2\tau A}$ . Thus

$$\max_{\tau} \bar{W} = \int_0^{\infty} [(1-\theta) \ln(\rho k_0 e^{[(1-\tau+\bar{r}\tau e)A-\rho]t})] e^{-\rho t} dt \quad (50)$$

$$+ \int_0^{\infty} \theta \ln\left(\tau A - \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2} k_0 e^{[(1-\tau+\bar{r}\tau e)A-c]t}\right) e^{-\rho t} dt \quad (51)$$

$$\Leftrightarrow \max_{\tau} \bar{W} = \int_0^{\infty} [(1-\theta) \ln(\rho k_0) + [(1-\tau+\bar{r}\tau e)A-\rho]t] e^{-\rho t} dt$$

$$+ \int_0^{\infty} \theta \ln\left(\tau A - \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2} k_0\right) e^{-\rho t} dt \quad (52)$$

$$+ \int_0^{\infty} [(1-\tau+\bar{r}\tau e)A-\rho]t e^{-\rho t} dt \quad (53)$$

$$\Leftrightarrow \max_{\tau} \bar{W} = \frac{1}{\rho} \left[ \left(1 - \tau + \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2}\right) A - \rho \right]$$

$$+ \int_0^{\infty} (1-\theta) \ln(\rho k_0) e^{-\rho t} dt \quad (54)$$

$$+ \int_0^{\infty} \theta \ln\left(\tau A - \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2} k_0\right) e^{-\rho t} dt \quad (55)$$

$$\Leftrightarrow \max_{\tau} \bar{W} = \frac{1}{\rho} \left[ \left(1 - \tau + \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2}\right) A - \rho \right] \quad (56)$$

$$+ \theta \ln\left(\tau A - \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2} k_0\right) \quad (57)$$

with first-order conditions:

$$-\frac{A}{\rho} + \frac{\theta}{\left(\tau - \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2 A}\right)} = 0 \quad (58)$$

$$\Leftrightarrow \tau = \frac{\theta \rho}{A} + \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2 A} > \tau \quad (59)$$

and growth rate

$$\begin{aligned}
\bar{\gamma}_k &= (1 - \tau + \bar{r}\tau e)A - \rho \\
&= \left( 1 - \frac{\theta\rho}{A} - \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2 A} + \frac{\bar{r}^2}{\pi(1-\pi)(1+\zeta)^2 A} \right) A - \rho \\
&= A - \rho(1 + \theta) = \gamma_k.
\end{aligned} \tag{60}$$

## 4 Conclusions

An endogenous growth model with the possibility of tax evasion was developed. It was shown that evasion is beneficial for growth. However, if government adjusts its tax policy the growth rate is unchanged by the possibility of evasion.

Further research should be undertaken to decide whether this result is robust to a generalization of the utility function (for more general constant relative risk aversion functions) and the introduction of evasion costs. Additionally, government might find it optimal to set a tax path instead of a tax rate once and for all. An empirical estimation of the model might be interesting.

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